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Abstract

Recent empirical studies have found substantial information rigidities faced by consumers and firms, when they forecast macro variables (Coibion and Gorodnichenko (2015) and Coibion et al. (2018)). In this study, we examine how information rigidities behave differently when it comes to forecasting industry- and firm-level variables. Using a firm-level panel dataset that contains *quantitative* forecasts of the (macro) inflation rate, the industry-specific inflation rate, and firm sales, we present evidence that the information rigidity associated with forecasting (macro) inflation is more pervasive than those associated with forecasting the other two variables. We back out the unobservable marginal cost of acquiring and processing information for the three target variables and find that the cost associated with digesting industry-level information is the highest among them.

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1 Introduction

How agents process information and form expectations are central to numerous economic questions ranging from households' portfolio choices to firm growth. Economists have paid ample attention to these research questions by focusing on the implications of information frictions on individual decisions and economic dynamics.¹ Recently, the empirical literature on information frictions and expectations formation began to take off, owing to the increasing availability of expectations survey data on macro variables.² Two common findings from the literature are: (1) there exist pervasive information rigidities (i.e., information frictions) faced by consumers and firms when they forecast macro variables; and (2) information rigidity, which leads to a systematic mis-forecasting of future macroeconomic outcomes, affects real economic decisions such as firms' hiring and investment decisions.

Surprisingly, existing literature mainly focuses on information rigidity concerning forecasting *macro* variables. This focus, in our opinion, is due to data constraints, as datasets used by papers in this literature usually contain firms' (quantitative) forecasts of macro variables only.³ This focus does not mean that only information rigidities associated with forecasting macro variables matter for firms' decisions. In fact, firms also have to acquire and process information concerning industry- and firm-level variables when making decisions, as the industry- and firm-specific shocks and macro shocks are far from being perfectly correlated. Moreover, several papers (e.g., Boivin et al. (2009), Maćkowiak and Wiederholt (2015) and Andrade et al. (2020)) argue that firms respond to macro-level and industry-level (and local-level) shocks differently, as choices are made endogenously, and therefore face different degrees of information rigidities concerning various variables. In this paper, we present a more complete picture of information rigidities faced by firms *at three levels*, which existing

¹Earlier contributions include Muth (1960), Muth (1961), Lucas (1972), Lucas (1973), among others.

²Seminal works include those by Coibion and Gorodnichenko (2012), Coibion and Gorodnichenko (2015), Coibion et al. (2018), and among others.

³Some papers such as Coibion et al. (2018) also investigate information rigidities concerning forecasting *industry-level* variables.

research has not yet explored.⁴

In this study, we empirically investigate the information rigidities faced by firms concerning forecasting three target variables. We achieve this goal by using a merged panel dataset from Japan (2004 – 2018) that contains firms’ *quantitative* forecasts of the (macro) inflation rate, the industry-specific inflation rate, and firm sales. We find that the information rigidity faced by firms concerning forecasting macro inflation is by far more pervasive than the one associated with forecasting firm’s sales and slightly more pervasive than the one associated with forecasting industry-specific inflation. Moreover, we estimate the processes of these three variables and find that the firm-level demand process is substantially more volatile than the processes of macro inflation and industry-specific inflation.

Guided by our model and empirical moments directly related to forecasting errors, we also back out the unobservable marginal cost of acquiring and processing information for the three target variables. We find that the cost associated with collecting and digesting industry-level information is the highest among the three target variables. In addition, we show that removing information rigidities concerning the industry-specific inflation rate and firm-specific demand increases the firm’s payoff substantially more than removing the information rigidity associated with forecasting the macro inflation. In summary, we find evidence that is consistent with the predictions of rational inattention models with elastic attention in the vein of Sims (2010), Luo and Young (2014), and Maćkowiak and Wiederholt (2015). Moreover, information rigidities concerning micro-level subjects such as industry-level inflation and firm demand lead to much higher payoff losses compared with the information rigidity concerning macro inflation.

The two-panel datasets that are merged into a combined dataset used in this study are obtained from the Japanese government. The first dataset is called the Annual Survey of Corporate Behavior (ASCB), conducted by the Economic and Social Research Institute

⁴Although several recent papers (e.g., Barrero (2020), Chen et al. (2020), Ma et al. (2020)) investigate information rigidities associated with forecasting firm-level variables such as firm sales and profits, they do not study how firms acquire and process information on industry and macro variables.

(ESRI) within the Cabinet Office over the period 2004 – 2018. This survey achieved a response rate of about 50% from all firms publicly listed at major stock exchanges in Japan, generating a panel sample of around 1000 firms per year. This survey is mandatory and asks each firm to report quantitative forecasts of nominal and real GDP growth rates (i.e., for the next one, three, and five fiscal years) in early January of each year.⁵ In the same survey, the firm is also asked to report quantitative forecasts of the nominal and real output growth rate of the industry it belongs to (i.e., the growth rate of the gross value of industrial output). The second dataset is a quarterly survey called the Business Outlook Survey (BOS), conducted by the Japanese Ministry of Finance (MOF) at the beginning of each quarter from 2004 – 2018. This survey is also mandatory and targets big firms as well as a randomly selected sample of medium-sized and small firms. The average response rate of this survey is 80%, which results in a sample of roughly 11,500 firms per quarter. The second survey asks firms to provide realized and expected sales and operating profits for each semi-year (i.e., April–September and October–March). As both datasets cover big firms, we merge them by matching firms’ Japanese names and their locations. As a result, we can construct a merged dataset that contains around 740 firms per year.

