Inter-Firm Network Growth over Firm Life Cycle and Its Macroeconomic Implications

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Inter-Firm Network Growth over Firm Life Cycle and Its Macroeconomic Implications*

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Abstract

This paper investigates the network growth pattern of young firms and its macroeconomic implications. Using panel data of firm-to-firm trade and financial surveys in Japan, we show that young firms face delays in acquiring new partners, even after accounting for typical age-dependent growth factors. To explain this pattern, we develop a general equilibrium model incorporating dynamic network formation of heterogeneous firms, distorted by an age-specific networking wedge. Using the calibrated model, we identify the macroeconomic significance of the wedge. Elimination of the wedge improves welfare by 2.4% by accelerating the network formation of young firms and restructuring supply chains in the economy.

Keywords: Firm Dynamics, Production Networks, Misallocation

JEL Classification: D21, D24, D57, D85, E22, E23, E61

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1 Introduction

Young firms are pivotal drivers of macroeconomic dynamism and productivity growth. Yet, their integration into supply chain networks—a cornerstone of modern economies—requires a gradual accumulation of business partners after market entry. Unlike established firms, young firms typically lack pre-existing connections, making the post-entry networking process crucial to their growth. These dynamics not only shape the growth trajectory of individual firms but also have broader aggregate implications because the structure of macro-level production networks ultimately arises from the cumulative networking decisions of individual firms.

Despite the ubiquity and macroeconomic importance of this mechanism, the literature has not fully explored its micro-level empirical foundations nor their corresponding macro-level implications. There are two primary reasons for this. The first reason is the limited availability of comprehensive data. Although the availability of network data has been improving in recent years, datasets that span a sufficient time horizon to estimate the life cycle of firms are still limited. Furthermore, to establish reliable stylized facts, network data must be combined with large-scale panel data to account for several factors commonly discussed in the firm dynamics literature. The second reason involves the inherent complexity of modeling. To analyze macroeconomic implications, it is necessary to model the dynamic decision-making processes underlying firms' network formation. However, production networks firms face are inherently infinite-dimensional, as a firm's business is influenced not only by its partners but also by the partners of those partners, and so forth. This leads to a curse of dimensionality, where the number of state variables in firms' decision-making becomes unmanageable.

This paper is the first to shed light on the network growth patterns of young firms and their macroeconomic implications through the use of unique datasets and an innovative modeling approach. First, leveraging unique panel data on firm-to-firm trade and financial statements, we empirically show that young firms exhibit several network growth patterns, which persist despite accounting for typical age-dependent growth factors. Second, building on these findings, we construct a novel general equilibrium model that incorporates dynamic network formation decisions of heterogeneous firms while overcoming the curse of dimensionality associated with network complexity. Finally, by calibrating the model to the empirical findings, we quantitatively identify misallocation stemming from young firms' networking decisions and room for welfare improvement through policies targeted at young firms.

We first present key findings regarding the network growth patterns over the firm life cycle, using proprietary datasets. We combine yearly transaction network data with firm survey panel data to build a comprehensive firm-year dataset covering nearly all Japanese firms with more than five employees from 2007 to 2022. Running fixed-effect regressions on the panel, we derive two main findings. First, while the number of suppliers and customers eventually converges to a mature level over the life cycle, young firms can only increase these connections slowly, even after controlling for typical age-dependent growth factors. This implies there are sources that prevent young firms from acquiring a sufficient number of partners, and the sources are different from the typical age-dependent variables like productivity or financial slackness. Second, once a supplier-buyer relationship is established, the churn rate remains stable throughout the firm's life cycle. This suggests that young firms' difficulty in achieving a mature network size stems not from post-matching mechanisms, such as high churn rates, but rather from pre-matching mechanisms related to matching with potential partners.

Next, we develop a general equilibrium (GE) model that incorporates dynamic network formation decisions of heterogeneous firms. Our model is a dynamic extension of Arkolakis et al. (2023), where firms search for suppliers and buyers using network advertisement. In our model, firms make dynamic advertising decisions to accumulate suppliers and buyers after market entry. We suppose two sources that prevent young firms from acquiring a sufficient number of partners immediately after entry. The first source comes from the convexity of advertisement cost. In order to advertise all at once when firms enter, they have to pay a higher marginal cost. This incentivizes firms to smooth their advertising over their life cycle, leading to a gradual expansion of their networks. For the second source, we assume a hypothetical age-specific networking wedge that creates a gap between perceived and actual advertisement costs, similar to the framework of Hsieh and Klenow (2009). If this wedge makes the perceived cost of young firms' advertising higher (as indicated by our calibration), then this hampers partner acquisition of the young firms. The dynamic network formation decisions introduce complexity that could lead to a curse of dimensionality in the value function. We overcome this challenge by adopting a truncation approach following Le Grand and Ragot (2022). The approach enables us to define the value function efficiently and solve it with a straightforward method.

Finally, we calibrate the model to the estimated network growth patterns and derive several macroeconomic implications. First, our model yields a good match to the empirical findings on network growth pattern over firm life-cycle. The R^2 for the network growth pattern between the data and the calibrated model is 0.99, indicating that 99% of the empirically estimated pattern is accounted for by our model. Next, we quantitatively analyze the welfare impact of distorted dynamic network formation. By comparing the baseline economy with a wedge-free economy (where the age-specific networking wedge is eliminated), we find a 2.4% welfare improvement in the wedge-free environment. Furthermore, we analytically decompose these welfare impacts. The results suggest that network-related channels—such as economy-wide production network restructuring and its spillovers—are more impactful than traditionally studied factors such as entry and wage adjustments. Lastly, we conduct policy simulations using the calibrated model. We first consider two policies: one that promotes supplier accumulation and another that promotes buyer accumulation for young firms. The two policies show an interesting asymmetric effect in that supporting supplier accumulation is more effective than supporting buyer accumulation. This is because supplier acquisition by young firms increases their exposure to the entire supply chain and their productivity by improved access to inputs or resources of the new suppliers at the same time, whereas buyer acquisition of young firms increases only their exposure to the entire supply chain without entailing productivity gain. Next, we consider a policy that promotes entry. Our experiment indicates that this policy becomes more effective under the existing distorted dynamic decisions of young firms than in a wedge-free economy. With the distortion, young firms have inefficiently low level of expected profits due to the lack of their partners, and this discourages entry. Hence, entry subsidies that make up for the discouragement become more effective in the environment distorted by the networking wedge.

Related Literature

This paper is linked to different strands of literature. First, this paper belongs to the extensive literature on firm dynamics (e.g., Hopenhayn, 1992; Foster et al., 2001; Klette and Kortum, 2004; Luttmer, 2007; Foster et al., 2016; Sterk et al., 2021). Our research particularly contributes to recent studies on customer accumulation (Alfaro-Urena et al., 2022; Eslava et al., 2023; Ignaszak and Sedlácek, 2023). Leveraging detailed panel data on firm-to-firm trades and financial surveys, we extend the previous studies by providing robust empirical findings on both supplier and customer accumulation in a unified framework.[1] Moreover, we demonstrate that these supplier- and customer-side dynamics have asymmetric macroeconomic effects from a network perspective. Our policy simulations indicate that promoting supplier accumulation in young firms can be twice as effective as those targeting customer accumulation.

Second, this paper is related to the recently growing literature on endogenous production network formation in general equilibrium (e.g., Oberfield, 2018; Eaton et al., 2022; Acemoglu and Azar, 2020; Lim, 2018; Bernard et al., 2022; Tintelnot et al., 2018; Dhyne et al., 2021; Kopytov et al., 2022; Elliott et al., 2022; Arkolakis et al., 2023; Miyauchi, 2024). Our model extends the literature by representing the dynamic decision-making processes behind firms' networking behavior. We address the curse of dimensionality associated with the complexity of the production networks considered by firms in their decision-making process, using a truncation approach adopted by Le Grand and Ragot (2022)[2] in HANK literature. Given that production networks exhibit rich dynamic patterns, as evidenced by empirical data from various industries and regions (Huneeus, 2020; Carvalho et al., 2021; Liu and Tsyvinski, 2024), our model has potential applications in a wide range of research in this field by capturing the firm-level decisions that yield the network dynamism.

Lastly, this paper is related to the extensive literature on resource misallocation.[3] Extending recent analyses of misallocation and its spillover effects in fixed production networks (Liu, 2019; Osotimehin and Popov, 2023), we endogenize network formation and examine how distorted networking decisions by young firms contribute to misallocation.[4] This extension allows us to analyze the wedge that distorts the networking pattern of individual firms. Our findings illustrate the broader macroeconomic implications of the wedge, since it not only affects networking behavior of individual firms, but also shapes the structure of production networks in the entire economy, through which distortions propagate.

Outline

The remainder of this paper proceeds as follows. Section 2 explains the data and empirical findings about networking behavior of young firms. Section 3 presents a model consistent with the findings. Section 4 calibrates the model parameters and explores the aggregate implications. Section 5 concludes.

[1] A contemporaneous study Aekka and Khanna (2024) explores partner acquisition patterns across age groups using cross-sectional firm-to-firm trade records from India in 2018, whereas our research utilizes long-term panel data to identify age-specific networking patterns that are not driven by typical age-related factors discussed in the literature.

[2] In the paper, they solve the planner's problem in the environment where both aggregate shocks and idiosyncratic shocks exist and the wealth distribution of household is a natural state variable for the planner. Instead of having wealth distribution of household, which is infinite-dimensional object, they adopt the history of aggregate shocks as a state variable with truncation, which is a finite-dimensional object up to truncated length.

[3] For an extensive survey of various channels, check Hopenhayn (2014) and Restuccia and Rogerson (2017).

[4] Contemporaneous studies Boehm et al. (2024) and Koike-Mori and Okumura (2024) analyze similar mechanisms through a model incorporating endogenous network formation among firms within the framework of a balanced growth path. By focusing on steady state analysis in our research, we provide a more detailed representation of firms' networking decision, enabling replication of empirically observed patterns in a calibrated model.

2 Empirical Findings

2.1 Data

We use a primary dataset obtained from Tokyo Shoko Research (TSR), a major credit reporting company in Japan.[5] TSR collects comprehensive information of firms through personal interviews or phone surveys, supplemented by public resources such as financial statements, corporate registrations, and public relations documents. The information is updated at an annual frequency, and the datasets compiled between 2007 and 2022 are used.

In addition to standard financial information of firms including the number of employees and sales, four-digit industry classification, year of establishment, and address, the TSR data have unique information about transaction partners. Each firm reports its suppliers, customers, and major shareholders up to 24 firms, respectively. Despite this truncation threshold, we do not regard the censoring as restrictive for two reasons. First, the share of firms that report exactly 24 suppliers or customers and can be potentially regarded as being bounded by the truncation is fewer than 0.1%. Second, we merge self-reported data and other-reported data following Bernard et al. (2019) and Carvalho et al. (2021). Specifically, we combine the list of suppliers (buyers) reported by the firm itself and the reports of others that report the firm as their buyer (supplier). Hence, even if the self-reported number of partner links is truncated, as long as the partners report the truncated links, the truncation issue is insignificant.

We build a firm-by-year panel including both financial information and network information for our analysis by combining the two sources. We use this panel data to examine the relationship between supplier/customer dynamics and firm age, and to calibrate a general equilibrium model using firm-level data. Descriptive statistics are shown in Appendix A.2.

2.2 Network Measure

To capture network dynamics by firm age, we first define several measures that summarize networking behavior of firms. For firm i and year t, using set notation $S_{i,t} :=$ set of suppliers$_{i,t}$, we define four scalar measures as follows.[6]

$$LogDegree^S_{i,t} := \log(\#S_{i,t}) \tag{2.1}$$

$$GrossGrowth^S_{i,t} := \frac{\#(S_{i,t} \backslash S_{i,t-1})}{\#S_{i,t-1}} \tag{2.2}$$

$$GrossDepreciation^S_{i,t} := \frac{\#(S_{i,t-1} \backslash S_{i,t})}{\#S_{i,t-1}} \tag{2.3}$$

$$NetGrowth^S_{i,t} := \frac{\#S_{i,t} - \#S_{i,t-1}}{\#S_{i,t-1}} \tag{2.4}$$

$$= \frac{\#(S_{i,t} \backslash S_{i,t-1}) - \#(S_{i,t-1} \backslash S_{i,t})}{\#S_{i,t-1}} \tag{2.5}$$

$$= GrossGrowth^S_{i,t} - GrossDepreciation^S_{i,t} \tag{2.6}$$

[5] This dataset is used in previous studies, including Fujii et al. (2017), Bernard et al. (2019), Carvalho et al. (2021) and Miyauchi (2024). Check Carvalho and Tahbaz-Salehi (2019) and Bacilieri et al. (2023) for comparison with production network data in other countries.

[6] # denotes cardinality of a set and \ denotes difference of sets.

Supplier log degree measure ($LogDegree^S_{i,t}$) indicates the number of suppliers firm i has at year t. Gross supplier growth measure ($GrossGrowth^S_{i,t}$) captures the ratio of the number of new suppliers firm i gained in year t to the number of suppliers last year $t-1$. Similarly, Gross supplier depreciation measure ($GrossDepreciatio^S_{i,t}$) captures the ratio of the number of suppliers firm i did not carry over from the previous year $t-1$ to the number of supplier last year $t-1$. Net supplier growth measure ($NetGrowth^S_{i,t}$) captures the ratio of net increase in the number of suppliers to the number of supplier last year $t-1$. This measure is identical to growth rate of the number of suppliers, and by definition, it is calculated as the gross supplier growth measure minus gross supplier depreciation measure. Also for $B_{i,t} :=$ set of buyers$_{i,t}$, we define corresponding four measures (buyer degree measure ($LogDegree^S_{i,t}$), gross buyer growth measure ($GrossGrowth^B_{i,t}$), gross buyer depreciation measure ($GrossDepreciation^B_{i,t}$) and net buyer growth measure ($NetGrowth^B_{i,t}$)) in the same procedure.

Table 1 presents the summary statistics of the network measures by firm age, defined as the years since registration as a corporation. Columns (1) and (2) report the mean and standard deviation (SD) for young firms (less than 20 years old) and old firms (20 years or older). Column (3) provides the difference between young and old firms and the corresponding standard error (SE).

Networking behavior differs clearly between young and old firms, as shown in Table 1. First, young firms have fewer partners than old firms as the log degree measures (the first and second rows) suggest. On average, young firms have 36% fewer suppliers and 26% fewer buyers. Second, the growth rate of the number of the partner is higher for young firms as the net growth measures (the third row and the sixth row) suggest. On average, the number of suppliers for young firms grows 4 percentage points faster than that of old firms, which is approximately twice as fast, and a similar pattern is observed for the number of buyers. Third, the age difference in the growth rate of the number of partners is mainly driven by the difference in partner acquisition, not partner churn, as suggested by the gross growth rate and the gross depreciation rate (from the third row to the fifth row, and from the sixth row to the eighth row). From its construction (2.6), large net growth is achieved by large gross growth or small gross depreciation. While both the gross growth and gross depreciation measures are higher for young firms and they have countervailing effects on the net growth, the net growth is larger for young firms because variation by age in gross depreciation is much milder than variation in gross growth; the latter is about 700% larger than the former in the supplier direction and about 250% larger in the buyer direction.

2.3 Key Facts

While the above observation highlights a clear relationship between production networks and firm age, it might just reflect other typical age-dependent patterns in firm performance measures. For example, accumulation of the internal reserves and associated relaxed financial constraints can allow firms to expand their business only gradually (Midrigan and Xu, 2014) and the observed pattern of network measure might be a simple consequence of it. Another possible explanation is learning-by-doing and the associated gradual productivity growth, as pointed out in the firm dynamics literature such as Haltiwanger et al. (2013) and Haltiwanger et al. (2016).

To isolate network-specific aging patterns from the effects of these typical age-dependent factors, we estimate the following fixed effect models for each $measure \in \{LogDegree,$

	(1) Young		(2) Old		(3) Young - Old	
	Mean	SD	Mean	SD	Difference	SE
Level-related Measure						
$LogDegree^S$	1.099	0.827	1.459	0.963	-0.360	0.001
$LogDegree^B$	1.238	0.823	1.501	1.000	-0.264	0.001
Growth-related Measure						
$NetGrowth^S$	0.083	0.600	0.042	0.306	0.041	0.000
$GrossGrowth^S$	0.150	0.606	0.103	0.305	0.047	0.000
$GrossDepreciation^S$	0.067	0.184	0.061	0.136	0.006	0.000
$NetGrowth^B$	0.090	0.670	0.049	0.400	0.040	0.000
$GrossGrowth^B$	0.177	0.684	0.121	0.404	0.056	0.000
$GrossDepreciation^B$	0.088	0.224	0.072	0.183	0.016	0.000

Table 1: Summary Statistics: Networking Measures

Notes: Summary statistics for networking measures by the age of the firms. Columns (1) and (2) provide mean and standard deviations (SD) for young firms and old firms, respectively. Column (3) shows the difference between young and old firms and standard error (SE). We define a firm as young if years that have passed after its birth is shorter than 20 years, and as an old firm otherwise.

$NetGrowth$, $GrossGrowth$, $GrossDepreciation\}$ and each $direction \in \{Supplier, Buyer\}$.

$$measure^{direction}_{i,s,t} = \sum_{a<50} \beta_a \mathbb{1}_{age_{i,t}=a} + X'_{it}\Theta + \alpha_i + \eta_{s,t} + \varepsilon_{i,s,t} \tag{2.7}$$

where i denotes the firm, t denotes the year, s denotes the industry (4-digit), X_{it} controls for typical growth factors of firms that affect the above age-dependent growth patterns independently of network-specific effects, α_i controls firm fixed effect, $\eta_{s,t}$ is the sector-year fixed effect, and $\varepsilon_{i,s,t}$ is an error term. As the growth factors of young firms, we include labor productivity (defined as value added divided by the number of employees) following Decker et al. (2020), financial leverage (defined as the amount of debt divided by the amount of equity) following Cavenaile and Roldan-Blanco (2021), and the inverse Mills ratio to correct for a sample selection bias following the procedure of Hansen (2022). The construction of the variables and their summary statistics are shown in Appendix A.1 and A.2, respectively.