In this paper, we present a rational inattention (RI) model with elastic attention to guide our empirical work. RI was first introduced in economics by Sims (2003). It argues that RI provides a single mechanism that generates stickiness pervasively. In RI theory, agents have limited information about the state of the world and learn slowly because they cannot process unlimited information.⁶ In this study, a typical firm needs to forecast the macro and industry-specific inflation rates as well as the change in firm-specific demand in order to adjust output. Individual firms in the model do not observe the values of the three target

⁵The fiscal year in Japan begins on April 1. and ends on March 31.

⁶The key innovation relative to standard noisy rational expectations models (e.g., Muth (1960), Lucas (1972), Lucas (1973)) is that the RI hypothesis permits agents to design the distribution of noise terms by focusing limited attention on certain variables at the expense of others. Under RI, agents respond to changes in the true underlying state slowly because it takes time for them to learn exactly what the new state is; they cannot learn without error because the information flow required to describe the state perfectly is larger than what their “Shannon channel” permits. Therefore, the distribution of the RI-induced noise is an outcome of optimal choice and will adapt to changing circumstances in the economy.

variables perfectly. Instead, they receive noisy signals of these variables every period and thus optimally filter (and forecast) the true values of these variables.

Following Sims (2010), Paciello and Wiederholt (2014), and Luo et al. (2017), we assume that the firm chooses the optimal degree of channel capacity to minimize the conditional variance of the forecast error (FE), given the marginal cost of acquiring and processing information. As a result, it equalizes the marginal benefit of reducing the variance of the FE and the marginal cost of acquiring and processing information. Importantly, we allow the constant information-processing cost to be variable-dependent and use our model (and the data moments) to back out its value for each of the three target variables. In addition, our model, which features a two-layer constant-elasticity-of-substitution (CES) demand function, generates a sensitivity parameter of the firm’s payoff with respect to a reduction in the variance of FEs (for each of the three target variables). This is also variable-dependent.⁷ Based on information costs and the sensitivity parameters, the model yields different values of the Kalman gain for the three target variables in the optimal filtering/forecasting problem. We obtain the values of the sensitivity parameters either from the literature (i.e., the two elasticities of substitution of the CES demand function) or by estimating the firm’s demand process (i.e., the persistence of the demand process). Then, we match the values of the Kalman gain for three target variables by calibrating the information cost parameters. Finally, we evaluate the extent to which the model can fit the other key non-targeted moments of the data (e.g., the standard deviation of the FEs).

In the empirical part of the study, we follow the literature (e.g., Coibion and Gorodnichenko (2012), Andrade and Le Bihan (2013) and Ryngaert (2017)) by using the estimated serial correlation of the FE to infer the degree of information rigidity. As agents know information perfectly in full-information-rational expectation (FIRE) models, ex-post FEs are random and therefore, serially uncorrelated. However, a positive serial correlation of the FEs exists in RI models, as the agent is facing informational constraints and thus absorbs new

⁷The sensitivity parameters depend on the two elasticities of substitution of our two-layer CES demand function as well as the persistence of the demand process.

information *gradually and with delay*. In particular, the first-order serial correlation of the FE equals the persistence of the AR(1) shock process, multiplied by the factor of one minus the Kalman gain.⁸ Therefore, the degree of information rigidity, which is inversely measured by the Kalman gain, becomes higher when the serial correlation of the FE is higher.

The regression analyses in our paper show that the (positive) serial correlation is the highest for the FE of (macro) inflation and the lowest for the FE of firm sales, while it is in the middle for the FE of the industry-specific inflation rate. The regression results also identify that (1) the process of the firm’s demand is more persistent than the (macro) inflation process, which, in turn, is more persistent than the process of industry-specific inflation, and (2) innovations to firm-specific demand are more volatile than innovations to the industry-specific inflation process, which, in turn, are more volatile than innovations to the process of macro inflation. In total, we find that the Kalman gain is the largest when the firm forecasts its demand (0.844), while its value is smaller when the firm forecasts the macro inflation rate (0.457) and the industry-specific inflation rate (0.567). We conclude that the degree of information rigidity concerning the macro target is more pervasive than the ones associated with forecasting the industry-specific inflation and firm’s demand process.⁹ Finally, although we do not aim to match the standard deviation of the three FEs in our calibration, the standard deviations of the three FEs implied by our calibrated model are quite close to those calculated from the data, with the percentage differences ranging from zero to 10%. In total, we conclude that our empirical results lend support to predictions of the RI models with elastic attention and the argument of variable-dependent information rigidities.

In the empirical section, we also back out the marginal cost of acquiring and processing information for each of the three target variables. We find that the marginal cost of acquir-

⁸Note that in the literature the Kalman gain is used to measure how much uncertainty can be removed upon receiving the new signals on unobservable factors.

⁹A recent paper by Meyer et al. (2021) shows that firms pay more attention to the evolution of their unit costs rather than aggregate inflation. Our results using forecasts at three levels *quantify* the difference in attention allocation among macro, industry-specific, and firm-specific variables.

ing and processing information is the highest for the industry-specific inflation process and the lowest for the process of macro inflation. Note that the degree of information rigidity associated with forecasting the industry-specific inflation process is revealed to be quite high. Additionally, its importance in the firm's payoff function and its volatility are quite high as well. The latter two findings usually imply a very low degree of information rigidity (conditioning on the information cost). The only way to rationalize all three findings is that the marginal cost of acquiring and processing industry-level information is so high that the firm is not incentivized to allocate substantial attention to industry-level information. Turning to macro inflation, we know that innovations to this process are extremely non-volatile and the importance of forecasting it (correctly) is low, which implies a small marginal benefit of reducing the perceived uncertainty. As the marginal cost and benefit are equalized when the firm chooses its channel capacity, the cost of acquiring and processing information concerning the macro inflation process must be very low. Finally, the cost of acquiring and processing information is relatively small for the process of firm-specific demand, as the revealed information rigidity is low and the importance of forecasting it (correctly) is low.

We understand the rationale for the cost of collecting and analyzing industry-level information to be the highest among the three target variables, as the firm in our dataset is large (i.e., publicly listed). The firm basically analyzes its competitors' pricing behavior (when it comes to forecasting industry-level inflation). Analyzing macro information is not as costly as analyzing industry-level information, as the firm analyzes the whole economy which is much more stable than each single industry. Analyzing firm-specific demand is probably less costly than analyzing the industry-level information, as the firm has enough internal data to carry this out (while it lacks its competitors' internal data). In summary, we uncover substantial heterogeneity with respect to the cost of acquiring and processing information at various levels, which is new to the literature.

In the final part of the paper, we implement a simple back-of-the-envelope calculation of the payoff gains, when we remove information rigidities (i.e., setting the marginal cost

of acquiring and processing information cost to zero). We do not want to emphasize the magnitudes of these gains, as our model is a stylized partial-equilibrium model. Rather, we emphasize the contribution made by removing the information rigidity concerning each target variable to the overall gain. The contribution depends on two factors positively: the importance of forecasting the target variable (correctly) and the variance of FE of the variable. We find that removing information frictions concerning forecasting the macro inflation increases the firm's payoff only slightly (less than 2% of the overall gain), as the importance of forecasting this variable (correctly) is low and the variance of its FE is small. In fact, the payoff gain from *only* removing information frictions associated with forecasting the macro inflation in our model ranges between 0.0144% and 0.0225%, which is consistent with the finding of a small welfare gain from the literature (e.g., Luo (2008) and Maćkowiak and Wiederholt (2015)). On the contrary, eliminating information frictions concerning forecasting the industry-specific inflation and firm demand accounts for most of the overall gain, as the importance of forecasting the former variable (correctly) is high and the variance of the FE of sales is much larger than that of the other two types of FEs. In summary, we find that helping firms collect and digest industry- and firm-level information is at least as important as helping them collect and digest macro information if the firm's payoff function (i.e., expected profit) is the objective function.

Literature Review Our paper builds on a large body of literature that studies the expectations formation of economic agents and how these expectations affect their optimal forecasts. This literature mainly focuses on empirically testing theories of information rigidities using survey data. For instance, Mankiw et al. (2003) use the cross-sectional distribution of forecasts to infer the degree of inattentiveness (i.e., the frequency of updating expectations). Using survey data, Coibion and Gorodnichenko (2012) study the conditional responses of forecasts to aggregate shocks and disagreements among forecasters in order to disentangle the sticky-information specification proposed by Mankiw and Reis (2002) and the noisy-information specification proposed in Sims (2003). They find mixed support for

the two theories. Andrade and Le Bihan (2013) use the ECB Survey of professional forecasters to characterize the formation of expectations and find that forecasters have predictable FEs, making different forecasts even when the forecasted target is the same. Coibion and Gorodnichenko (2015) propose a new approach to quantify the degree of information rigidity using the US and international data of professional forecasters and other agents. Our paper complements this literature by empirically showing that the information rigidity concerning forecasting macro inflation is more pervasive than the ones associated with forecasting the firm’s own demand and the industry-specific inflation rate. This difference implies that although the effectiveness of macro policies (e.g., monetary and fiscal policies) might be limited due to high degrees of information rigidity at the macro level, policies that target certain industries or firms can be effective as firms allocate more attention to industry-level and firm-specific information. Moreover, we show that reducing the cost of collecting and digesting industry-level and firm-specific information can lead to much larger gains, compared with the case in which the information rigidity concerning macro inflation is eliminated.

Our paper is closely related to a recent paper by Andrade et al. (2020) who study how firms respond to macro and industry-level shocks by adjusting their expectations and prices. Our paper complements their study by providing *quantitative* estimates of the information rigidities *and* costs of acquiring and processing information concerning macro-, industry-specific and firm-specific variables. We also quantify the payoff losses due to the existence of information costs.