From its construction, β_a can be interpreted as an average value of the measure[7] of firm with age a relative to that of firm with age 50 after eliminating effects from the age-dependent growth variables and the firm's unique characteristics. Hence, age-specific pattern in β_a indicates that as a firm ages, the firm changes its networking behaviors captured by our measures even without the change in the typical age-specific growth factors.

Figure 1 shows the estimation results for panel regression of equation (2.7) for log degree measures. For each direction of links, we plot estimated coefficients on each age $\hat{\beta}_a$ with its 99% corresponding confidence intervals shaded in gray. As the figure shows, age-dependent patterns clearly survive the control of typical age-specific growth variables. Even if a hypothetical young firm has the same level of productivity and financial slackness as old firms, it experiences a significant shortage of suppliers and buyers and it takes time to accumulate a mature number of partners. Quantitatively, the hypothetical firm starts its business with 80% fewer suppliers and 60% fewer buyers than in its matured level. In Appendix B.1, we show the details of the regression results.

Figure 2 shows the estimation results for panel regression of equation (2.7) for remaining growth-related measures, $NetGrowth$, $GrossGrowth$, and $GrossDepreciation$. Gross depreciation measures are multiplied by -1, so small values in the figure imply large shares of partner churn. The age-dependent patterns observed in the summary statistics survive the control of typical age-dependent variables also among the growth measures. First, as the age-specific pattern of the net growth measure implies, the number of partners grows rapidly when firms are young, which is consistent with the regression result for the log degree measures. Next, the mild dynamics in the gross depreciation measure and the corresponding similarity between the net growth measure and the gross growth measure from its construction (2.6) are also confirmed. This suggests that the difficulty for young firms to reach a mature network size is not due to post-matching mechanisms such as high churn rates, but to pre-matching mechanisms related to the matching with potential partners. In Appendix B.1, we show the details of the regression results.

In summary, the regression results with firm fixed effects and controlled by the growth factor confirm that it takes time for young firms to establish a sufficient number of partners,

[7] Note that for a dependent variable vector Y and an independent variable matrix X with each column representing dummy for each age (so the row sum does not exceed 1) we obtain

$$\hat{\beta} = (X'X)^{-1}X'Y = \left(diag^{-1}\left(\sum_{a_{it}=0} 1, \cdots, \sum_{a_{it}=\mathrm{A}} 1\right)\right)^{-1}\left(\sum_{a_{it}=0} Y_{it}, \cdots, \sum_{a_{it}=\mathrm{A}} Y_{it}\right)' = (\bar{Y}_{a=0}, \cdots, \bar{Y}_{a=A})'.$$

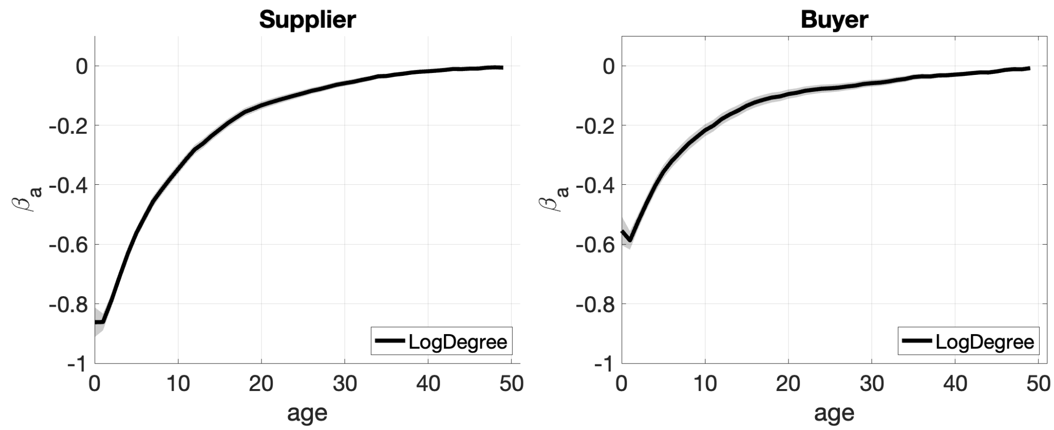


Figure 1: Number of Networks and Age

Notes: Estimation results for panel regression expressed in equation (2.7) using $LogDegree$ measure. For each direction, we plot the estimated coefficient on each age $\hat{\beta}_a$ with its 99% confidence interval in a shadow. Controls include firm fixed effect, sector * year fixed effect, typical growth factors of young firms including labor productivity (value added per employee) and leverage (debt/equity).

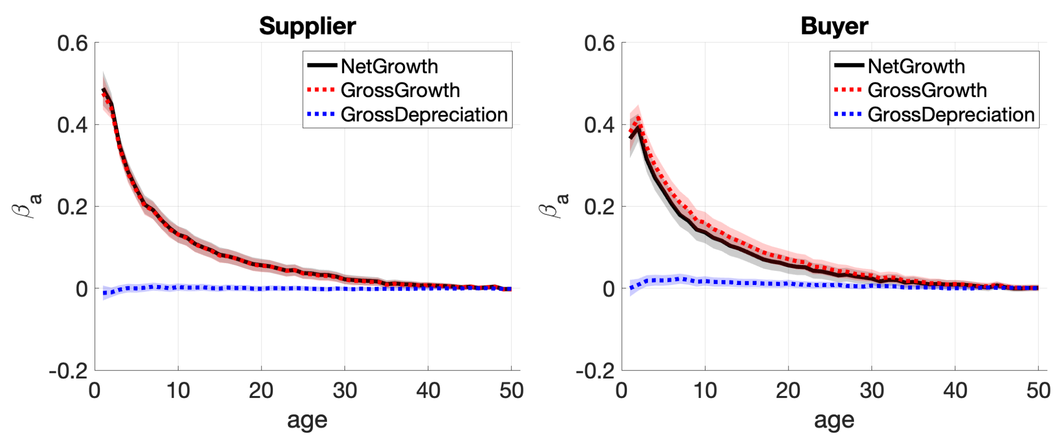


Figure 2: Growth of Networks and Age

Notes: Estimation results for panel regression expressed in equation (2.7) using the three growth-related measure, $NetGrowth$, $GrossGrowth$, and $GrossDepreciation$. For each measure and direction, we plot the estimated coefficient on each age $\hat{\beta}_a$ with its 99% confidence interval in a shadow. For clarity, we plot drop measures after multiplying -1. Controls include firm fixed effect, sector * year fixed effect, typical growth factors of young firms including labor productivity (value added per employee) and leverage (debt/equity).

mainly due to the difficulty in matching with potential partners. This finding is robust to a series of robustness checks. We show that our empirical results are robust to the exclusion of firms in financial industries, to the exclusion of links between firms with capital ownership, and to alternative measures of financial constraints and productivity. The details of those robustness checks are found in Appendix B.2.

3 Model

In this section, we develop a dynamic general equilibrium model that incorporates dynamic network formation decisions of heterogeneous firms over their life cycle.

3.1 Model environment

The economy consists of a representative household, representative advertisement goods producer, and a continuum of firms, each producing a unique good. Firms are owned by the household, and heterogeneous across states $\chi = (\phi, a)$, where ϕ and a are the *fundamental productivity* of a firm's production process and the age of the firm (years passed after its entry), respectively. The measure of firms over the states is denoted by $F(\chi)$, and its corresponding density is given by $f(\chi)$. For brevity, a firm with state χ is also referred to as χ-firm. We assume household income is numeraire.

3.2 Household

The representative household supplies a unit of labor inelastically and has constant-elasticity-of-substitution (CES) preferences over all goods in the economy, given by:

$$U = \left[\int x^H(\chi)^{\frac{\sigma-1}{\sigma}} dF(\chi)\right]^{\frac{\sigma}{\sigma-1}}. \tag{3.1}$$

Here, σ denotes the elasticity of substitution across varieties, and $x^H(\chi)$ is the household's consumption of χ-firm varieties. Given the price $p^H(\chi)$ charged by χ-firms to the household, household demand is given by:

$$x^H(\chi) = \Delta^H p^H(\chi)^{-\sigma} \tag{3.2}$$

where Δ^H denotes the household demand shifter:

$$\Delta^H = \left(\frac{1}{P^H}\right)^{-\sigma} U \tag{3.3}$$

and P^H denotes the consumer price index:

$$P^H = \left[\int \left(p^H(\chi)\right)^{1-\sigma} dF(\chi)\right]^{\frac{1}{1-\sigma}}. \tag{3.4}$$

Note that since household income is the numeraire, household utility is the inverse of the CPI:

$$U = \frac{1}{P^H}. \tag{3.5}$$

3.3 Production

Each firm produces its output using labor and an input bundle combining intermediate goods sourced from its suppliers. The production function is given by:

$$X(\chi) = \zeta\phi l(\chi)^{\alpha} B(\chi)^{1-\alpha} \tag{3.6}$$

Here, $X(\chi)$ is output (in quantities) of χ-firm, ϕ is the fundamental productivity, $l(\chi)$ is the amount of labor used by χ-firm, α is the labor share, and ζ is a normalization constant.[8]

$B(\chi)$ is a CES input bundle given by:

$$B(\chi) = \left[\int (x(\chi,\chi'))^{1-\frac{1}{\sigma}} k^S(\chi,\chi')d\chi'\right]^{\frac{\sigma}{\sigma-1}} \tag{3.7}$$

Here, $x(\chi,\chi')$ is the quantity that χ-buyer purchases from χ'-supplier, $k^S(\chi,\chi')$ is the density of χ'-supplier χ-buyer has, and σ is the elasticity of substitution across supplies.

The unit cost of χ-firm is given by:

$$\eta(\chi) = \frac{1}{\phi} w^{\alpha} P(\chi)^{1-\alpha} \tag{3.8}$$

Here, w is the wage rate and the producer price index is equal to:

$$P(\chi) = \left[\int (p(\chi,\chi'))^{1-\sigma} k^S(\chi,\chi')d\chi'\right]^{\frac{1}{1-\sigma}} \tag{3.9}$$

Here, $p(\chi,\chi')$ is the price charged by χ'-suppliers to χ-buyer.

Hence, the unit cost becomes

$$\eta(\chi) = \frac{1}{\phi}\left[\int (p(\chi,\chi'))^{1-\sigma} k^S(\chi,\chi')d\chi'\right]^{\frac{1-\alpha}{1-\sigma}}. \tag{3.10}$$

3.4 Advertisement Goods Producer

The representative advertisement goods producer combines all goods in the economy and produce advertisement goods with its production function being

$$X^{Adv} = \left[\int \left(x^{Adv}(\chi)\right)^{\frac{\sigma-1}{\sigma}} dF(\chi)\right]^{\frac{\sigma}{\sigma-1}}. \tag{3.11}$$

Given the price $p^{Adv}(\chi)$ charged by χ-firms to the advertisement goods producer, advertisement goods producer demand is given by:

$$x^{Adv}(\chi) = \Delta^{Adv} p^{Adv}(\chi)^{-\sigma} \tag{3.12}$$

where Δ^{Adv} denotes the advertisement demand shifter

$$\Delta^{Adv} = \left(\frac{1}{P^{Adv}}\right)^{-\sigma} X^{Adv}. \tag{3.13}$$

[8] $\zeta = \dfrac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$. This normalization constant simplifies the expresssion for the cost function, without any bearing on the equilibrium results.

and P^{Adv} is its unit cost given by

$$P^{Adv} = \left[\int \left(p^{Adv}(\chi) \right)^{1-\sigma} dF(\chi) \right]^{\frac{1}{1-\sigma}}. \tag{3.14}$$

3.5 Market Structure

We assume monopolistic competition when a χ-firm sells its good to the representative household, the representative advertisement goods producer, or its buyers. Given the identity of the elasticity of substitution over varieties, the markups that the χ-firm charges over marginal cost becomes identical. The profit-maximizing prices charged by each firm are given by:

$$p^H(\chi) = p^{Adv}(\chi) = p(\chi', \chi) = \mu\eta(\chi) \tag{3.15}$$

where $\mu = \dfrac{\sigma}{\sigma - 1}$ is the markup rate over marginal cost.[9] For the advertisement goods market, we assume perfect competition so the original price charged by the representative advertisement goods producer to each χ-firms is P^{Adv}.

3.6 Network Formation

There are search and matching frictions to obtain new suppliers and buyers. To search for buyers and suppliers, firms post advertisements by purchasing advertisement goods to maximize their expected lifetime profit. The probability of successful matches depends on an aggregate matching technology and the number of searching suppliers and buyers. Our model extends the static framework in Arkolakis et al. (2023) by incorporating dynamic decision-making, leading to an age-specific networking pattern over firm life cycle.

3.6.1 Timeline

Time is discrete in the model. At the beginning of each period, firms face an exogenous exit shock with probability $1 - \varphi$. Links that existed in the last period are exogenously terminated on a link termination shock with probability $1 - \delta$, on exit of its counterpart, or on its terminal link age Al large enough.[10] After that, firms post advertisement n^S, n^B respectively for suppliers and buyers. For firms newly entering the market ($a = 0$), we simply assume fixed initial number of supplier and buyer advertisements, determined by their fundamental productivity ϕ, at no cost. Links newly created in the period are determined by the amount of advertisements and the aggregate matching rates m^S and m^B. Combined with the links succeeded from the last period, the networks in the period realize. Then, firms produce goods given the network structure at the period.

[9]Constant markup is derived from the assumption that each firm is an atomic object for its partners, which implies that its behavior does not change the partners behavior as in Lim (2018) and Huneeus (2020). We assume it since strategic behavior over the network is out of our scope. For these strategic environment, please check Oberfield (2018), Acemoglu and Tahbaz-Salehi (2020) and Kopytov et al. (2022).

[10]Note that due to the structure of the exogenous shocks, this model yields a stable gross depreciation measure over firm life cycle, which is observed in the empirical part.

3.6.2 Profit from Network

We define a history of advertisement $\mathbf{n}^{a-1} = (n^S_{a-1}, n^S_{a-2}, ..., n^S_{a-Al}, n^B_{a-1}, n^B_{a-2}, ..., n^B_{a-Al})$ where $n^S_{a-\tau}$ is the advertisement for suppliers paid by the firm when firm is $a-\tau$ years old and $n^B_{a-\tau}$ is that for buyers. We also suppose that firms cannot make advertisement before they enter the market, i.e., $n^S_{a'<0} = n^B_{a'<0} = 0$.

Then, one-period operating profit of a firm can be written as:

$$\pi(\phi, \mathbf{n}^{a-1}, \mathbf{n}_a) = C\phi^{\sigma-1} w^{\alpha} \underbrace{\left[m^S \sum_{\tau=0}^{Al} w^S_\tau n^S_{a-\tau} \right]^{1-\alpha}}_{\text{upstream factor}} \times \underbrace{\left[\Delta + m^B \sum_{\tau=0}^{Al} w^B_\tau n^B_{a-\tau} \right]}_{\text{downstream factor}} \tag{3.16}$$

where C is a constant scalar, Δ is an aggregate demand shifter defined by $\Delta := \Delta^H + \Delta^{Adv}$. m^S and m^B are aggregate matching rate for supplier and buyer endogenously determined in equilibrium. Weight variables w^S_τ and w^B_τ are also endogenously determined in equilibrium, and they govern the profitability of a link created at τ-period ago each for supplier and buyer. We put closed-form expressions of them in Appendix C.1.

Intuitively, the (nominal) profit of a firm is determined by 4 factor: its own fundamental productivity ϕ, wage level w, the upstream factor, and the downstream factor. The upstream factor captures the supplier-side networks, which determine production efficiency, and the downstream factor captures the buyer-side networks and the aggregate demand, which determine how widely a firm can access its customer base. The profit is large when the upstream factor $\left[m^S \sum_{\tau=0}^{Al} w^S_\tau n^S_{a-\tau} \right]^{1-\alpha}$ is large, which happens when the aggregate matching rate to the supplier m^S is high, the weight variable for supplier w^S_τ is large,[11] or the firms have invested in the supplier advertisement a lot. Similarly, the profit is large when the downstream factor $\left[\Delta + m^B \sum_{\tau=0}^{Al} w^B_\tau n^B_{a-\tau} \right]$ is large, which happens when the aggregate demand shifter Δ is large, the aggregate matching rate to the buyer m^B is high, the weight variable for buyer w^B_τ is large, or the firms have invested in the buyer advertisement a lot.

3.6.3 Cost for Network Formation

To search for partners, firms must purchase advertisement goods from the advertisement goods producer. We assume a convex structure for advertisement technology, where posting n^S units of supplier advertisements requires $f_S \frac{(n^S)^{\gamma^S}}{\gamma^S}$ units of advertisement goods with $\gamma_S > 1$.[12] Additionally, we incorporate an age-specific *upstream networking wedge* ξ^S_a, which creates a gap between the original advertisement goods price P^{adv} and the price faced by firms of age a, following Hsieh and Klenow (2009). Under this setting, young firms face difficulties in acquiring sufficient partners after entry due to the two factors. First, the convexity of the cost structure results in higher marginal costs for firms that attempt to

[11] As Appendix C.1 shows, we can interpret w^S reflects suppliers' average productivity and w^B reflects buyers' average demand.

[12] In the calibration section, we show $\gamma_S > 1$ holds true in the data using variation of the number of suppliers among firms.

advertise intensively upon entry. Consequently, young firms are discouraged from acquiring a large number of partners at once. Second, the age-specific networking wedge, which is estimated as a decreasing function in age in the calibration part, increases the effective advertisement cost for young firms. This also hampers their partner acquisition. Similarly, we assume that n^B units of buyer advertisements requires $f_B \frac{(n^B)^{\gamma^B}}{\gamma^B}$ units of advertisement goods with an age-specific *downstream networking wedge* ξ_a^B.

3.6.4 Firm Problem

Given the one-period operating profit function (3.16) and advertisement costs above, the firm problem for $a > 0$ becomes

$$V(\phi, a, \mathbf{n}^{a-1}) = \max_{\mathbf{n}_a=(n_a^S, n_a^B)} \pi(\phi, \mathbf{n}^{a-1}, \mathbf{n}_a) - \xi_a^S P^{Adv} f_S \frac{(n_a^S)^{\gamma^S}}{\gamma^S} - \xi_a^B P^{Adv} f_B \frac{(n_a^B)^{\gamma^B}}{\gamma^B} + \beta\varphi V(\phi, a+1, \mathbf{n}^a) \tag{3.17}$$

where β denotes the time discount rate, and φ denotes the exogenous survival rate. The key technique we adopt is to use the history of advertisement $\mathbf{n}^{a-1}$, a finite-dimensional object, instead of storing all network information around the firm, which would render the state space of the value function infinite-dimensional. This approach is analogous to the truncation method used by Le Grand and Ragot (2022) to solve high-dimensional value functions. In the paper, they solve the planner's problem in the environment where both aggregate shocks and idiosyncratic shocks exist and the wealth distribution of household is a natural state variable for the planner. Rather than using the wealth distribution of households, an infinite-dimensional object, they adopt a truncated history of aggregate shocks as a finite-dimensional state variable. This approach is particularly effective in our network formation context due to the observed churn rate of approximately 0.08 for each link. This relatively high churn rate ensures that the cumulative contribution of links exceeding an age of Al periods, proportional to $(1-0.08)^{Al}$, diminishes rapidly, making truncation computationally valid with negligible error. Hence, we can solve the above firm problem using the standard value function iteration defined on the finite dimensional state space. Further details are provided in Appendix D.1.

Note that, conditional on survival to age a, firms differ only in their fundamental productivity. Therefore, we can specify value function $V(\chi)$, policy function $n^S(\chi)$ and $n^B(\chi)$ and its history $n_{a'}^S(\chi)$ and $n_{a'}^B(\chi)$ using $\chi = (\phi, a)$.[13]

3.7 Matching

Following a long tradition in the literature of labor search and matching (Diamond, 1982; Mortensen, 1986; Pissarides, 1985) and its application to supplier-buyer matching by Arkolakis et al. (2023), we assume that the aggregate number of successful matches between a supplier advertisement and a buyer advertisement is determined by matching technology represented by a matching function with constant elasticity

$$M = (M^S)^{\lambda^S} (M^B)^{\lambda^B} \tag{3.18}$$

[13] In the stationary equilibrium we are going to focus on, $n_{a'}^S((\phi, a))q = n^S((\phi, a'))$ and $n_{a'}^B((\phi, a)) = n^B((\phi, a'))$ hold true for $\chi = (\phi, a)$ firm.

where λ^S and λ^B are elasticity of the matching to each of the aggregate advertisement $M^S = \int n^S(\chi)dG(\chi)$ and $M^B = \int n^B(\chi)dG(\chi)$.

Then, matching rates can be written as:

$$m^S = M/M^B = (M^B)^{\lambda^B-1}(M^S)^{\lambda^S} \tag{3.19}$$

$$m^B = M/M^S = (M^B)^{\lambda^B}(M^S)^{\lambda^S-1}. \tag{3.20}$$

3.8 Entry

We assume potential entrants can enter the market by paying a sunk cost f^e in units of labor. Zero profit condition for entry yields

$$wf^e = E[V(\chi)|a=0] \tag{3.21}$$

$$= \int V(\phi, a=0)dH(\phi) \tag{3.22}$$

where $h(\phi)$ is a distribution of fundamental productivity ϕ.

3.9 Market Clearing

Market clearing for the output of a χ-firm is given by:

$$X(\chi) = x^H(\chi) + x^{Adv}(\chi) + \int x(\chi', \chi)k^B(\chi, \chi')d\chi' \tag{3.23}$$

where $k^B(\chi, \chi')$ is the mass of χ'-buyer χ-supplier has.

Market clearing for the advertisement goods is given by:

$$X^{Adv} = \int f_S \frac{(n^S(\chi))^{\gamma^S}}{\gamma^S} dF(\chi) + \int f_B \frac{(n^B(\chi))^{\gamma^B}}{\gamma^B} dF(\chi). \tag{3.24}$$

Combined with demand shifter of household and demand shifter of advertisement goods producer, (3.23) can be reduced to

$$X(\chi) = p(\chi)^{-\sigma}\Delta + \int x(\chi', \chi)k^B(\chi, \chi')d\chi'. \tag{3.25}$$

Market clearing for labor is given by:

$$1 = \int l^p(\chi)dG(\chi) + f^e M^e. \tag{3.26}$$

Lastly, we assume a direct transfer of a margin generated by the networking wedge to household income as follows

$$T = \int (\xi_a^S - 1)f_S P^{Adv} \frac{(n^S(\chi))^{\gamma^S}}{\gamma^S} dF(\chi) + \int (\xi_a^B - 1)f_B P^{Adv} \frac{(n^B(\chi))^{\gamma^B}}{\gamma^B} dF(\chi). \tag{3.27}$$

3.10 Distribution and Network Structure

Every period, M_e mass of firms enter the market following the zero profit condition (3.22), and incumbents exit with probability $1-\varphi$. Combined with the assumption that fundamental productivity does not grow after entry, we have a τ-period transition equation $g_\tau((\phi,a)|(\phi,a-1)) = \varphi^\tau$.[14] Then, the measure of firm over χ follows

$$f(\chi_{a=0}) = M_e h(\phi) \tag{3.28}$$

$$f(\chi_{a\geq 1}) = g_\tau((\phi,a)|(\phi,a-1))f(\chi_{a-1}) = \varphi f(\chi_{a-1}). \tag{3.29}$$

We also define the distribution of χ-suppliers (buyers) that a firm posting supplier (buyer) advertisement faces in a period. From the duality of matching structure, the probability that χ-suppliers (buyers) match a buyer (supplier) is proportional to the number of the buyer (supplier) advertisement the χ-suppliers (buyers) make. Hence, we obtain

$$g_B(\chi) = n^S(\chi)f(\chi)/\int n^S(\chi)dF(\chi) \tag{3.30}$$

$$g_S(\chi) = n^B(\chi)f(\chi)/\int n^B(\chi)dF(\chi). \tag{3.31}$$

Since links from the last period remain connected with probability δ, the entire production network structure of the economy is given by

$$k^S(\chi,\chi') = \sum_{\tau=0}^{Al} n^S_{a-\tau}(\chi)\int m^S g_S(\chi'')\delta^\tau g_\tau(\chi'|\chi'')d\chi'' = \sum_{\tau=0}^{Al} n^S_{a-\tau}(\chi)m^S(\varphi\delta)^\tau g_S(\chi') \tag{3.32}$$

$$k^B(\chi,\chi') = \sum_{\tau=0}^{Al} n^B_{a-\tau}(\chi)\int m^B g_S(\chi'')\delta^\tau g_\tau(\chi'|\chi'')d\chi'' = \sum_{\tau=0}^{Al} n^B_{a-\tau}(\chi)m^B(\varphi\delta)^\tau g_B(\chi'). \tag{3.33}$$

3.11 Two Fixed Points over Production Networks

It is worth discussing how production networks shape the structure of the entire economy. First, from the unit cost of firm (3.10) and markup pricing (3.15), we obtain

$$\eta(\chi)^{1-\sigma} = \left(\frac{1}{\phi}\right)^{1-\sigma} w^{\alpha(1-\sigma)}\left[\int(\mu\eta(\chi'))^{1-\sigma}k^S(\chi,\chi')d\chi'\right]^{1-\alpha}. \tag{3.34}$$

By defining an inverse measure of unit cost as *network productivity* $\Phi(\chi) = \eta^{1-\sigma}$, (3.34) becomes

$$\Phi(\chi) = \left(\frac{1}{\phi}\right)^{1-\sigma} w^{\alpha(1-\sigma)}\mu^{1-\alpha}\left[\int \Phi(\chi')k^S(\chi,\chi')d\chi'\right]^{1-\alpha}. \tag{3.35}$$

This corresponds to the backward fixed point problem in Bernard et al. (2022). Network productivity $\Phi(\chi)$ of χ-firm depends on its fundamental productivity ϕ and the wage rate w, and network productivity of all its suppliers χ' and the number of supplier $k^S(\chi,\chi')$.

[14] $\chi_{a=a'}$ is identical to (ϕ, a'). For visibility, we use both expressions interchangeably.

Note that an increase in the number of suppliers enhances productivity even without an increase in the productivity of each suppliers. This stems from love of variety of production function, and empirically observed by Bernard et al. (2019).

Second, from the unit cost of firm (3.10) and goods market clearing condition (3.25), we obtain

$$X(\chi) = p(\chi)^{-\sigma} \left(\Delta + (1-\alpha) \int \left(\frac{\eta(\chi')}{P(\chi')^{1-\sigma}} \right) X(\chi') k^B(\chi, \chi') d\chi' \right). \tag{3.36}$$

This corresponds to the forward fixed point problem in Bernard et al. (2022). Demands for χ-firm depends on its price $p(\chi)$, aggregate demand Δ, its buyers' demand determined by $\left(\frac{\eta(\chi')}{P(\chi')^{1-\sigma}} \right) X(\chi')$, and the number of buyers $k^B(\chi, \chi')$. Note that an increase in the number of buyers increases demand for $X(\chi)$ even without an increase in the demand of each buyers. This is empirically observed by papers analyzing the contribution of demand accumulation to firm growth like Ignaszak and Sedlácek (2023) and Alfaro-Urena et al. (2022).

3.12 Equilibrium

We can now define the equilbirium. For tractability, this paper focuses on stationary equilibrium.

Definition 1 (Stationary Equilibrium)**.** Given a household income as a numeraire ($I = 1$), a stationary equilibrium is w, production network structure $k^S(\chi, \chi'), k^B(\chi, \chi')$, price function $p(\chi)$ and output function $X(\chi)$ such that the production network structure is consistent with policy function of firms, price and output of each firms respectively is a solution to the backward fixed point equation and the forward fixed point equation, zero profit condition for potential entrants satisfy, and all the markets clear.

Our model addresses two well-known difficulties in solving the equilibrium of a general equilibrium model with endogenous network formation: the curse of dimensionality and the simultaneity of decision making. First, as discussed in Section 3.6.4, the curse of dimensionality in firms' network formation decisions is addressed by a truncation approach, leveraging the observed churn rate of links. Second, we handle the simultaneity of decision making by assuming that each firm is treated as an atomic entity by its partners. This assumption allows decision of each firm independent of the decision of other firms while keeping the interconnectedness over production networks as in the backward fixed point (3.35) and forward fixed point (3.36) like Lim (2018) and Bernard et al. (2022).[15]

Leveraging these tractable structures, our computational strategy is both efficient and conceptually straightforward. First, given an initial guess for the equilibrium object, we solve the firms' value function problem to determine their advertising investment and the resulting production network structure. Second, solving backward fixed point equation and the forward fixed point equation over production networks, we obtain price and allocation

[15] Several approaches that can directly treat formation process of discrete network are worth mentioning. Taschereau-Dumouchel (2022) proposes a new solution technique to solve the nonconvex optimization problems with binary simultaneous decision of firms utilizing *relaxed problems*. Elliott et al. (2022) models continuous investment choice that determines the probability of each discrete link being active, which avoids the computational intractability of discrete networks. Both of the approaches are attractive to solve one-period problem, but hard to apply to our situation where firms' decision making is dynamic.

over firms. Third, we adjust mass of entrants and the wage level so that the zero profit condition of potential entrants and the labor market clearing condition hold, and continue to its convergence. For the detail of the algorithm, see Appendix D.

4 Quantitative Analysis

In this section, we combine the empirical findings in Section 2 and the model developed in Section 3 to quantitatively assess the macroeconomic significance of network frictions young firms face and derive implications for welfare-improving industry policies. Section 4.1 calibrates the model, and section 4.2 performs several counterfactual simulations.

4.1 Calibration

4.1.1 Aggregate Parameters

Table 2 summarizes aggregate parameter values, their source/reference, and data for setting targets. The time discount rate β is set to 0.94 which matches the average cost of capital for Japanese firms shown by Suto and Takehara (2017). The elasticity of substitution between varieties of the household consumption bundle and the intermediate good bundle of firms is set to 5 following Bernard et al. (2022). This yields markup rate $\mu = 1.25$, which is close but slightly higher than the previous estimation of the markup rate of Japanese firms estimated by Nakamura and Ohashi (2022). The labor inputs share α in the production function is set to 0.45 so that the ratio of the aggregate intermediate inputs sales to the sum of aggregate intermediate inputs sales and aggregate consumption sales matches the value in input-output table in Japan at 2015 (0.46).[16]

Following Lim (2018) and Bernard et al. (2022), we assume fundamental productivity follows a log-normal distribution with mean 0, i.e., $\log(\phi) \sim \mathcal{N}(0, (\sigma^{\phi})^2)$. The dispersion parameter σ^{ϕ} is set to 0.46 so that the standard deviation of the log sales match the data at 2015 in our sample (2.37).

The exogenous survival rate φ is set to 0.962 so that the entry rate in the model matches the annual business entry rate (5.2%) in Japan at 2015.[17] The exogenous network survival rate is set to 0.92, which is an average probability of a link surviving two consecutive periods when both supplier and buyer continue to operate in the data at 2015 in our sample.

The elasticity of advertisement costs for supplier γ^S and buyer γ^B are set to 2.73 and 3.51 so that the standard deviation of the log number of suppliers and customers matches the data in our sample. (0.92 and 0.93, respectively) The matching function elasticity λ^S and λ^B are borrowed from Miyauchi (2024) and set to 0.9 and 1.0, respectively. Since $\lambda^S + \lambda^B = 1.9 > 1$, the matching function exhibits increasing-return-to-scale characters, which is consistent with the observed empirical patterns about supplier acquisition pattern after an anticipated supplier bankruptcy shown in Miyauchi (2024). We normalize advertisement cost $f^S = f^B = 1$ and entry cost $f^e = 1$.

4.1.2 Wedge Parameters

We determine remaining parameters related to network growth so that the model yields a matched pattern observed in the empirical part. To make estimation simple, we assume

[16] See https://www.soumu.go.jp/english/dgpp_ss/data/io/io15_00001.htm.
[17] See https://www.chusho.meti.go.jp/pamflet/hakusyo/H28/PDF/2016shohaku_eng.pdf.

Parameter	Description	Value	Target
β	Time discounting	0.94	Suto and Takehara (2017)
σ	Elasticity of substitution	5	Bernard et al. (2022)
α	Labor share	0.45	expenditure share of IMD goods
σ^{ϕ}	Dispersion of fundamental prod.	0.46	dispersion of log sales
φ	Exogenous exit rate	0.962	aggregate exit rate
δ	Network destruction rate	0.92	average network destruction rate
γ^S	Elast. of ad. cost for new suppliers	2.73	dispersion of log # suppliers
γ^B	Elast. of ad. cost for new buyers	3.51	dispersion of log # buyers
λ^S	Elast. of matching for supplier	0.9	Miyauchi (2024)
λ^B	Elast. of matching for buyer	1.0	Miyauchi (2024)

Table 2: Summary of aggregate parameter values, their source/reference, and data for setting targets.

the age-specific network formation cost parameters ξ_a^S and ξ_a^B have a following exponential parametric form for $a > 0$:

$$\xi_a^S = 1 + \alpha_\xi^S \left(\beta_\xi^S\right)^a \tag{4.1}$$

$$\xi_a^B = 1 + \alpha_\xi^B \left(\beta_\xi^B\right)^a \tag{4.2}$$

where α_ξ^S and α_ξ^B determine the size of wedge, and $\beta_\xi^S(< 1)$ and $\beta_\xi^B(< 1)$ determine the speed of its decay. We also assume that a fixed initial number of suppliers/buyer advertisement is proportional to its (firm-level) steady state level of advertisement (i.e., $n^S(a = 0, \phi) = \iota^S n^S(a = \infty, \phi)$ and $n^B(a = 0, \phi) = \iota^B n^B(a = \infty, \phi)$ given a coefficient parameter ι^S and ι^B).

We calibrate these parameters so that the model yields the same network growth pattern of young firms with the data. Remember that the age pattern of β_a in equation (2.7) for LogDegree measures can be interpreted as a network growth pattern. Hence, we estimate the wedge parameters so that β_a in equation (2.7) for LogDegree measures using the real data and model-generated data yield matched result. Let $\hat{\beta}_a^{S,data}$ denote the estimated coefficient in equation (2.7) for the supplier LogDegree measure using real data, $\bar{\hat{\beta}}^{S,data}$ denote the average across all ages, and $\hat{\beta}_a^{S,model}$ for the estimated coefficient using model-generated data, with a corresponding notation for the buyer direction. Then, we set an objective function below that consists of population-weighted R^2 between model and data for supplier and buyer, R_S^2 and R_B^2, respectively.

$$\begin{aligned} Obj. &:= R_S^2 + R_B^2 \qquad (4.3) \\ &:= \left(1 - \frac{\sum_{a\geq 1}(1-\varphi)\varphi^{a-1}\left(\hat{\beta}_a^{S,model} - \hat{\beta}_a^{S,data}\right)^2}{\sum_{a\geq 1}(1-\varphi)\varphi^{a-1}\left(\hat{\beta}_a^{S,data} - \bar{\hat{\beta}}^{S,data}\right)^2}\right) \\ &\quad + \left(1 - \frac{\sum_{a\geq 1}(1-\varphi)\varphi^{a-1}\left(\hat{\beta}_a^{B,model} - \hat{\beta}_a^{B,data}\right)^2}{\sum_{a\geq 1}(1-\varphi)\varphi^{a-1}\left(\hat{\beta}_a^{B,data} - \bar{\hat{\beta}}^{B,data}\right)^2}\right) \qquad (4.4) \end{aligned}$$

Table 3 summarize the estimated parameters that minimize the above objective function.

We observe larger α_ξ and β_ξ in supplier direction compared to buyer direction. This implies the wedge that young firms are facing to build networks is more serious and more prolonged with respect to supplier acquisition.