Our paper is also related to the literature on elastic attention proposed in Sims (2003) (e.g., Luo (2008), Paciello and Wiederholt (2014), Luo and Young (2016), Baker et al. (2020), Afrouzi and Yang (2021), and Miao et al. (2022), etc). Luo and Young (2014) find that RI models with elastic attention better replicate different consumption behaviors in emerging and developing small open economies. Paciello and Wiederholt (2014) show that optimal monetary policies under fixed attention and elastic attention differ significantly because the monetary authority can manipulate firms’ decisions on how much attention they devote

to aggregate conditions. Luo et al. (2017) find that households' elastic attention can help explain the observed decline in the relative inequality of consumption to income in the US economy. We contribute to this literature by presenting evidence that allocated attention is heterogeneous across the macro-, industry- and firm-specific target variables.

This paper is organized as follows. Section 2 describes how we construct the dataset. Section 3 presents a simple RI model with elastic attention and derives theoretical predictions for the predictable FEs and disagreement among firms. Section 4 presents our main empirical results and examines the extent to which our calibrated model can match non-targeted moments in the data. Section 5 concludes the paper.

2 Data

The first dataset employed is the Annual Survey of Corporate Behavior (ASCB), conducted by the Economic and Social Research Institute in the Cabinet Office of Japan.¹⁰ Each year, the survey questionnaire was sent to all listed firms on the Tokyo and Nagoya Stock Exchanges. A total of 2000 firms, on average, were surveyed during these years. Of them, 50% on average responded to the survey each year. The survey is conducted annually in January. Respondents are required to answer the questions regarding their quantitative forecasts for the (real and nominal) GDP growth rate, the growth rate of (real and nominal) industrial output, and the expected average (percentage) change in their input and output prices for the next fiscal year.¹¹

The second dataset we use is called the BOS, implemented by Japanese MOF every quarter. The survey covers all big firms (i.e., firms with registered capital of more than 2 billion JPY or, equivalently, 20 million USD) and a representative sample of medium-sized and small firms.¹² We have obtained the second dataset from 2004/Q2 to 2018/Q4. The

¹⁰This is the same dataset as the one used in Tanaka et al. (2019).

¹¹A fiscal year in Japan (nendo in Japanese) spans from April/1 of the current year to March/31 of the next year.

¹²For firms with registered capital between 0.5 billion JPY and 2 billion JPY, 50% of them are randomly

average response rate of this survey is 80%, which results in a panel sample of roughly 11,500 firms per quarter. The second survey asks firms to report forecasted sales and operating profits for each semi-year ahead (i.e., April–September and October–March). It also asks the firm to report realized sales and operating profits for the past two half-year periods.

We merge the BOS conducted at the beginning of every second quarter (April) with ASCB, as the timing of the two surveys is close, and there are a large number of firms that report their forecasted sales and operating profits in BOS conducted in April. We use this merged dataset as the main data to conduct our analysis. On average, we are able to match 73% of observations (around 740 firms per year) in the ASCB datasets with the observations in the BOS dataset conducted in April and the matching rate is relatively stable over the years as shown in Table 1.¹³

Ideally, we would want to merge BOS conducted at the beginning of every first quarter with ASCB, as both are conducted in January. However, there are fewer firms that report their forecasted sales and operating profits in BOS conducted in January than the one conducted in April.¹⁴ We merge BOS conducted at the beginning of every first quarter with ASCB to create an alternative dataset for our analysis. However, due to many missing values of forecasts in BOS conducted in January, we only use this alternative dataset for robustness checks. We will show our findings are robust to using this alternative dataset.¹⁵

sampled every quarter. For firms with registered capital between 0.1 billion JPY and 0.5 billion JPY, 10% of them are randomly sampled every quarter. For firms with registered capital less than 0.1 billion JPY, 1% of them are randomly sampled every quarter. The random sample is redrawn at the beginning of every fiscal year; that is, as long as a medium-size or small firm is selected for the survey in a given fiscal year, it appears in the survey for all four quarters of that fiscal year.

¹³As BOS data end in 2018/Q4 (i.e., realized variables are unavailable for the fiscal year of 2018), we end up with a panel dataset that contains forecast errors made between April 2004 and April 2017 over 14 years (in terms of the timing of forecasting).

¹⁴Roughly 40% firms that answered the survey reported their forecasted sales and operating profits in January, while roughly 75% firms that answered the survey reported their forecasted sales and operating profits in April.

¹⁵As BOS data starts from 2004/Q2 and ends in 2018/Q4, the alternative dataset contains forecast errors made between January 2005 and January 2017 over 13 years (in terms of the timing of forecasting).