Parameter	Description	Value
α_ξ^S	Level of age-specific ad. cost for new supplier	2
β_ξ^S	Decay of age-specific ad. cost for new supplier	0.85
α_ξ^B	Level of age-specific ad. cost for new buyer	0.65
β_ξ^B	Decay of age-specific ad. cost for new buyer	0.235
ι^S	Coeff. of the initial ad. for new supplier	2.6
ι^B	Coeff. of the initial ad. for new buyer	4

Table 3: Summary of network-specific parameter values

We plot the estimated coefficients of β_a in equation (2.7) for LogDegree measures using the real data and model-generated data, respectively. While we make a strong parametric assumption on ξ_a, Figure 3 shows a good fit between the data and the model. Quantitatively, the weighted R^2 is 0.994 for supplier direction and 0.986 for buyer direction, which implies approximately 99% of the empirically estimated network growth pattern is accounted for by our calibrated model.

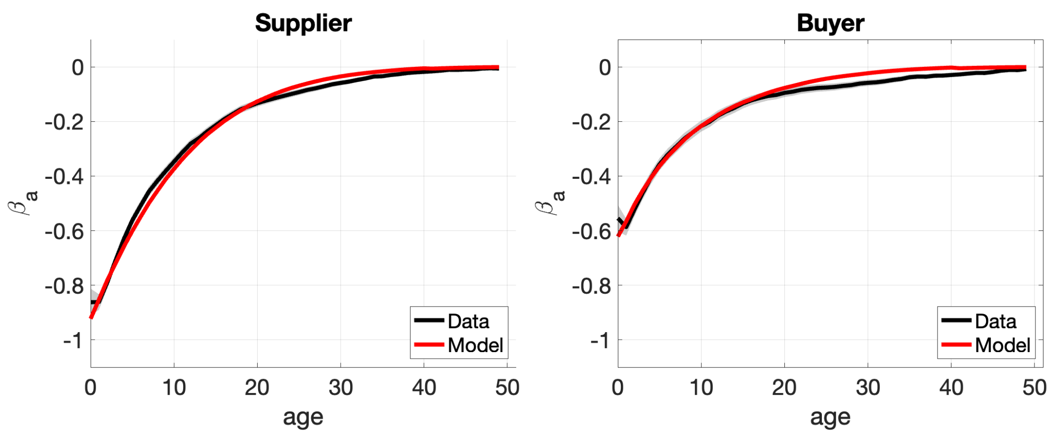


Figure 3: $\hat{\beta}_a$ in Data and Model

Notes: The comparison of observed/model-generated relationships between LogDegree measures and firm age. We plots estimated coefficients of β_a in equation (2.7) for LogDegree measures using the real data and model-generated data, respectively. The shadow is the 99% confidence interval for the coefficients using real data.

4.2 Counterfactual Simulations

4.2.1 Evaluation of the size of wedge

In this first simulation, we analyze *wedge-free economy*, a counterfactual economy without networking wedge, i.e., $\xi_a^S = \xi_a^B = 1$ for all a. Comparing it with the baseline economy calibrated above, we quantitatively assess the significance of the estimated networking wedge on networking behavior of firms and macroeconomy.

Figure 4 compares network growth patterns in the two economies. We plot estimated coefficients of β_a in equation (2.7) for LogDegree measures using data generated by the baseline model and one by the counterfactual model, respectively. As clearly shown in the left figure, supplier accumulation becomes much faster in the wedge-free economy. While somewhat milder, buyer accumulation is also accelerated in the wedge-free economy. These results confirm the networking wedge in the real economy certainly affects micro-level networking decisions of young firms and delays their partner accumulation.

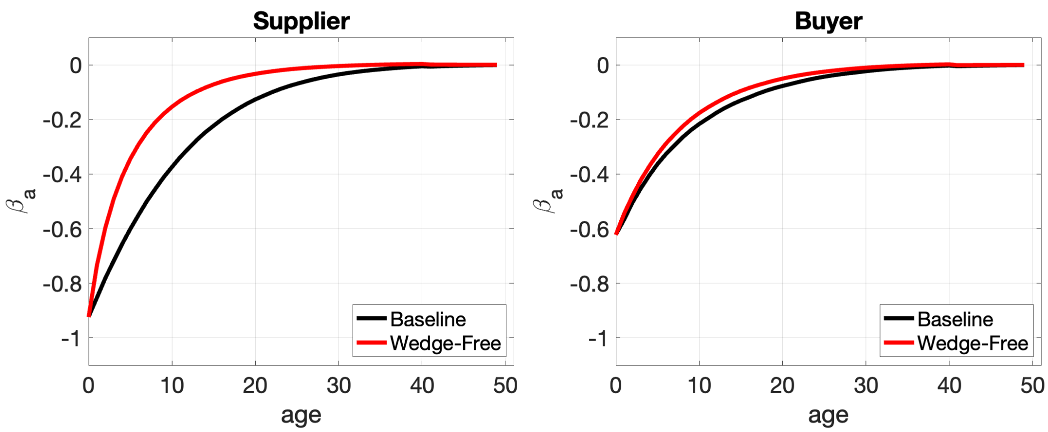


Figure 4: $\hat{\beta}_a$ in the Baseline and Wedge-Free Economy

Table 4 shows the effects on macroeconomic variables as percentage changes from the baseline economy. k is the average number of suppliers (or equivalently the average number of buyers, due to network duality). k_y^S and k_o^S are the average number of suppliers for young firms ($a < 20$) and old firms ($a \geq 20$), respectively. k_y^B and k_o^B are the same for the number of buyers. First, we observe a sizable impact on aggregate variables. Welfare improves by 2.35% and the mass of entrants increases by 2.91% and wage rises by 1.29%. Network-related measures also increase, but with asymmetry with respect to its direction and beneficiary. Since the wedge in the baseline mainly hampers supplier accumulation of young firms, eliminating it fosters supplier acquisition efforts by young firms. This results in the highest increase in the average number of suppliers for young firms k_y^S, directly. At the same time in the matching market, increased mass of supplier advertisement increase the buyer matching rate. This equally increase the number of buyer for both young k_y^B and old firms k_o^B. To summarize, this result highlights the macroeconomic significance of networking wedge. While its nature is totally micro-level wedge that distorts the networking decision of each of the young firms, its macroeconomic effect become sizable in the equilibrium.

	U	M^e	w	k	m^S	m^B	k_y^S	k_o^S	k_y^B	k_o^B
Wedge-Free	2.351	2.914	1.289	9.955	1.882	9.912	20.149	2.301	11.380	8.690

Table 4: Wedge-Free Economy (percentage change from the baseline economy)

Next, to investigate the mechanism of the welfare improvement, we develop Proposition

1, which decomposes welfare changes by discretizing the state space of χ and representing variables in the linear space using bold symbols. Here, $\mathbb{1}$ is an unit column vector, and we abuse the fraction bar $-$ as $\dfrac{\boldsymbol{x}}{\boldsymbol{y}} := \left(\dfrac{x_1}{y_1}, \cdots, \dfrac{x_n}{y_n}\right)$ or equivalently $:= \boldsymbol{x} \circ \boldsymbol{y}^{\circ -1}$ in the hadamard expression. The proof is in Appendix C.2.

Proposition 1 (Decomposition of change in U): Given the expenditure share of household $\boldsymbol{w}^E$ and network matrix weighted by transaction value $\boldsymbol{W}^S$, change in the welfare U can be decomposed as follows.

$$\frac{dU}{U} = -\frac{dP^H}{P^H} = \underbrace{\frac{1}{\sigma-1}\frac{dM^e}{M^e}}_{\text{LofV effect}} + \underbrace{\frac{1}{\sigma-1}\boldsymbol{w}^{E\prime} \cdot \frac{d\boldsymbol{\Phi}}{\boldsymbol{\Phi}}}_{\text{Price effect}} \tag{4.5}$$

$$\frac{d\boldsymbol{\Phi}}{\boldsymbol{\Phi}} = \underbrace{-\alpha(\sigma-1)\frac{dw}{w}}_{\text{Wage effect}} + \underbrace{(1-\alpha)\left(\boldsymbol{W}^S \circ \frac{d\boldsymbol{K}^s}{\boldsymbol{K}^s}\right)\mathbb{1}}_{\text{Networking effect}} + \underbrace{(1-\alpha)\boldsymbol{W}^S\frac{d\boldsymbol{\Phi}}{\boldsymbol{\Phi}}}_{\text{Spillover effect}} \tag{4.6}$$

The interpretation is quite intuitive. The first equality in (4.5) applies because the household income is numeraire in this economy.[18] The second equality decomposes the change in the price index into the love of variety effect, the increase in the mass of firms (extensive margin), and the price effect, the price change of each firm weighted by the household's exposure (intensive margin).

The second equation (4.6) further decomposes the price change of each firm. The first term is a direct path-through of labor cost. The second term captures the effect of acquisition of new suppliers.[19] Since the increase in the number of suppliers enable firms to produce goods more efficiently due to the love of variety structure in the production function, the unit costs of production decrease. Furthermore, due to the spillover effects of supply chain networks captured by the third term, the wage and networking effects are amplified by multiplier $(1-\alpha)\boldsymbol{W}^S$.

Table 5 shows the decomposition of the welfare change U based on Proposition 1.[20] First, we observe a sizable impact from the love of variety effect. Since firms in the wedge-free economy can access abundant suppliers and customers from an early stage, the value of entry is improved and mass of entrant increases. The second, and more significant impact comes from the price effect. While the increased mass of firms raises the wage w and negatively impacts welfare U, the networking effect, which is approximately three times larger than the wage effect in absolute terms and twice as large as the love of variety effect, can surpass the negative effect. Furthermore, these effects propagate through the supply chain and amplified as the spillover effect, which is also larger than the love of variety effect and the wage effect.

[18] This technique to set the nominal GDP as numeraire and focus on the change in the price index is widely used in the literature of aggregation of shocks like Baqaee (2018), Baqaee and Farhi (2020) and Baqaee and Rubbo (2023).

[19] When the production network structure remain unchanged, i.e., $\dfrac{d\boldsymbol{K}^s}{\boldsymbol{K}^s} = 0$, this equation simply captures the effect of the change in wage (labor costs) and productivity of upstream firms (intermediate goods costs) on the productivity of firms over supply chains, as studied in a wide range of the literature on production networks such as Oberfield (2018).

[20] There is a slight difference in the change of U between Table 4 and 5, which stems from the linearization in the proposition. While we do not investigate the difference since its out of the scope of our analysis, check Baqaee and Farhi (2019) for the detail.

To summarize, this decomposition result demonstrates the macroeconomic importance of accounting for the production network. The networking effect and the spillover effects, which arises only when we consider production network, have at least comparable impacts on welfare compared to the conventional channels like the love of variety effect and wage effect.

	U	$LofV$	$Price$	$Wage$	$Networking$	$Spillover$
Wedge-Free	2.387	0.728	1.658	-0.580	1.465	0.833

Table 5: Wedge-Free Economy (Decomposition of the change in U)

We further decompose the expression in Proposition 1 by age as follows. Suppose the state space is indexed by age in an ascending order, without loss of generality. Then, given a generic bivariate function $A(\chi, \chi')$ and its associated matrix $\boldsymbol{A}$, and a generic univariate function $x(\chi)$ and its associated vector $\boldsymbol{x}$, we have block matrix expression as follows.

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{A}_{y,y} & \boldsymbol{A}_{y,o} \\ \boldsymbol{A}_{o,y} & \boldsymbol{A}_{o,o} \end{pmatrix} \tag{4.7}$$

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_y \\ \boldsymbol{x}_o \end{pmatrix} \tag{4.8}$$

where $\boldsymbol{A}_{y,y}, \boldsymbol{A}_{o,y}, \boldsymbol{A}_{y,o}, \boldsymbol{A}_{o,o}$ correspond to discretized expression of a bivariate function $A(\chi, \chi')$ with $a < 20 \wedge a' < 20$, $a \geq 20 \wedge a' < 20$, $a < 20 \wedge a' \geq 20$, and $a \geq 20 \wedge a' \geq 20$, respectively, and $\boldsymbol{x}_y$ and $\boldsymbol{x}_o$ correspond to a univariate function $x(\chi)$ with $a < 20$ and $a \geq 20$, respectively. This block matrix expression leads to the next proposition.

Proposition 2 (Decomposition of change in U by firm age): Suppose the state space is indexed by its age element a in an ascending order. Then, change in the welfare U can be decomposed as follows.

$$\begin{aligned}
\frac{dU}{U} = & \underbrace{\frac{1}{\sigma-1}\frac{dM_e}{M_e}}_{\text{LofV Effect}} - \underbrace{\alpha\frac{dw}{w}}_{\text{Wage Effect}} \\
& + \underbrace{\frac{1-\alpha}{\sigma-1}(\boldsymbol{w}_y^{E'}\,\boldsymbol{w}_o^{E'})\left(\begin{pmatrix} \boldsymbol{W}_{y,y}^S \circ \frac{d\boldsymbol{K}_{y,y}^S}{\boldsymbol{K}_{y,y}^S} & \boldsymbol{W}_{y,o}^S \circ \frac{d\boldsymbol{K}_{y,o}^S}{\boldsymbol{K}_{y,o}^S} \\ \boldsymbol{W}_{o,y}^S \circ \frac{d\boldsymbol{K}_{o,y}^S}{\boldsymbol{K}_{o,y}^S} & \boldsymbol{W}_{o,o}^S \circ \frac{d\boldsymbol{K}_{o,o}^S}{\boldsymbol{K}_{o,o}^S} \end{pmatrix}\mathbb{1}\right)}_{\text{Networking Effect}} \\
& + \underbrace{\frac{1-\alpha}{\sigma-1}(\boldsymbol{w}_y^{E'}\,\boldsymbol{w}_o^{E'})\begin{pmatrix} \boldsymbol{W}_{y,y}^S & \boldsymbol{W}_{y,o}^S \\ \boldsymbol{W}_{o,y}^S & \boldsymbol{W}_{o,o}^S \end{pmatrix}\begin{pmatrix} \frac{d\boldsymbol{\Phi}_y}{\boldsymbol{\Phi}_y} \\ \frac{d\boldsymbol{\Phi}_o}{\boldsymbol{\Phi}_o} \end{pmatrix}}_{\text{Spillover Effect}}
\end{aligned} \tag{4.9}$$

By examining the elements of the block matrix in (4.9), we can clarify from which age group to which age group network growth contributed to the welfare change (networking effect) and from which age group to which age group propagation of these changes contributes to the welfare change (spillover effect) going beyond the aggregate-level analysis of network growth and propagation. The proof is in Appendix C.2.

Table 6 presents the age decomposition results of the networking effect and spillover effect. The results exhibit asymmetry across both firm age and network directions, but remain intuitive. First, the age decomposition of networking effect shows that the networking effect arises mainly from the growth in the number of young and old suppliers of young firms (the first and second columns). Second, this asymmetry leads to the next asymmetry in the spillover effect. Since the young firms can increase their productivity due to the increase in the number of suppliers, their buyers benefit from the spillover effect due to cheaper intermediate goods. The buyers could be either young firms and old firms, so the spillover effect from young firms is shared equally between young and old buyers (the fifth and seventh columns).

	Networking				*Spillover*			
	yy	yo	oy	oo	yy	yo	oy	oo
Wedge-Free	0.624	0.665	0.078	0.098	0.339	0.075	0.326	0.093

Table 6: Wedge-Free Economy (Decomposition of the change in U by age group)

Lastly, to clarify which direction of networking wedge is driving the above result, we analyze *upstream wedge-free economy* where only the upstream networking wedge is removed ($\xi_a^S = 1$ and $\xi_a^B = 1 + \alpha_\xi^B \left(\beta_\xi^B\right)^a$) and *downstream wedge-free economy* where only the downstream networking wedge is removed ($\xi_a^S = 1 + \alpha_\xi^S \left(\beta_\xi^S\right)^a$ and $\xi_a^B = 1$).

Figure 5 compares the network growth pattern in the several economies. Consistent with the previous analysis, the elimination of the upstream networking wedge yields a similar pattern to the wedge-free economy. In contrast, the elimination of the downstream networking wedge leads to a pattern nearly identical to the baseline economy. The macroeconomic implication is also similar. As Table 7 shows, elimination of the upstream networking wedge has a similar impact on macro variables, whereas the elimination of the downstream networking wedge leads to only minor changes, reflecting differences in the magnitude of changes in the networking behavior of young firms. To summarize, the upstream networking wedge creates greater distortions in young firms' networking behavior, leading to worse macroeconomic outcomes compared to the downstream networking wedge.

	U	M^e	w	k	m^S	m^B	k_y^S	k_o^S	k_y^B	k_o^B
Wedge-Free	2.351	2.914	1.289	9.955	1.882	9.912	20.149	2.301	11.380	8.690
Upstream Wedge-Free	2.209	2.459	1.208	9.510	1.343	9.595	19.663	1.887	10.649	8.499
Downstream Wedge-Free	0.131	0.384	0.071	0.425	0.529	0.253	0.429	0.422	0.732	0.152

Table 7: Several Wedge-Free Economies (percentage change from the baseline economy)

4.2.2 Network Subsidy

In this exercise, we examine which direction of subsidy has greater macroeconomic impacts from perspectives of policy makers. The observed larger impacts of the elimination of upstream networking wedge compared to the elimination of downstream networking wedge

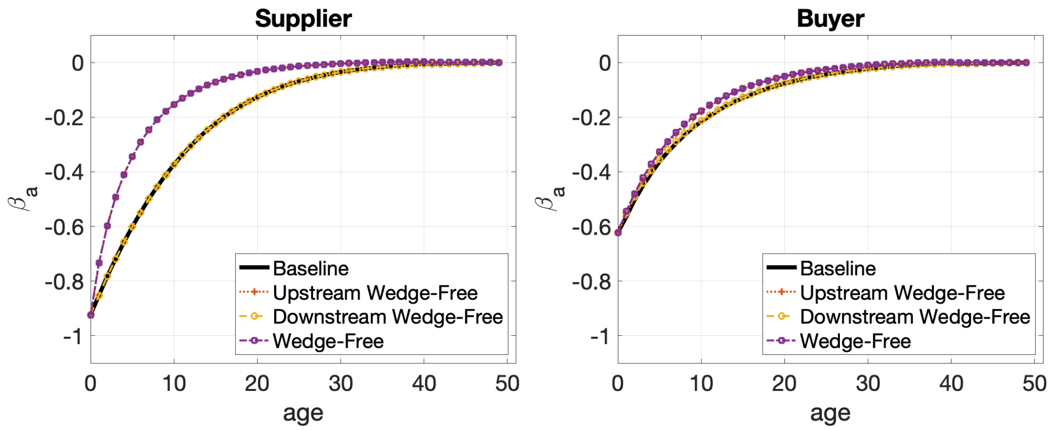


Figure 5: $\hat{\beta}_a$ in several Wedge-Free Economies

may directly result from the larger size of the existing wedge, or the different effectiveness of the wedge on the macroeconomy. If the former mechanism alone explains the previous result and the effectiveness between the two directions is identical, subsidies in either direction should be able to improve welfare to the same degree by giving the same level of subsidies. To test this, we conduct two network subsidy policy experiments, financed by a lump-sum tax.

We consider age- and direction-dependent network subsidy policies denoted by s_a^S and s_a^B. This changes firm problem as follows.

$$V(\phi, a, \mathbf{n}^{a-1}) = \max_{\mathbf{n}_a=(n_a^S, n_a^B)} \pi(\phi, \mathbf{n}^{a-1}, \mathbf{n}_a) - (1-s_a^S)\xi_a^S P^{Adv} f_S \frac{(n_a^S)^{\gamma^S}}{\gamma^S} - (1-s_a^B)\xi_a^B P^{Adv} f_B \frac{(n_a^B)^{\gamma^B}}{\gamma^B}$$
$$+ \beta\varphi V(\phi, a+1, \mathbf{n}^a) \tag{4.10}$$

The first policy we consider is to subsidize supplier acquisition of young firms by $1 - s_a^S = \frac{1}{\xi_a^S} = \frac{1}{1+\alpha_\xi^S\left(\beta_\xi^S\right)^a}$ with $s_a^B = 1$ (*upstream subsidy policy*), and the second one is to subsidize buyer acquisition of young firms by $1 - s_a^B = \frac{1}{\xi_a^S} = \frac{1}{1+\alpha_\xi^S\left(\beta_\xi^S\right)^a}$ with $s_a^S = 1$ (*downstream subsidy policy*). From its construction, the upstream subsidy yields exactly the same result as the upstream wedge-free economy. On the other hand, the downstream subsidy yields a different result from the downstream wedge-free economy because its subsidy level aligned with the size of the upstream networking wedge surpasses that of the existing downstream networking wedge.

Figure 6 compares the effects of the two policies on the network growth. As the figure clearly shows, the two contrasting policies achieve symmetric patterns, i.e., subsidizing supplier/buyer acquisition promotes faster supplier/buyer accumulation.

Table 8 shows the macroeconomic consequences. While both policies improve welfare and promote entry, the upstream subsidy has twice the impact of the downstream subsidy. The network-related variables exhibit symmetry consistent with the growth patterns in Figure 6.

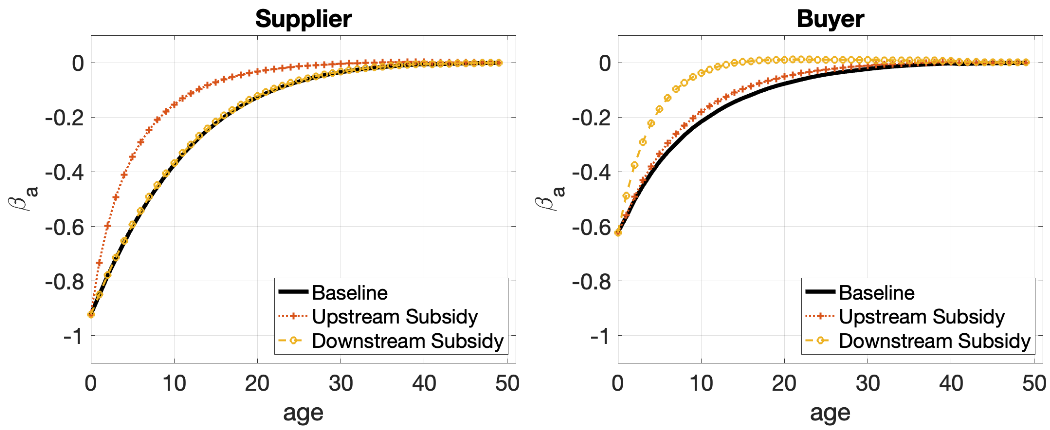


Figure 6: $\hat{\beta}_a$ under Network Subsidy Poilcy

	U	M^e	w	k	m^S	m^B	k_y^S	k_o^S	k_y^B	k_o^B
Upstream Subsidy	2.209	2.459	1.208	9.510	1.343	9.595	19.663	1.887	10.649	8.499
Downstream Subsidy	1.238	1.171	1.154	7.996	8.027	1.020	8.409	7.687	15.082	1.710

Table 8: Network Subsidy (percentage change from the baseline economy)

To identify the source of the quantitative differences in welfare impacts, we compare the associated decomposition results in Table 9. First, reflecting the different impacts in the mass of entrants M^e, the love of variety effect is larger for the upstream subsidy. Furthermore, the networking and spillover effects are much larger for the upstream subsidy. One explanation of the difference is the different efficiency of the newly created networks structure. The upstream subsidy encourages young firms to acquire both more young and old suppliers, boosting their productivity by leveraging the new suppliers' productivity. On the other hand, under the downstream subsidy policy, young firms accelerate its buyer acquisition, without increasing their suppliers and the corresponding productivity gain. From the perspective of the buyers added by the young firms, the newly added young suppliers have not accumulated their own suppliers yet and have low productivity. Hence, the productivity gains of new buyers remain limited due to their reliance on low-productivity suppliers. The difference in the productivity gain is further amplified by the spillover effect across the entire production networks.

	U	$LofV$	$Price$	$Wage$	$Networking$	$Spillover$
Upstream Subsidy	2.243	0.615	1.628	-0.543	1.412	0.816
Downstream Subsidy	1.250	0.293	0.957	-0.519	0.969	0.522

Table 9: Network Subsidy (Decomposition of the change in U)

We can confirm the mechanism explained above by age decomposition result in Table 10. The downstream subsidy mainly increases the young buyer-young supplier links (second row, first column) and the old buyer-young supplier links (second row, third column). Since these young suppliers have not accumulated sufficient suppliers yet, their contribution to buyer productivity is limited compared to that in the upstream subsidy economy. The difference is amplified as shown in the spillover effect.

	Networking				*Spillover*			
	yy	yo	oy	oo	yy	yo	oy	oo
Upstream Subsidy	0.603	0.659	0.060	0.090	0.336	0.071	0.322	0.088
Downstream Subsidy	0.459	0.042	0.399	0.070	0.127	0.121	0.124	0.151

Table 10: Network Subsidy (Decomposition of the change in U by age group)

4.2.3 Implication for Entry Subsidy

In this counterfactual simulation, we analyze the effect of entry subsidy. As the previous simulation results suggest, the existing networking wedge hinders partner acquisition during young ages and leads to weaker business performance, thereby suppressing entry. Here, by comparing the effects of entry subsidy in the baseline economy and counterfactual wedge-free economy, we evaluate the margin of the entry subsidy impact that is attributable to the networking wedge.

Table 11 shows the effects of the entry subsidy that increases entry by 10% for each economy. We observe higher growth in welfare U in the baseline economy. In the baseline economy, where the decision of young firms are distorted by the existing networking wedge, the suppressed original entry level leads to milder wage growth from the additional labor demand yielded by the subsidy. This leads to greater growth in network-related variables in the baseline economy due to the lower advertisement cost. Note that this entry subsidy policy does not alter age-specific growth pattern of network as Figure 6 shows. So the growth in k_y^S, k_o^S, k_y^B and k_o^B is proportional in the both economies.

	U	M^e	w	k	m^S	m^B	k_y^S	k_o^S	k_y^B	k_o^B
Baseline	2.250	10.000	1.419	4.179	6.998	6.368	4.179	4.179	4.179	4.179
Wedge-Free	2.191	10.000	1.466	4.141	6.981	6.346	4.141	4.141	4.141	4.141

Table 11: Entry Subsidy (percentage change)

Table 12 shows the decomposed change in welfare U. Since we compare the policies that achieve the same level of growth in entry, the love of variety effect is the same. The smaller negative wage effect, larger positive networking effect, and corresponding spillover effect contribute to the lower negative price effects. As a result, the higher growth in welfare U is achieved in the baseline economy. To summarize, the entry subsidy policy can be more effective under the distorted network formation of young firms because the original entry level without the policy is inefficiently low in the environment.

	U	$LofV$	$Price$	$Wage$	$Networking$	$Spillover$
Baseline	2.343	2.500	-0.157	-0.638	0.575	-0.087
Wedge-Free	2.285	2.500	-0.215	-0.660	0.569	-0.118

Table 12: Entry Subsidy (Decomposition of the change in U)

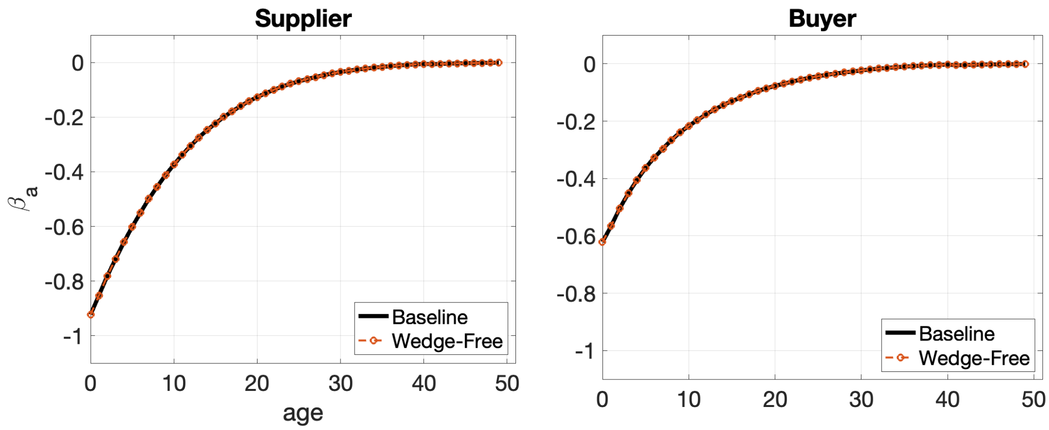


Figure 7: $\hat{\beta}_a$ under Entry Subsidy

5 Conclusion

This paper presents the network growth patterns of young firms and their macroeconomic implications. First, using fixed-effect regressions on a comprehensive firm-by-year dataset, which combines yearly transaction network data and firm survey panel data, we derive two main findings. The first one is that while the number of suppliers and customers eventually converges to a mature level over the firm's life cycle, young firms can only increase these connections slowly, even after accounting for typical age-dependent growth factors. This implies there are sources that prevent young firms from acquiring a sufficient number of partners immediately after entry, and these sources differ from the typical age-dependent components like productivity or financial slackness. The second one is that the churn rate of supplier and buyer relationships remains stable across the firm life cycle once the relationships are established. This suggests that the difficulty for young firms to reach a mature network size is not due to post-matching mechanisms such as high churn rates, but to pre-matching mechanisms related to the matching with potential partners.

Second, we develop a general equilibrium model that incorporates dynamic network formation decisions of heterogeneous firms. We suppose two sources prevent young firms from acquiring a sufficient number of partners immediately after entry. The first factor arises from convexity of advertisement cost. In order to advertise all at once when firms enter, they have to pay a higher marginal cost. The second factor is a hypothetical age-specific networking wedge that yields a gap between the original advertisement goods price and the actual price paid by firms. Although the dynamic network formation decision yields a curse of dimensionality issue due to the complexity of the network, a truncation approach allows us to properly define the value function of the firms and apply a simple method to solve it.

Finally, we calibrate the model to the estimated growth patterns of networks, and we derive several macroeconomic implications using the model. First, our model yields a fairly good match to the empirical findings. The R^2 measure for the network growth pattern (comparing the data and the calibrated model) is 0.99, implying that our model accounts for 99% of the empirically observed growth pattern. Next, we analyze the welfare impact of the distorted dynamic network formation. Comparing the baseline economy to a wedge-free economy where the age-specific networking wedge is removed, we observe a 2.4% welfare improvement in the wedge-free economy. In particular, our propositions on the decomposition of welfare impacts reveal that unconventional channels, such as changes in the network structure and their spillovers, can have a larger effect than conventional channels like entry and wage adjustments. Lastly, we conduct numerical analyses to evaluate the effects of different industry policies. The first policy experiment that promotes supplier/customer acquisition of young firms shows that supporting supplier acquisition of young firms is more effective than supporting buyer acquisition, which reflects differences in the efficiency of the newly created network structure. The second policy experiment that promotes entry shows that the policy can be more effective when network formation is distorted for young firms because the original entry level without the policy is inefficiently low in the environment.

This paper highlights the critical role of an age-specific wedge in shaping young firms' network formation and its macroeconomic impacts. Identifying the sources of this wedge remains a key challenge for future research, promising not only to enhance our understanding of firm dynamics and supply chain networks, but also to support policymakers in crafting effective industry policies.

References

Acemoglu, Daron, and Pablo D Azar. 2020. "Endogenous production networks." *Econometrica* 88 (1): 33–82.

Acemoglu, Daron, and Alireza Tahbaz-Salehi. 2020. "Firms, failures, and fluctuations: the macroeconomics of supply chain disruptions."Technical report, National Bureau of Economic Research.

Aekka, Anuraag, and Gaurav Khanna. 2024. "Endogenous Production Networks and Firm Dynamics."

Alfaro-Urena, Alonso, Isabela Manelici, and Jose P Vasquez. 2022. "The effects of joining multinational supply chains: New evidence from firm-to-firm linkages." *The Quarterly Journal of Economics* 137 (3): 1495–1552.

Allen, Treb, Costas Arkolakis, and Xiangliang Li. 2015. "On the existence and uniqueness of trade equilibria." *Manuscript, Yale Univ.*

Arkolakis, Costas, Federico Huneeus, and Yuhei Miyauchi. 2023. "Spatial production networks."Technical report, National Bureau of Economic Research.

Bacilieri, Andrea, András Borsos, Pablo Astudillo-Estevez, and François Lafond. 2023. "Firm-level production networks: what do we (really) know." *INET Oxford Working Paper* 2023.

Baqaee, David, and Emmanuel Farhi. 2020. "Entry vs. Rents: Aggregation with Economies of Scale." *NBER Working Paper* (w27140): .

Baqaee, David Rezza. 2018. "Cascading failures in production networks." *Econometrica* 86 (5): 1819–1838.

Baqaee, David Rezza, and Emmanuel Farhi. 2019. "The macroeconomic impact of microeconomic shocks: Beyond Hulten's theorem." *Econometrica* 87 (4): 1155–1203.

Baqaee, David, and Elisa Rubbo. 2023. "Micro propagation and macro aggregation." *Annual Review of Economics* 15 (1): 91–123.

Bernard, Andrew B, Emmanuel Dhyne, Glenn Magerman, Kalina Manova, and Andreas Moxnes. 2022. "The origins of firm heterogeneity: A production network approach." *Journal of Political Economy* 130 (7): 000–000.

Bernard, Andrew B, Andreas Moxnes, and Yukiko U Saito. 2019. "Production networks, geography, and firm performance." *Journal of Political Economy* 127 (2): 639–688.

Boehm, Johannes, Ruairidh South, Ezra Oberfield, and Mazhar Waseem. 2024. "The Network Origins of Firm Dynamics: Contracting Frictions and Dynamism with Long-Term Relationships."

Brunnermeier, Markus K, and Yuliy Sannikov. 2016. "The I theory of money."Technical report, National Bureau of Economic Research.

Carvalho, Vasco M, Makoto Nirei, Yukiko U Saito, and Alireza Tahbaz-Salehi. 2021. "Supply chain disruptions: Evidence from the great east japan earthquake." *The Quarterly Journal of Economics* 136 (2): 1255–1321.

Carvalho, Vasco M, and Alireza Tahbaz-Salehi. 2019. "Production networks: A primer." *Annual Review of Economics* 11 635–663.

Cavenaile, Laurent, and Pau Roldan-Blanco. 2021. "Advertising, innovation, and economic growth." *American Economic Journal: Macroeconomics* 13 (3): 251–303.

Decker, Ryan A, John Haltiwanger, Ron S Jarmin, and Javier Miranda. 2020. "Changing business dynamism and productivity: Shocks versus responsiveness." *American Economic Review* 110 (12): 3952–3990.

Dhyne, Emmanuel, Ayumu Ken Kikkawa, Magne Mogstad, and Felix Tintelnot. 2021. "Trade and domestic production networks." *The Review of Economic Studies* 88 (2): 643–668.

Diamond, Peter A. 1982. "Aggregate demand management in search equilibrium." *Journal of Political Economy* 90 (5): 881–894.

Eaton, Jonathan, Samuel S Kortum, and Francis Kramarz. 2022. "Firm-to-Firm Trade: Imports, exports, and the labor market."Technical report, National Bureau of Economic Research.

Elliott, Matthew, Benjamin Golub, and Matthew V Leduc. 2022. "Supply network formation and fragility." *American Economic Review* 112 (8): 2701–47.

Eslava, Marcela, John Haltiwanger, and Nicolas Urdaneta. 2023. "The Size and Life-Cycle Growth of Plants: The Role of Productivity, Demand, and Wedges." *The Review of Economic Studies* 259–300.

Foster, Lucia, John C Haltiwanger, and Cornell John Krizan. 2001. "Aggregate productivity growth: Lessons from microeconomic evidence." In *New developments in productivity analysis*, 303–372, University of Chicago Press.

Foster, Lucia, John Haltiwanger, and Chad Syverson. 2016. "The slow growth of new plants: Learning about demand?" *Economica* 83 (329): 91–129.