Table 1: Percentage of successful matching of the main dataset

year	obs. in ASCB	matched obs.	percentage
2004	1,243	794	63.9%
2005	1,031	679	65.9%
2006	1,123	780	69.5%
2007	1,042	756	72.6%
2008	1,035	711	68.7%
2009	1,027	721	70.2%
2010	1,032	756	73.3%
2011	863	629	72.9%
2012	890	674	75.7%
2013	815	631	77.4%
2014	867	672	77.5%
2015	982	766	78.0%
2016	1,062	826	77.8%
2017	1,168	901	77.1%
2018	1,107	858	77.5%
Total	15,287	11,154	73.0%

Notes: The number of observations in ASCB dataset is reported in the second column, and the number of observations in the matched dataset is reported in the third column. Note that in each year t , forecasters in the ASCB dataset are reported in January (i.e., the first quarter), while forecasts in the BOS dataset are reported in April or early May (i.e., the second quarter). Both forecasts are made for the fiscal year of t , and the fiscal year begins in April.

2.1 Forecasts and FEs

We construct the FE of macro and industry-specific inflation rates as follows. First, we obtain the time-series data of the (nominal and real) GDP growth rate and that of the (nominal and real) growth rate of industrial output from ESRI's website. Second, we calculate the macro inflation rate by taking the difference between the nominal GDP growth rate and the real GDP growth rate. We implement the same exercise for the industry-specific inflation rate. Then, we define the FE of the macro-level inflation rate and that of industry-specific inflation rate as

$$\mathbb{F}\mathbb{E}_{\omega,t-1}^{\pi}(t) \equiv \pi_t - \mathbb{E}_{\omega,t-1}[\pi_t], \quad (1)$$

and

$$\mathbb{F}\mathbb{E}_{\omega,t-1}^{\pi^i}(t) \equiv \pi_t^i - \mathbb{E}_{\omega,t-1}[\pi_t^i], \quad (2)$$

where ω indicates the firm, t denotes the year, and i refers to the industry the firms belong to. The macro and industry-specific inflation rates are denoted by π and π^i , respectively.

$\mathbb{E}_{\omega,t-1}\pi_t$ and $\mathbb{E}_{\omega,t-1}\pi_t^i$ are the forecasted macro and industry-specific inflation rates from fiscal year $t - 1$ to t , while π_t and π_t^i are the realized macro-level and industry-specific inflation rates from fiscal year $t - 1$ to t .

Next, as firms report both realized and forecasted sales, we define the percentage FE and the logarithm of FE as follows:

$$\mathbb{F}\mathbb{E}_{\omega,t-1}^{pct,sales}(t) \equiv \frac{R_{\omega,t} - \mathbb{E}_{\omega,t-1}[R_{\omega,t}]}{\mathbb{E}_{\omega,t-1}[R_{\omega,t}]}, \quad (3)$$

Alternatively, we define the (logarithm) FE of the total cost as

$$\mathbb{F}\mathbb{E}_{\omega,t-1}^{log,sales}(t) \equiv \log(R_{\omega,t}) - \log(\mathbb{E}_{\omega,t-1}[R_{\omega,t}]). \quad (4)$$

where $R_{\omega,t}$ and $\mathbb{E}_{\omega,t-1}[R_{\omega,t}]$ are realized and forecasted sales of firms ω respectively.

In Table 2, we present the summary statistics of the forecasted and realized inflation rate both at the macro level and at the industry level. Several observations are worth mentioning. First, Japan has experienced deflation from the beginning of our dataset, as the average inflation rate is negative from 2004-2018. This can be seen from the average realized (macro) inflation rate in Tables 2. Second, the average realized (industry-specific) inflation rate is slightly positive, which is higher than the average realized (macro) inflation rate. This is possible as the (industry-specific) inflation rate is the price change of industrial output (i.e., not value added).¹⁶ Third, the variation of (industry-specific) inflation rates is larger than that of macro inflation rates, which substantiates the fact that industry-level shocks (to inflation) are more volatile than macro shocks (to inflation). In Table 3, we report the summary statistics of the FEs defined as above. In this table, we observe that Japanese firms had over-predicted the macro inflation rate and under-predicted the industry-specific

¹⁶In the data, average inflation rate of manufacturing industries is higher than that of service sectors. Since we have 13 manufacturing industries out of 23 industries in total, inflation rates of manufacturing industries are over-represented in the calculation of the average (industry-specific) inflation rate. This also explains why the average realized (industry-specific) inflation rate is higher than the average macro inflation rate.

inflation rate over the period 2004-2017. Note that the standard deviation of the FE of the industry-specific inflation rate is larger than that of the macro inflation rate. This is true, even when we use the residual FE of the industry-specific inflation rate where we have removed the aggregate component and the size effect from the original FE. For FE of sales, the average is close to zero, while its standard deviation is much larger than that of the two inflation rates.

Table 2: Summary statistics of the inflation rates

	Obs.	mean	std. dev.	median
realized macro-level inflation rate	10296	-0.40%	1.13%	-0.70%
forecasted macro-level inflation rate	8881	-0.12%	0.80%	0.00%
realized industry-specific inflation rate	10296	0.38%	3.12%	0.39%
forecasted industry-specific inflation rate	7770	-0.04%	0.77%	0.00%

Notes: Realized macro-level and industry-specific inflation rates (23 industries) are obtained from the website of the Economic and Social Research Institute (ESRI) within the Cabinet Office and refer to the fiscal year (April to March). To exclude outliers, we trim the top and bottom one percent of observations of the forecasts. Time span: 2004-2017 (fiscal years).