Fujii, Daisuke, Yukiko Umeno Saito, and Tatsuro Senga. 2017. *The dynamics of inter-firm networks and firm growth.* RIETI.

Haltiwanger, John, Ron S Jarmin, Robert Kulick, and Javier Miranda. 2016. "High growth young firms: contribution to job, output, and productivity growth." In *Measuring entrepreneurial businesses: Current knowledge and challenges*, 11–62, University of Chicago Press.

Haltiwanger, John, Ron S Jarmin, and Javier Miranda. 2013. "Who creates jobs? Small versus large versus young." *Review of Economics and Statistics* 95 (2): 347–361.

Hansen, Bruce. 2022. *Econometrics.* Princeton University Press.

Hopenhayn, Hugo A. 1992. "Entry, exit, and firm dynamics in long run equilibrium." *Econometrica: Journal of the Econometric Society* 1127–1150.

Hopenhayn, Hugo A. 2014. "Firms, misallocation, and aggregate productivity: A review." *Annual Review of Economics* 6 (1): 735–770.

Hsieh, Chang-Tai, and Peter J Klenow. 2009. "Misallocation and manufacturing TFP in China and India." *The Quarterly Journal of Economics* 124 (4): 1403–1448.

Huneeus, Federico. 2020. "Production Network Dynamics and the Propagation of Shocks." *Working Papers.*

Ignaszak, Marek, and Petr Sedlácek. 2023. "Customer acquisition, business dynamism and aggregate growth."Technical report, Working Paper.

Klette, Tor Jakob, and Samuel Kortum. 2004. "Innovating firms and aggregate innovation." *Journal of political economy* 112 (5): 986–1018.

Koike-Mori, Yasutaka, and Koki Okumura. 2024. "Production Networks and R&D Misallocation."

Kopytov, Alexandr, Kristoffer Nimark, Bineet Mishra, and Mathieu Taschereau-Dumouchel. 2022. "Endogenous Production Networks under Supply Chain Uncertainty."

Le Grand, François, and Xavier Ragot. 2022. "Refining the truncation method to solve heterogeneous-agent models." *Annals of Economics and Statistics* (146): 65–92.

Lim, Kevin. 2018. "Endogenous production networks and the business cycle." *Working Papers.*

Liu, Ernest. 2019. "Industrial policies in production networks." *The Quarterly Journal of Economics* 134 (4): 1883–1948.

Liu, Ernest, and Aleh Tsyvinski. 2024. "A Dynamic Model of Input–Output Networks." *Review of Economic Studies* 3608—-3644.

Luttmer, Erzo GJ. 2007. "Selection, growth, and the size distribution of firms." *The Quarterly Journal of Economics* 122 (3): 1103–1144.

Midrigan, Virgiliu, and Daniel Yi Xu. 2014. "Finance and misallocation: Evidence from plant-level data." *American Economic Review* 104 (2): 422–58.

Miyauchi, Yuhei. 2024. "Matching and Agglomeration: Theory and Evidence From Japanese Firm-to-Firm Trade." *Econometrica* 92 (6): 1869–1905.

Mortensen, Dale T. 1986. "Job search and labor market analysis." *Handbook of labor economics* 2 849–919.

Nakamura, Tsuyoshi, and Hiroshi Ohashi. 2022. "Japanese Firms' Markups and Firm-to-firm Transactions." Discussion papers 22083, Research Institute of Economy, Trade and Industry (RIETI), `https://ideas.repec.org/p/eti/dpaper/22083.html`.

Oberfield, Ezra. 2018. "A theory of input–output architecture." *Econometrica* 86 (2): 559–589.

Osotimehin, Sophie, and Latchezar Popov. 2023. "Misallocation and intersectoral linkages." *Review of Economic Dynamics* 51 177–198.

Pissarides, Christopher A. 1985. "Short-run equilibrium dynamics of unemployment, vacancies, and real wages." *The American Economic Review* 75 (4): 676–690.

Restuccia, Diego, and Richard Rogerson. 2017. "The causes and costs of misallocation." *Journal of Economic Perspectives* 31 (3): 151–74.

Sterk, Vincent, Petr Sedláček, and Benjamin Pugsley. 2021. "The nature of firm growth." *American Economic Review* 111 (2): 547–579.

Stokey, Nancy L. 1989. *Recursive methods in economic dynamics.* Harvard University Press.

Suto, Megumi, and Hitoshi Takehara. 2017. "CSR and cost of capital: evidence from Japan." *Social Responsibility Journal* 13 (4): 798–816.

Taschereau-Dumouchel, Mathieu. 2022. "Cascades and fluctuations in an economy with an endogenous production network." *Available at SSRN 3115854.*

Tintelnot, Felix, Ayumu Ken Kikkawa, Magne Mogstad, and Emmanuel Dhyne. 2018. "Trade and domestic production networks."Technical report, National Bureau of Economic Research.

A Data

A.1 Variable Construction

A.1.1 Inverse Mill's Ratio

To consider selection issue in the FE estimation, we include inverse Mill's ratio for our regression following Hansen (2022). We assume a selection model below,

$$\begin{aligned}
Y_{it}^* &= X_{it}'\beta + e_{it} \\
S_{it}^* &= Z_{it-1}'\gamma + u_{it} \\
S_{i,t} &= 1\{S_{it}^* > 0\} \\
Y_{it} &= \begin{cases} Y_{it}^* & if\ S_{it} > 0 \\ missing & if\ S_{it} = 0 \end{cases} \\
\begin{pmatrix} e \\ u \end{pmatrix} &\sim \left(0, \begin{pmatrix} \sigma^2 & \sigma_{21} \\ \sigma_{21} & 1 \end{pmatrix}\right)
\end{aligned}$$

with $Z = \{$Leverage, Labor productivity, Firm age, Industry (2-digit), Year, Prefecture$\}$. Then, using the inverse Mills ratio $\lambda(x)$, we obtain

$$E[Y_{i,t}|X_{i,t}, Z_{i,t}, S_{i,t} = 1] = X_{it}'\beta + \sigma_{21}\lambda(Z_{it}'\gamma). \tag{A.1}$$

Hence, by running regression with the constructed inverse Mills ratio $\hat{\lambda}_{it} = \hat{\lambda}(Z_{it}'\hat{\gamma})$ using the observed sample ($S_{it} = 1$), we can control the selection issues. The detail is shown in Hansen (2022).

A.2 Summary Stats

	count	mean	sd	p50
Young				
employee	1484874	6655.054	592407.709	4
sales	5125881	795876.177	39350398.689	69104
number of buyer	2309366	4.462	43.079	3
number of supplier	2248193	4.063	19.896	2
age	5233641	10.549	5.346	11
Old				
employee	3058463	19753.461	452436.798	9
sales	9623011	1721322.729	48069623.245	120000
number of buyer	6422389	7.986	42.536	4
number of supplier	7044355	7.166	44.583	3
age	9734155	38.502	13.909	36
Total				
employee	4543337	15472.580	502527.728	7
sales	14748892	1399689.830	45232433.493	99941
number of buyer	8731755	7.054	42.709	3
number of supplier	9292548	6.416	40.054	3
age	14967796	28.728	17.706	27

Table 13: Summary Statistics

	count	mean	sd	p50
Young				
employee (log)	1301260	1.866	1.415	1.609
sales (log)	5125881	10.877	2.367	11.143
number of buyer (log)	2309366	0.983	0.823	1.099
number of supplier (log)	2248193	0.892	0.827	0.693
age (log)	5233641	2.299	0.600	2.485
Old				
employee (log)	2912746	2.608	1.790	2.303
sales (log)	9623011	11.823	1.844	11.695
number of buyer (log)	6422389	1.314	1.001	1.386
number of supplier (log)	7044355	1.258	0.966	1.099
age (log)	9734155	3.618	0.341	3.611
Total				
employee (log)	4214006	2.379	1.718	2.079
sales (log)	14748892	11.494	2.090	11.512
number of buyer (log)	8731755	1.227	0.968	1.099
number of supplier (log)	9292548	1.169	0.948	1.099
age (log)	14967796	3.157	0.772	3.332

Table 14: Summary Statistics (log)

	count	mean	sd	p50
Young				
RLP	1301260	38622.781	173724.767	18921.000
LP	1301260	3741.336	21290.612	2091.333
leverage	1484849	9.461	1011.451	1.263
netDE	1484752	2.552	329.746	-0.034
LP (log)	1120579	7.650	1.441	7.884
RLP (log)	1295686	9.874	1.141	9.853
LP (growth)	721720	0.033	1.083	0.025
RLP (growth)	889490	0.022	0.530	0.012
leverage (growth)	789721	-0.033	0.584	-0.033
netDE (growth)	892717	-5.154	3680.716	-0.132
Old				
RLP	2912745	37239.534	228436.371	20367.000
LP	2912745	3230.063	20896.667	1958.364
leverage	3058437	7.064	2292.903	0.968
netDE	3058369	1.743	249.888	-0.073
LP (log)	2542834	7.543	1.449	7.782
RLP (log)	2909085	9.910	1.148	9.923
LP (growth)	1918701	0.019	1.005	0.012
RLP (growth)	2325202	-0.010	0.438	-0.003
leverage (growth)	2015153	-0.031	0.402	-0.024
netDE (growth)	2353037	-0.184	263.949	-0.060
Total				
RLP	4214005	37666.672	213047.447	19930.176
LP	4214005	3387.941	21020.427	1996.000
leverage	4543286	7.847	1968.125	1.043
netDE	4543121	2.008	278.517	-0.064
LP (log)	3663413	7.576	1.447	7.812
RLP (log)	4204771	9.899	1.146	9.903
LP (growth)	2640421	0.023	1.027	0.015
RLP (growth)	3214692	-0.001	0.466	0.000
leverage (growth)	2804874	-0.031	0.461	-0.026
netDE (growth)	3245754	-1.551	1943.368	-0.074

Table 15: Summary Statistics: Control Variables

B Estimation

B.1 Estimation Results (Detail)

	(1) LogDegree^S	(2) LogDegree^B
age=0	-0.680***	-0.481***
age=1	-0.710***	-0.510***
age=2	-0.636***	-0.458***
age=3	-0.556***	-0.403***
age=4	-0.489***	-0.352***
age=5	-0.433***	-0.306***
age=6	-0.384***	-0.272***
age=7	-0.336***	-0.241***
age=8	-0.303***	-0.213***
age=9	-0.269***	-0.186***
age=10	-0.243***	-0.169***
age=11	-0.215***	-0.153***
age=12	-0.188***	-0.132***
age=13	-0.168***	-0.120***
age=14	-0.150***	-0.110***
age=15	-0.134***	-0.096***
age=16	-0.116***	-0.086***
age=17	-0.100***	-0.075***
age=18	-0.088***	-0.072***
age=19	-0.080***	-0.066***
age=20	-0.072***	-0.064***
age=21	-0.068***	-0.060***
age=22	-0.060***	-0.054***
age=23	-0.055***	-0.052***
age=24	-0.047***	-0.049***
age=25	-0.041***	-0.043***
age=26	-0.038***	-0.044***
age=27	-0.036***	-0.044***
age=28	-0.028***	-0.041***
age=29	-0.023***	-0.037***
age=30	-0.021***	-0.034***
age=31	-0.016***	-0.032***
age=32	-0.014***	-0.032***
age=33	-0.010***	-0.027***
age=34	-0.007**	-0.026***
age=35	-0.006**	-0.021***
age=36	-0.007**	-0.019***
age=37	-0.005*	-0.019***
age=38	-0.002	-0.016***
age=39	-0.001	-0.016***
age=40	-0.000	-0.016***
age=41	-0.001	-0.014***
age=42	-0.000	-0.012***
age=43	0.001	-0.011***
age=44	0.002	-0.010***
age=45	-0.000	-0.009***
age=46	0.001	-0.005**
age=47	0.001	-0.005**
age=48	0.000	-0.005**
age=49	0.001	-0.003
leverage	0.250***	0.188***
LP	0.016***	0.012***
imr	29.110***	21.906***
Observations	2718383	2826112
R^2	0.938	0.907

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 16: Regression Result (detail): Number of Networks and Age

	(1)	(2)	(3)	(4)	(5)	(6)
	NetGrowth^S	GrossGrowth^S	GrossDepreciation^S	NetGrowth^B	GrossGrowth^B	GrossDepreciation^B
age=1	0.459***	0.466***	-0.044***	0.383***	0.386***	-0.052***
age=2	0.500***	0.504***	-0.041***	0.417***	0.456***	-0.015***
age=3	0.360***	0.366***	-0.035***	0.336***	0.380***	-0.005
age=4	0.298***	0.307***	-0.027***	0.284***	0.329***	-0.002
age=5	0.253***	0.262***	-0.029***	0.255***	0.297***	-0.002
age=6	0.216***	0.226***	-0.026***	0.221***	0.265***	0.002
age=7	0.195***	0.205***	-0.024***	0.196***	0.238***	-0.001
age=8	0.177***	0.188***	-0.023***	0.179***	0.223***	0.002
age=9	0.154***	0.163***	-0.025***	0.157***	0.196***	-0.001
age=10	0.140***	0.152***	-0.022***	0.146***	0.184***	-0.001
age=11	0.133***	0.142***	-0.024***	0.136***	0.173***	0.000
age=12	0.119***	0.129***	-0.022***	0.131***	0.165***	-0.003
age=13	0.111***	0.121***	-0.021***	0.121***	0.154***	-0.001
age=14	0.100***	0.110***	-0.021***	0.111***	0.141***	-0.004
age=15	0.091***	0.100***	-0.022***	0.102***	0.133***	-0.002
age=16	0.085***	0.095***	-0.019***	0.095***	0.124***	-0.002
age=17	0.079***	0.088***	-0.020***	0.087***	0.116***	-0.002
age=18	0.076***	0.085***	-0.019***	0.080***	0.107***	-0.002
age=19	0.068***	0.077***	-0.019***	0.075***	0.100***	-0.004
age=20	0.064***	0.072***	-0.018***	0.071***	0.096***	-0.003
age=21	0.060***	0.068***	-0.017***	0.064***	0.089***	-0.002
age=22	0.056***	0.064***	-0.017***	0.064***	0.086***	-0.003
age=23	0.048***	0.055***	-0.017***	0.057***	0.079***	-0.003
age=24	0.050***	0.057***	-0.016***	0.052***	0.072***	-0.003
age=25	0.045***	0.052***	-0.016***	0.051***	0.071***	-0.003
age=26	0.044***	0.049***	-0.016***	0.046***	0.066***	-0.003
age=27	0.036***	0.042***	-0.015***	0.043***	0.061***	-0.003
age=28	0.038***	0.042***	-0.016***	0.040***	0.056***	-0.004*
age=29	0.035***	0.039***	-0.014***	0.037***	0.051***	-0.005**
age=30	0.029***	0.035***	-0.013***	0.035***	0.051***	-0.002
age=31	0.026***	0.031***	-0.013***	0.030***	0.044***	-0.004*
age=32	0.024***	0.029***	-0.012***	0.030***	0.044***	-0.002
age=33	0.023***	0.027***	-0.012***	0.028***	0.040***	-0.004**
age=34	0.022***	0.026***	-0.010***	0.023***	0.034***	-0.005**
age=35	0.018***	0.022***	-0.011***	0.024***	0.033***	-0.005**
age=36	0.017***	0.023***	-0.009***	0.020***	0.030***	-0.003*
age=37	0.015***	0.019***	-0.009***	0.017***	0.026***	-0.004**
age=38	0.016***	0.019***	-0.009***	0.018***	0.027***	-0.002
age=39	0.012***	0.015***	-0.008***	0.014***	0.020***	-0.006***
age=40	0.011***	0.015***	-0.007***	0.016***	0.022***	-0.004***
age=41	0.009***	0.013***	-0.006***	0.013***	0.018***	-0.003**
age=42	0.008***	0.010***	-0.006***	0.012***	0.017***	-0.003**
age=43	0.007***	0.010***	-0.005***	0.008**	0.012***	-0.003*
age=44	0.005*	0.007***	-0.005***	0.007**	0.012***	-0.002
age=45	0.003	0.007***	-0.003***	0.008**	0.013***	-0.001
age=46	0.003	0.005**	-0.003***	0.005*	0.009***	-0.001
age=47	0.004*	0.006***	-0.003***	0.003	0.009***	0.002
age=48	0.004*	0.006***	-0.002**	0.003	0.006**	-0.000
age=49	0.002	0.002	-0.003***	0.004	0.005*	-0.001
leverage	0.010***	0.008***	-0.009***	-0.006**	-0.013***	-0.012***
LP	0.003***	0.002***	-0.001***	0.001***	0.000	-0.001***
imr	1.246***	0.951***	-0.993***	-0.632**	-1.461***	-1.406***
Observations	2425776	2425776	2465580	2510808	2510808	2550151
R^2	0.177	0.227	0.306	0.155	0.219	0.319

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 17: Regression Result (detail): Growth of Networks and Age

B.2 Robustness Check

In this section, we run a series of robustness checks for the key facts observed in Section 2.3. Figure 8 shows the estimation results after dropping firms in financial industries. (4.0% of the sample is dropped.) Figure 8 shows the estimation result when we do not count the links with capital relationships. (0.8% of the links become uncounted.) Figure 10 shows the results controlled by net DE ratio instead of leverage. Figure 11 shows the results controlled by RLP (defined by revenue divided by employee) instead of LP following Bernard et al. (2019). All of the results are quite similar, and that supports the hypothesis that even without productivity growth or loosening of financial constraints of young firms, their networks measures show typical age-speficic patterns.

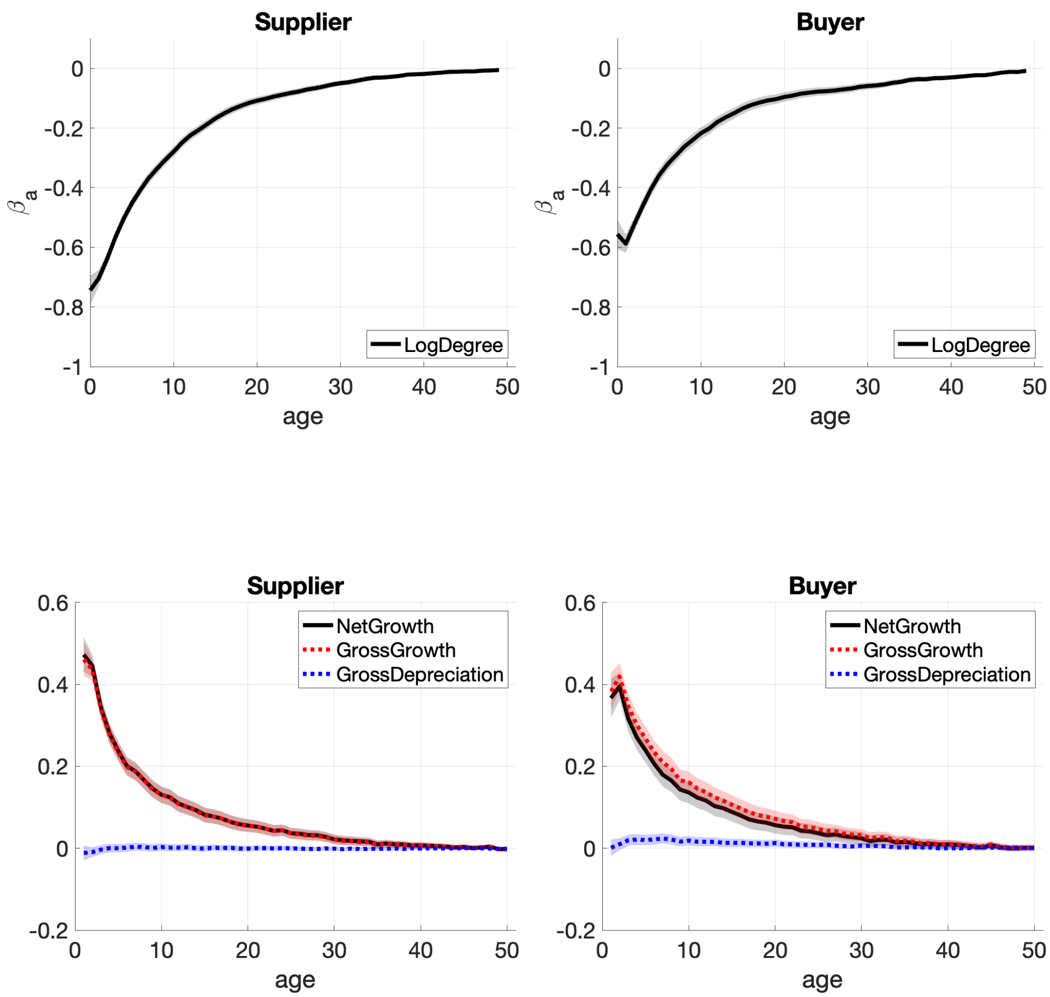
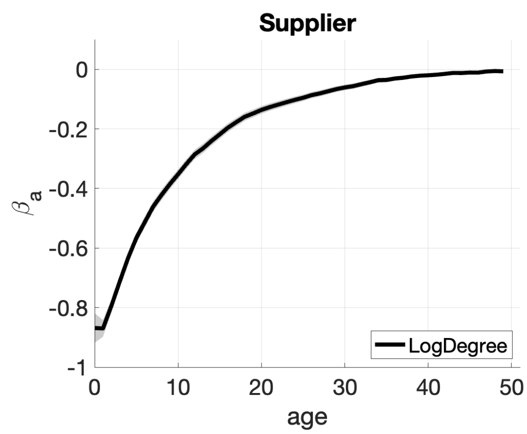


Figure 8: Networking Measures and Firm Aging without Firms in Financial Sectors

Notes: Estimation results for panel regression expressed in equation (2.7) without firms in financial sectors.



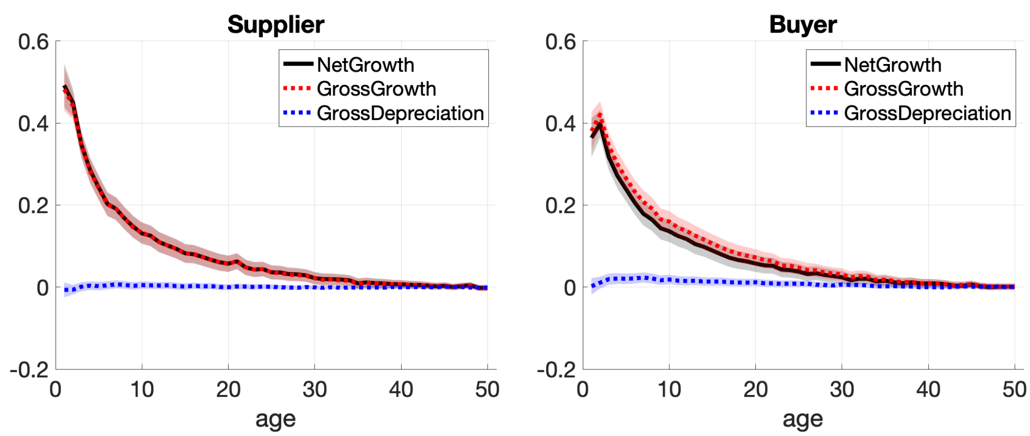


Figure 9: Networking Measures and Firm Aging with Counting Links Backed up by Capital-relationship

Notes: Estimation results for panel regression expressed in equation (2.7) with counting links backed up by capital-relationship.

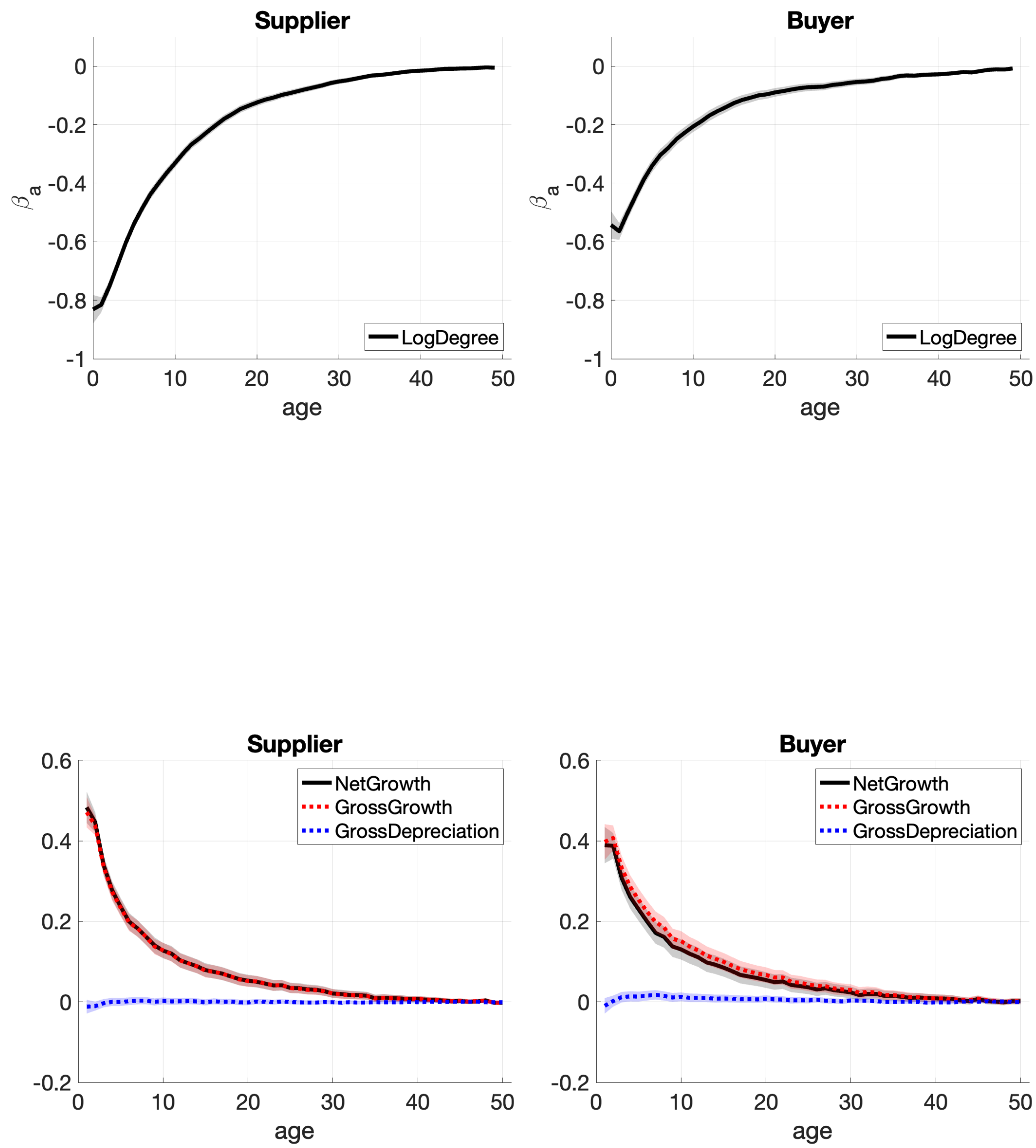
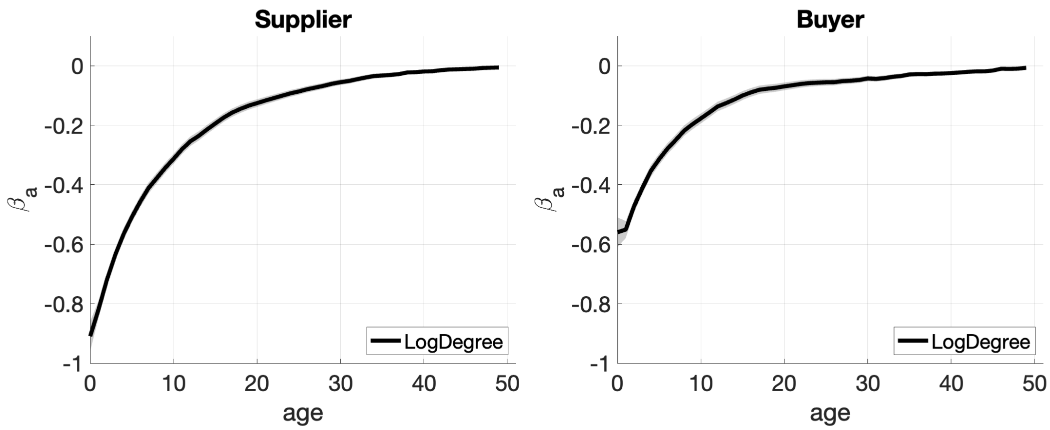


Figure 10: Networking Measures and Firm Aging Controlled by Net DE Ratio

Notes: Estimation results for panel regression expressed in equation (2.7) controlled by net DE ratio instead of leverage.



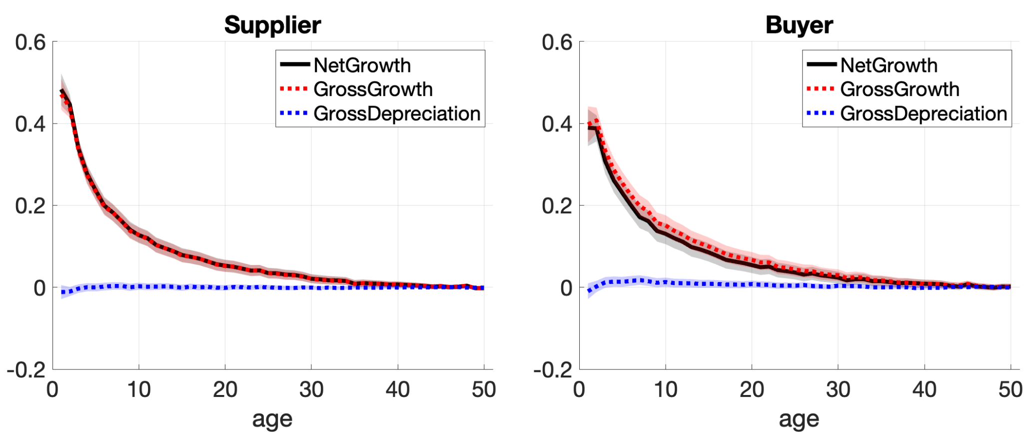


Figure 11: Networking Measures and Firm Aging Controlled by RLP

Notes: Estimation results for panel regression expressed in equation (2.7) controlled by RLP instead of LP.

C Derivation

C.1 Derivation of profit function

Profit of χ-firm is

$$\pi(\chi) = (\mu-1)\eta(\chi)X(\chi) \tag{C.1}$$

$$= (\mu-1)\eta(\chi)(p(\chi))^{-\sigma}\left(\Delta + \int \left(\frac{1}{P(\chi')}\right)^{-\sigma}(1-\alpha)\frac{\eta(\chi')}{P(\chi')}X(\chi')k^B(\chi,\chi')d\chi'\right) \tag{C.2}$$

$$= (\mu-1)\eta(\chi)(\mu\eta(\chi))^{-\sigma}\left(\Delta + \int \left(\frac{1}{P(\chi')}\right)^{-\sigma}(1-\alpha)\frac{\eta(\chi')}{P(\chi')}X(\chi')k^B(\chi,\chi')d\chi'\right) \tag{C.3}$$

$$= (\mu-1)\mu^{-\sigma}\eta(\chi)^{1-\sigma}\left(\Delta + \int \left(\frac{1}{P(\chi')}\right)^{-\sigma}(1-\alpha)\frac{\eta(\chi')}{P(\chi')}X(\chi')k^B(\chi,\chi')d\chi'\right) \tag{C.4}$$

$$\begin{aligned}= (\mu-1)\mu^{-\sigma}\left(\frac{1}{\phi}\right)^{1-\sigma}&\left[\int (p(\chi'))^{1-\sigma}k^S(\chi,\chi')d\chi'\right]^{1-\alpha}\\ &\times\left(\Delta + \int \left(\frac{1}{P(\chi')}\right)^{-\sigma}(1-\alpha)\frac{\eta(\chi')}{P(\chi')}X(\chi')k^B(\chi,\chi')d\chi'\right)\end{aligned} \tag{C.5}$$

$$\begin{aligned}= (\mu-1)\mu^{-\sigma+(1-\sigma)(1-\alpha)}\left(\frac{1}{\phi}\right)^{1-\sigma}&\left[\int (\eta(\chi'))^{1-\sigma}k^S(\chi,\chi')d\chi'\right]^{1-\alpha}\\ &\times\left(\Delta + \int \left(\frac{1}{P(\chi')}\right)^{-\sigma}(1-\alpha)\frac{\eta(\chi')}{P(\chi')}X(\chi')k^B(\chi,\chi')d\chi'\right).\end{aligned} \tag{C.6}$$

Remember that the χ-firm can control $k^S(\chi,\chi')$ and $k^B(\chi,\chi')$ via advertisement cost n^S and n^B following (3.32) and (3.33). Hence, we obtain

$$\begin{aligned}\pi(\phi,\mathbf{n}^a) =&(\mu-1)\mu^{-\sigma+(1-\sigma)(1-\alpha)}\left(\frac{1}{\phi}\right)^{1-\sigma}\\ &\times\left[\int (\eta(\chi'))^{1-\sigma}\sum_{\tau=0}^{Al} n^S_{a-\tau}m^S\delta^\tau g^\tau_S(\chi')d\chi'\right]^{1-\alpha}\\ &\times\left(\Delta + \int \left(\frac{1}{P(\chi')}\right)^{-\sigma}(1-\alpha)\frac{\eta(\chi')}{P(\chi')}X(\chi')\sum_{\tau=0}^{Al} n^B_{a-\tau}m^B\delta^\tau g^\tau_B(\chi')d\chi'\right).\end{aligned} \tag{C.7}$$

For readability, define a constant coefficient and two weighting variables that are determined endogenously in equilibrium as follows.

$$C = (\mu-1)\mu^{-\sigma+(1-\sigma)(1-\alpha)} \tag{C.8}$$

$$w^S_\tau = \delta^\tau\int_{\chi'}(\eta(\chi'))^{1-\sigma}g^\tau_S(\chi')d\chi' \tag{C.9}$$

$$w^B_\tau = \delta^\tau(1-\alpha)\int_{\chi'}\left(\frac{1}{P(\chi')}\right)^{-\sigma}\frac{\eta(\chi')}{P(\chi')}X(\chi')g^\tau_B(\chi')d\chi' \tag{C.10}$$

Note that all the terms do not include $\mathbf{n}^a$ nor ϕ, so all of them are exogenously given in the decision making process of firms.

Using these expressions, we obtain

$$\pi(\phi, \mathbf{n}^a) = C\phi^{\sigma-1}\left[m_s \sum_{\tau=0}^{Al} w_\tau^S n_{a-\tau}^S\right]^{1-\alpha} \times \left[\Delta_H + m_B \sum_{\tau=0}^{Al} w_\tau^B n_{a-\tau}^B\right]. \tag{C.11}$$

C.2 Decomposition

Talking log of (3.35) for both sides, we obtain

$$\log(\Phi(\chi)) = (1-\sigma)\log\left(\frac{1}{\phi}\right) + \alpha(1-\sigma)\log w + (1-\alpha)\log\mu + (1-\alpha)\log\left[\int \Phi(\chi')k^S(\chi,\chi')d\chi'\right]^{1-\alpha}. \tag{C.12}$$

Total differentiation on both sides yield

$$\frac{d\Phi(\chi)}{\Phi(\chi)} = \alpha(1-\sigma)\frac{dw}{w} + (1-\alpha)\frac{\int d\Phi(\chi')k^S(\chi,\chi') + \Phi(\chi')dk^S(\chi,\chi')d\chi'}{\int \Phi(\chi')k^S(\chi,\chi')d\chi'} \tag{C.13}$$

$$= \alpha(1-\sigma)\frac{dw}{w} + (1-\alpha)\frac{\int \left(\frac{d\Phi(\chi')}{\Phi(\chi')} + \frac{dk^S(\chi,\chi')}{k^S(\chi,\chi')}\right)\Phi(\chi')k^S(\chi,\chi')d\chi'}{\int \Phi(\chi')k^S} \tag{C.14}$$

$$= \alpha(1-\sigma)\frac{dw}{w} + (1-\alpha)\int W^S(\chi,\chi')\left(\frac{d\Phi(\chi')}{\Phi(\chi')} + \frac{dk^S(\chi,\chi')}{k^S(\chi,\chi')}\right)d\chi' \tag{C.15}$$

where $W^S(\chi,\chi') := \dfrac{\int \Phi(\chi')k^S(\chi,\chi')d\chi'}{\int \Phi(\chi')k^S}$ is network matrix weighted by transaction value.