Table 3: Summary statistics of forecast errors

	Obs.	mean	std. dev.	median
forecast error of macro-level inflation rate	8151	-0.25%	1.00%	-0.40%
forecast error of industry-specific inflation rate	7128	0.56%	2.83%	0.40%
(percentage) forecast error of sales	5910	-0.90%	8.70%	-0.45%
(logarithm) forecast error of sales	5911	-0.0131	0.0896	-0.00457
residual forecast error of industry-specific inflation rate	7126	-0.03%	2.43%	-0.13%

Notes: Realized macro-level and industry-specific inflation rates (23 industries) are obtained from the website of the Economic and Social Research Institute (ESRI) within the Cabinet Office and refer to the fiscal year (April to March). The forecast error is defined as the difference between the realized value from the forecasted value. To construct the residual forecast error of the industry-specific inflation rate, we tease out the aggregate component and the size effect from the original forecast error of industry-specific inflation rate. To exclude outliers, we trim the top and bottom one percent of observations of the FEs. Time span: 2004-2017 (fiscal years).

In Table 4, we present the list of industries included in our dataset and the number of observations (of the industry-specific inflation forecast) that belong to each industry. All firms are grouped into 23 (broad) industries, and more than half of them (13) are manufacturing industries. The fact that the ASCB dataset has broad industry classifications helps firms answer the survey, as most firms in our dataset are large firms with businesses across several

small industries. It is clear from the table that manufacturing firms are over-represented in the sample (compared with their contribution to the GDP of Japan), as more than half of the observations are from manufacturing industries. However, several non-manufacturing industries such as construction, wholesale/retail, finance, and transportation also have many observations.

Table 4: Number of observations from each industry

industry name	obs. of industry-specific inflation forecasts
Fisheries and Agriculture	76
Mining	42
Construction	819
Food	442
Textiles	299
Pulp and paper	108
Chemicals	1025
Coals and oil	61
Ceramics products	318
Primary metal	612
Metal products	282
General machineries	868
Electronic machineries	1052
Transportation equipments	455
Precision machineries	148
Other manufacturing	356
Wholesale/retail	1690
Finance	717
Real estate	185
Transportation	647
Information and Communication	310
Electricity and Gas	194
Other services	448

Notes: This table presents the number of observations of industry-specific inflation expectations for 23 industry in merged dataset. The industry-specific inflation is the inflation rate of industrial output (i.e., not value added).

Tables 8 and 9 in Online Appendix 6.1 show the same statistics for the alternative dataset (i.e., the BOS conducted in January). Naturally, the standard deviation of the forecast error of sales is larger than that of the main dataset, as firms are asked to forecast their sales three

months earlier than the timing of the main dataset.

3 A Simple Model with Elastic Attention

In order to guide our empirical analysis, we present a simple model that a firm needs to forecast the changes in macro-, industry- and firm-level variables in order to adjust its output.¹⁷ In the model, the firm does not observe the macro inflation rate, the industry-specific inflation rate, and its demand shifter perfectly. Instead, it receives noisy signals of these three variables every period and thus has to filter the true values of these variables at the end of each period. Accordingly, the firm forecasts the changes in these three variables (from the current period to the next period) based on their values filtered at the end of the current period.

Following the assumptions made in RI models, we assume that the firm in our model economy chooses the channel capacity in order to minimize the variance of the FE subject to a constant marginal cost of acquiring and processing information. As a result, the firm equalizes the marginal benefit of reducing the variance of the FE (by increasing the channel capacity) and the marginal cost of acquiring and processing information. Since there are three variables, the information-processing cost can vary across target variables. In addition, the sensitivity of the firm's payoff to the reduction of the variance of the FE can be different across the three target variables, depending on the structure of the firm's payoff function. In RI models, the relationship between the marginal cost of acquiring and processing information and the optimal channel capacity (and the Kalman gain in the filtering problem) is a one-to-one mapping. Therefore, we perform a simple calibration exercise by matching the implied Kalman gain (based on the constant information cost and the sensitivity parameters) from the model with the estimated Kalman gain obtained from the empirical section. Moreover, we evaluate the performance of our calibrated model in terms of matching

¹⁷As our macro-level and industry-level forecasts represent changes, we assume that the firm forecasts the change in its demand to make micro-level forecasts consistent with macro-level as well as industry-level forecasts.

non-targeted moments.

3.1 Environment

3.1.1 Demand and Supply

In our model, there are N industries in the economy. Each firm produces a differentiated variety within an industry. In the economy, the representative consumer has the following nested-CES preferences, where the first nest is among the composite goods produced by firms from different industries, indexed by i ,

$$U_t = \left(\sum_{i=1}^N Q_{it}^{\frac{\delta-1}{\delta}} \right)^{\frac{\delta}{\delta-1}},$$

and the second nest is among the varieties $\omega \in \Omega_{it}$ produced by firms from each industry i ,

$$Q_{it} = \left(\int_{\omega \in \Omega_{it}} e^{\frac{a_t(\omega)}{\sigma}} q_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}.$$

In the first nest, δ is the elasticity between goods produced by firms from different industries. In the second nest, σ is the elasticity between different varieties within the same industry, and $a_t(\omega)$ is the demand shifter for variety ω . We assume that firms differ in their demand shifters, $a_t(\omega)$, and need to have a higher elasticity of substitution within the industry than between industries (i.e., $\sigma > \delta$) in order to have an interior solution for the representative consumer's optimization problem. After denoting consumers' total (nominal) expenditure as Y_t , we can express the demand for a particular variety, ω , as:

$$q_t(\omega) = Y_t P_t^{\delta-1} P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} p_t(\omega)^{-\sigma}, \quad (5)$$

where P_t is the aggregate price index for all goods, and $P_{i,t}$ is the ideal price index of industry i . After substituting the real consumption $C_t \equiv \frac{Y_t}{P_t}$ into equation (5), we obtain

$$q_t(\omega) = C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} p_t(\omega)^{-\sigma}.$$

The ideal price index of goods industry i can be expressed as

$$P_{i,t} \equiv \left(\int_{\Omega_{i,t}} e^{a_t(\omega)} p_t(\omega)^{1-\sigma} d\omega \right)^{1/(1-\sigma)},$$

and the aggregate price index can be written as

$$P_t \equiv \left(\sum_{i=1}^N P_{i,t}^{1-\delta} \right)^{1/(1-\delta)}.$$

Each variety ω is produced by a firm whose production function is simply

$$q_t(\omega) = l_t(\omega), \tag{6}$$

where $l_t(\omega)$ is the amount of labor it hires and $q_t(\omega)$ is its real output. Firms hire labor in a perfectly competitive labor market and sell output in monopolistically competitive goods markets.

We assume that the firm chooses output at the beginning of each period in order to maximize the expected profit. Specifically, the objective function is

$$\max_{q_t(\omega)} q_t(\omega) \left[\left(C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} \right)^{\frac{1}{\sigma}} q_t(\omega)^{-\frac{1}{\sigma}} - w_t \right], \tag{7}$$

where w_t is the prevailing wage rate. If the firm knew all the information concerning various variables in equation (7) *perfectly*, solving the above optimization problem would be straightforward. However, the assumptions we make are that firms do not know the aggregate price level, P_t , the industry-specific price level, $P_{i,t}$, and the demand shifter, $a_t(\omega)$ perfectly. We

now discuss how various variables evolve over time and the information environment concerning those variables.

3.1.2 Dynamic Processes

We have four dynamic processes that show up in the profit function in equation (7). First, we have estimated the persistence of the real GDP growth rate of Japan at the annual level, which turns out to be extremely low (0.075) and statistically insignificant. Therefore, we assume that the growth rate of real consumption (i.e., GDP) follows a random walk:

$$g_{t+1}^c = \epsilon_{c,t+1},$$

where $g_{t+1}^c \equiv \log(C_{t+1}) - \log(C_t)$. $\epsilon_{c,t+1}$ is the innovation to the (logarithm of) real consumption and distributed normally with the mean \bar{g}_c and variance $\sigma_{g_c}^2$ where $\bar{g}_c > 0$.¹⁸

Second, we assume the two inflation rates (macro and industry-specific), $\pi_{t+1} \equiv \log(P_{t+1}) - \log(P_t)$ and $\pi_{t+1}^i \equiv \log(P_{i,t+1}) - \log(P_{i,t})$, all follow the AR(1) process:

$$x_{t+1} = \rho_x x_t + \epsilon_{x,t+1},$$

where $x \in \{\pi, \pi^i\}$ and $\epsilon_{x,t+1}$ is an independently and identically distributed (iid) innovation and distributed normally with mean 0 and variance σ_x^2 , which implies that the long-run variance of x is $\sigma_x^2 / (1 - \rho_x^2)$. In Section 4.1, we will show that the persistence of both processes is below one.

Third, we assume that the firm-specific demand shifter $a_t(\omega)$, follows an AR(1) process as well:

$$a_{t+1}(\omega) = \rho_a a_t(\omega) + \epsilon_{a(\omega),t+1}, \tag{8}$$

where $\epsilon_{a(\omega),t+1}$ is an iid innovation and distributed normally with mean 0 and variance σ_a^2 .

¹⁸In the model, there are no investments, government expenditure and net exports. Therefore, consumption equals GDP.