In a linear form, we obtain

$$\frac{d\mathbf{\Phi}}{\mathbf{\Phi}} = \alpha(1-\sigma)\frac{dw}{w} + (1-\alpha)\left(\boldsymbol{W}^S \circ \frac{d\boldsymbol{K}^S}{\boldsymbol{K}^S}\right)\mathbb{1} + (1-\alpha)\boldsymbol{W}^S\left(\frac{d\mathbf{\Phi}}{\mathbf{\Phi}}\right). \tag{C.16}$$

Here, we suppose the state space is indexed by its age in a ascending manner WOLG. Then, for a generic matrices $\boldsymbol{A}$ and a generic vector $\boldsymbol{x}$, we can have a decomposed expression as follows.

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{A}_{y,y} & \boldsymbol{A}_{y,o} \\ \boldsymbol{A}_{o,y} & \boldsymbol{A}_{o,o} \end{pmatrix} \tag{C.17}$$

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_y \\ \boldsymbol{x}_o \end{pmatrix} \tag{C.18}$$

where $\boldsymbol{A}_{y,y}, \boldsymbol{A}_{o,y}, \boldsymbol{A}_{y,o}, \boldsymbol{A}_{o,o}$ correspond to discretized expression of a bivariate function $A(\chi,\chi')$ with $a < 20 \wedge a' < 20$, $a \geq 20 \wedge a' < 20$, $a < 20 \wedge a' \geq 20$, and $a \geq 20 \wedge a' \geq 20$, respectively, and $\boldsymbol{x}_y$ and $\boldsymbol{x}_o$ correspond to a univariate function $x(\chi)$ with $a < 20$ and $a' < 20$, respectively.

Using this expression, (C.16) becomes

$$\begin{pmatrix} \frac{d\mathbf{\Phi}_y}{\mathbf{\Phi}_y} \\ \frac{d\mathbf{\Phi}_o}{\mathbf{\Phi}_o} \end{pmatrix} = \alpha(1-\sigma)\frac{dw}{w} + (1-\alpha)\left(\begin{pmatrix} \boldsymbol{W}^S_{y,y} & \boldsymbol{W}^S_{y,o} \\ \boldsymbol{W}^S_{o,y} & \boldsymbol{W}^S_{o,o} \end{pmatrix} \circ \begin{pmatrix} \frac{d\boldsymbol{K}^S_{y,y}}{\boldsymbol{K}^S_{y,y}} & \frac{d\boldsymbol{K}^S_{y,o}}{\boldsymbol{K}^S_{y,o}} \\ \frac{d\boldsymbol{K}^S_{o,y}}{\boldsymbol{K}^S_{o,y}} & \frac{d\boldsymbol{K}^S_{o,o}}{\boldsymbol{K}^S_{o,o}} \end{pmatrix}\right)\mathbb{1}$$

$$+ (1-\alpha)\begin{pmatrix} \boldsymbol{W}^S_{y,y} & \boldsymbol{W}^S_{y,o} \\ \boldsymbol{W}^S_{o,y} & \boldsymbol{W}^S_{o,o} \end{pmatrix}\begin{pmatrix} \frac{d\mathbf{\Phi}_y}{\mathbf{\Phi}_y} \\ \frac{d\mathbf{\Phi}_o}{\mathbf{\Phi}_o} \end{pmatrix}. \tag{C.19}$$

Substituting $\Phi(\chi) = \eta(\chi)^{1-\sigma}$ into (3.4) yields

$$P^H = \mu\left[\int \Phi(\chi)dF(\chi)\right]^{\frac{1}{1-\sigma}} \tag{C.20}$$

$$= \mu\left[\int \Phi(\chi)dE(\chi)\right]^{\frac{1}{1-\sigma}} M_e^{\frac{1}{1-\sigma}} \tag{C.21}$$

where $E(a,\phi) = \phi^a g(\phi)$. Taking log for both sides, we obtain

$$\log P^H = \log\mu + \frac{1}{1-\sigma}\log\left[\int \Phi(\chi)dE(\chi)\right] + \frac{1}{1-\sigma}\log M_e \tag{C.22}$$

Total differentiation on both sides yield

$$\frac{dP^H}{P^H} = \frac{1}{1-\sigma}\frac{dM_e}{M_e} + \frac{1}{1-\sigma}\frac{\int d\Phi(\chi)dE(\chi)}{\int \Phi(\chi)dE(\chi)} \tag{C.23}$$

$$= \frac{1}{1-\sigma}\frac{dM_e}{M_e} + \frac{1}{1-\sigma}\frac{\int \frac{d\Phi(\chi)}{\Phi(\chi)}\Phi(\chi)dE(\chi)}{\int \Phi(\chi)dE(\chi)} \tag{C.24}$$

$$= \frac{1}{1-\sigma}\frac{dM_e}{M_e} + \frac{1}{1-\sigma}\int w^E(\chi)\frac{d\Phi(\chi)}{\Phi(\chi)}d\chi \tag{C.25}$$

where $w^E(\chi) \coloneqq \dfrac{\Phi(\chi)e(\chi)}{\int \Phi(\chi)dE(\chi)}$ is the expenditure of household on χ-firm goods.

In a linear form, we obtain

$$\frac{dP_H}{P_H} = \frac{1}{1-\sigma}\frac{dM_e}{M_e} + \frac{1}{1-\sigma}\boldsymbol{w}^{E'}\frac{d\mathbf{\Phi}}{\mathbf{\Phi}} \tag{C.26}$$

Furthermore, we obtain

$$\frac{dP_H}{P_H} = \frac{1}{1-\sigma}\frac{dM_e}{M_e} + \frac{1}{1-\sigma}(\boldsymbol{w}_y^{E'}\boldsymbol{w}_o^{E'})\begin{pmatrix}\frac{d\boldsymbol{\Phi}_y}{\boldsymbol{\Phi}_y}\\ \frac{d\boldsymbol{\Phi}_o}{\boldsymbol{\Phi}_o}\end{pmatrix} \tag{C.27}$$

$$= \frac{1}{1-\sigma}\frac{dM_e}{M_e} + \alpha\frac{dw}{w}$$
$$+ \frac{1-\alpha}{1-\sigma}(\boldsymbol{w}_y^{E'}\boldsymbol{w}_o^{E'})\left\{\begin{pmatrix}\boldsymbol{W}_{y,y}^S \circ \frac{d\boldsymbol{K}_{y,y}^S}{\boldsymbol{K}_{y,y}^S} & \boldsymbol{W}_{y,o}^S \circ \frac{d\boldsymbol{K}_{y,o}^S}{\boldsymbol{K}_{y,o}^S}\\ \boldsymbol{W}_{o,y}^S \circ \frac{d\boldsymbol{K}_{o,y}^S}{\boldsymbol{K}_{o,y}^S} & \boldsymbol{W}_{o,o}^S \circ \frac{d\boldsymbol{K}_{o,o}^S}{\boldsymbol{K}_{o,o}^S}\end{pmatrix}\mathbb{1} + \begin{pmatrix}\boldsymbol{W}_{y,y}^S & \boldsymbol{W}_{y,o}^S\\ \boldsymbol{W}_{o,y}^S & \boldsymbol{W}_{o,o}^S\end{pmatrix}\begin{pmatrix}\frac{d\boldsymbol{\Phi}_y}{\boldsymbol{\Phi}_y}\\ \frac{d\boldsymbol{\Phi}_o}{\boldsymbol{\Phi}_o}\end{pmatrix}\right\} \tag{C.28}$$

$$= \underbrace{\frac{1}{1-\sigma}\frac{dM_e}{M_e}}_{\text{LofV Effect}} + \underbrace{\alpha\frac{dw}{w}}_{\text{Wage Effect}}$$
$$+ \underbrace{\frac{1-\alpha}{1-\sigma}(\boldsymbol{w}_y^{E'}\boldsymbol{w}_o^{E'})\left(\begin{pmatrix}\boldsymbol{W}_{y,y}^S \circ \frac{d\boldsymbol{K}_{y,y}^S}{\boldsymbol{K}_{y,y}^S} & \boldsymbol{W}_{y,o}^S \circ \frac{d\boldsymbol{K}_{y,o}^S}{\boldsymbol{K}_{y,o}^S}\\ \boldsymbol{W}_{o,y}^S \circ \frac{d\boldsymbol{K}_{o,y}^S}{\boldsymbol{K}_{o,y}^S} & \boldsymbol{W}_{o,o}^S \circ \frac{d\boldsymbol{K}_{o,o}^S}{\boldsymbol{K}_{o,o}^S}\end{pmatrix}\mathbb{1}\right)}_{\text{Networking Effect}}$$
$$+ \underbrace{\frac{1-\alpha}{1-\sigma}(\boldsymbol{w}_y^{E'}\boldsymbol{w}_o^{E'})\begin{pmatrix}\boldsymbol{W}_{y,y}^S & \boldsymbol{W}_{y,o}^S\\ \boldsymbol{W}_{o,y}^S & \boldsymbol{W}_{o,o}^S\end{pmatrix}\begin{pmatrix}\frac{d\boldsymbol{\Phi}_y}{\boldsymbol{\Phi}_y}\\ \frac{d\boldsymbol{\Phi}_o}{\boldsymbol{\Phi}_o}\end{pmatrix}}_{\text{Spillover Effect}} \tag{C.29}$$

Since the income is numeraire, we also obtain

$$\frac{dU}{U} = \underbrace{\frac{1}{\sigma-1}\frac{dM_e}{M_e}}_{\text{LofV Effect}} - \underbrace{\alpha\frac{dw}{w}}_{\text{Wage Effect}}$$
$$+ \underbrace{\frac{1-\alpha}{\sigma-1}(\boldsymbol{w}_y^{E'}\boldsymbol{w}_o^{E'})\left(\begin{pmatrix}\boldsymbol{W}_{y,y}^S \circ \frac{d\boldsymbol{K}_{y,y}^S}{\boldsymbol{K}_{y,y}^S} & \boldsymbol{W}_{y,o}^S \circ \frac{d\boldsymbol{K}_{y,o}^S}{\boldsymbol{K}_{y,o}^S}\\ \boldsymbol{W}_{o,y}^S \circ \frac{d\boldsymbol{K}_{o,y}^S}{\boldsymbol{K}_{o,y}^S} & \boldsymbol{W}_{o,o}^S \circ \frac{d\boldsymbol{K}_{o,o}^S}{\boldsymbol{K}_{o,o}^S}\end{pmatrix}\mathbb{1}\right)}_{\text{Networking Effect}}$$
$$+ \underbrace{\frac{1-\alpha}{\sigma-1}(\boldsymbol{w}_y^{E'}\boldsymbol{w}_o^{E'})\begin{pmatrix}\boldsymbol{W}_{y,y}^S & \boldsymbol{W}_{y,o}^S\\ \boldsymbol{W}_{o,y}^S & \boldsymbol{W}_{o,o}^S\end{pmatrix}\begin{pmatrix}\frac{d\boldsymbol{\Phi}_y}{\boldsymbol{\Phi}_y}\\ \frac{d\boldsymbol{\Phi}_o}{\boldsymbol{\Phi}_o}\end{pmatrix}}_{\text{Spillover Effect}} \tag{C.30}$$

C.3 Firm Level Steady State

Since there is no shock on fundamental productivity of each firm after its entry, the number of partners it has approaches its steady state optimal level as in the standard model of capital accumulation like Midrigan and Xu (2014) as a firm ages. In this section, we show how to derive it. While it does not have a closed form solution, but we can solve it computationally easily and utilize this characteristics to make computation algorithms efficient as discussed in D.1.

When a firm with productivity ϕ is in its ss (where the firm's state variables are constant across periods as $n^S = n_a^S$ and $n^B = n_a^B$), the F.O.C. w.r.t. n_a^S must satisfy

$$\sum_{\tau=0}^{Al}(\beta\varphi)^{\tau}C\phi^{\sigma-1}(1-\alpha)m_s w_\tau^s (K^s)^{-\alpha}(\Delta + K^B) = f_s(n_a^s)^{\gamma^s-1} \tag{C.31}$$

where

$$K^S := m_s \sum_{\tau'=0}^{Al} n^S w_{\tau'}^s = m_s W^s n^S \tag{C.32}$$

$$K^B := m_b \sum_{\tau'=0}^{Al} n^B w_{\tau'}^b = m_b W^b n^B \tag{C.33}$$

with $W^s = \sum_{\tau'=0}^{Al} w_{\tau'}^s$ and $W^b = \sum_{\tau'=0}^{Al} w_{\tau'}^b$, and

$$k^S := m_s \sum_{\tau'=0}^{Al} (\varphi\delta)\tau' n^S = m_s X n^S \tag{C.34}$$

$$k^B := m_b \sum_{\tau'=0}^{Al} (\varphi\delta)\tau' n^B = m_b X n^B \tag{C.35}$$

with $X = \sum_{\tau'=0}^{Al}(\varphi\delta)\tau' = \dfrac{1-(\varphi\delta)^{Al+1}}{1-\varphi\delta}$.

F.O.C. w.r.t. n_a^b must satisfy

$$\sum_{\tau=0}^{Al}(\beta\varphi)^{\tau}C\phi^{\sigma-1}(K^S)^{1-\alpha}m_b w_\tau^b = f_b(n_a^b(\chi))^{\gamma^b-1} \tag{C.36}$$

$$\tag{C.37}$$

Since $n^S = n_a^S$ and $n^B = n_a^B$ hold in a firm-lvel ss, we obtain

$$\sum_{\tau=0}^{Al}(\beta\varphi)^{\tau}C\phi^{\sigma-1}(1-\alpha)m_s w_\tau^s m_s^{-\alpha}(n^S)^{\alpha}\left(\sum_{\tau=0}^{Al} w_\tau^s\right)^{-\alpha}\left(\Delta + m_b n^B \sum_{\tau=0}^{Al} w_\tau^b\right) = f_s(n^S)^{\gamma^s-1} \tag{C.38}$$

$$\sum_{\tau=0}^{Al}(\beta\varphi)^{\tau}C\phi^{\sigma-1}(n^S)^{1-\alpha}\left(\sum_{\tau=0}^{Al} w_\tau^s\right)^{1-\alpha} m_b w_\tau^b = f_b(n^B)^{\gamma^b-1}. \tag{C.39}$$

The system above can be easily solved by nonlinear solvers in standard packages in programming languages since they compose a system of polynomial about n^S and n^B. In our implementation, we solve it by fsolve function in Matlab. Note that we have as many of these equations as the number of grids in ϕ.

D Computation Algorithm

D.1 Value Function Iteration

As a preparation, first obtain ss value of n^S, n^B for each ϕ by solving a nonlinear system of FOCs shown in Appendix C.3. Second, make a grid space of n^S, n^B for each ϕ around its ss value. (The construction of state space in this way can save the memory, often used in two-asset heterogeneous-agent-new-keynesian (HANK) literature like Brunnermeier and Sannikov (2016).) We set terminal age of firms Af that is large enough.

For each ϕ, keep the iteration below until it converges.

0. Guess $n_a^{i=0}$ for $a \in \{0, ..., Af\}$
1. Construct V_a^i backward from $a = Af$ to $a = 0$
2. Calculate $n_a^{i=i+1}$ forward from $a = 0$ to $a = Af$, and iterate it until convergence.

With a discretization of state space n^S, n^B into a grid with 41 points (1681 in total) and tolerance levels 10^{-4}, executing the algorithm above takes around 30 seconds.

D.2 Fixed Point Problem

We can solve the two fixed point problems, backward fixed point problem (3.35) and forward fixed point problem (3.36) by iteration of the mappings given the network structure as in Lim (2018) in the following way. About the backward fixed point problem,

0. Guess $P(\chi)$.
1. By calculating integration in the right hand side of (3.35) given $P(\chi)$, update $P(\chi)$ and iterate it until convergence.

Since the right hand side is concave in $\Phi(\chi)$, we can easily show that Blackwell's sufficient condition is satisfies as discussed in Bernard et al. (2022), and the contraction mapping theorem guarantees uniqueness and consistency of this iteration as shown in Stokey (1989). We can solve for $X(\chi)$ similarly. While the linearity about $X(\chi)$ in the forward fixed point problem makes hard to show the sufficient condition, under our calibrated parameters, the algorithm above returns unique fixed points from several initial guess about $X(\chi)$. About the existence and uniqueness of this class of the equation, check Allen et al. (2015).

D.3 Equilibrium

Using iteration for two fixed points as an inner loop, we update outer variables (functions).

0. Guess $m_j, \Delta_H, U, w_j^\tau, M_e$.
1. Given m_j, Δ_H, w_j^τ, solve the value function of firm using VFI D.1 to obtain $n_j(\chi), eV(\chi)$
2. If the entry surplus is positive (negative), increase (decrease) M_e.
3. Calculate $g(\chi)$ and $g_\tau^j(\chi)$
4. Calculate m_j
5. Calculate $k^j(\chi, \chi')$

6. Compute fixed point of $P(\chi)$ using iteration of mapping D.2 and calculate associated price index.

7. Compute fixed point of $X(\chi)$ using iteration of mapping D.2.

8. Calculate w_j^{τ} and U and iterate it until convergence.

With a discretization of state space S_{χ} into a grid with $25 * 25$ points and tolerance levels 10^{-4}, executing the algorithm above takes around 10 minutes.

While we do not have a formal proof of existence and uniqueness, the nested fixed point algorithm is numerically well behaved and converges to the same solution irrespective of the chosen starting values.