3.1.3 Information Environment

The key assumption in this paper is that individual firms cannot observe the target variables perfectly due to limited information-processing capacity. Specifically, with finite capacity $\kappa \in (0, \infty)$, a random variable $\{x_t\}$ following a continuous distribution cannot be observed without an error and thus, the information set at time $t + 1$, denoted \mathcal{I}_{t+1} , is generated by the entire history of noisy signals $\{x_j^*\}_{j=0}^{t+1}$. Following the literature, we assume the noisy signal takes the following additive form:

$$x_{t+1}^* = x_{t+1} + \eta_{t+1},$$

where η_{t+1} is the endogenous noise caused by the finite capacity. We further assume that η_{t+1} is an iid idiosyncratic shock and is independent of the fundamental shocks affecting the economy. The reason why the RI-induced noise is idiosyncratic is that the endogenous noise arises from the firm's own internal information-processing constraint. Firms with finite capacity choose a new signal $x_{t+1}^* \in \mathcal{I}_{t+1} = \{x_1^*, x_2^*, \dots, x_{t+1}^*\}$ that reduces the uncertainty about the variable x_{t+1} as much as possible. Formally, this idea can be described by the information constraint

$$\mathbb{H}(x_{t+1}|\mathcal{I}_t) - \mathbb{H}(x_{t+1}|\mathcal{I}_{t+1}) \leq \kappa, \quad (9)$$

where κ is the firm's information channel capacity, $\mathbb{H}(x_{t+1}|\mathcal{I}_t)$ denotes the entropy of the state *prior to* observing the new signal at $t + 1$, and $\mathbb{H}(x_{t+1}|\mathcal{I}_{t+1})$ is the entropy *after* observing the new signal. κ imposes an upper bound on the amount of information flow—that is, the change in the entropy—that can be transmitted in any given period. In this paper, we assume that the prior distribution of x_{t+1} is Gaussian.

In the linear-quadratic-Gaussian (LQG) framework, as has been shown in Sims (2003) and Sims (2010), the true state under RI also follows a normal distribution $s_t|\mathcal{I}_t \sim N(\mathbb{E}[s_t|\mathcal{I}_t], \Sigma_t)$, where $\Sigma_t = \mathbb{E}_{\omega,t}[(x_t - \hat{x}_t)^2]$ and $\hat{x}_t = \mathbb{E}_{\omega,t}[x_t]$. In addition, given that the noisy signal takes the additive form $x_{t+1}^* = x_{t+1} + \eta_{t+1}$, the noise $\eta_{t+1} \sim N(0, \Lambda)$ will also be Gaussian. In this

case, equation (9) is reduced to

$$\log (|\Psi_t|) - \log (|\Sigma_{t+1}|) \leq 2\kappa,$$

where $\Psi_t = \mathbb{E}_{\omega,t} [(x_{t+1} - \mathbb{E}_{\omega,t} [x_{t+1}])^2]$ and Σ_{t+1} are the conditional variances prior to and after observing the new signal, respectively. As more information about the state becomes available in single-agent models, this constraint will be binding.¹⁹ The conditional variance is updated according to the following standard formula in the steady state:

$$\Lambda^{-1} = \Sigma^{-1} - \Psi^{-1}.$$

The evolution of the estimated state, $\hat{x}_t = \mathbb{E}_{\omega,t} [x_t]$ is governed by the Kalman filtering equation:

$$\hat{x}_{t+1} = (1 - G_x) \rho_x \hat{x}_t + G_x x_{t+1}^*,$$

where $G_x = \Sigma \Lambda^{-1}$ is the Kalman gain,

$$x_t - \hat{x}_t = \frac{(1 - G_x) \epsilon_{x,t}}{1 - (1 - G_x) \rho_x \cdot L} - \frac{G_x \eta_t}{1 - (1 - G_x) \rho_x \cdot L} \quad (10)$$

is the estimation error with $\mathbb{E}_{\omega,t} [x_t - \hat{x}_t] = 0$, and L is the standard lag operator. Given the definition of the FE in equation (1), we can rewrite the forecast error as follows:

$$\begin{aligned} \mathbb{F}\mathbb{E}_{\omega,t}(t+1) &= x_{t+1} - \mathbb{E}_{\omega,t} [x_{t+1}] \\ &= \rho_x (x_t - \hat{x}_t) + \epsilon_{x,t+1}. \end{aligned} \quad (11)$$

Now, we discuss the information environment concerning the growth rate of real GDP and the wage rate. First, as the growth rate of real GDP, g_t^c is an iid random variable, the firm's optimal forecast is simply \bar{g}_c which is the prior mean of g_t^c . Therefore, whether the

¹⁹By "better" we mean that conditional on the draws by nature for the true state, the expected utility of the agent increases if information about that state is improved.

firm knows the past noisy signals of g_t^c is irrelevant. Next, we assume that the firm knows the wage rate, w_t , perfectly, as it is the firm itself that sets the wage rate and pays wages to workers.

Finally, we discuss the time assumption. First, the firm decides output at the beginning of each period based on its forecasts made (at the end of last period). At the same time, it chooses the channel capacity for each of the three target variables. Then, the goods are sold in the market, which leads to the realized price. The realized price differs from the expected price in general. Finally, the firm receives signals of the three target variables in the current period. As a result, the firm filters the current three target variables and forms its forecasts for those three target variables in the next period.

3.1.4 Discussions of Modeling Choices

In the model, we assume that firms differ in the demand shifters that they need to learn, while firms can also differ in their cost shifters in reality. We make such a modeling choice for two reasons. First, recent literature on firm heterogeneity reveals that it is mainly the demand-side factors that lead to firm heterogeneity (see Hottman et al. (2016)). Second, we think that the demand shifter is *more* likely to be exogenous to the firm compared to the cost shifter and the wage rate, as demand is determined by consumers' tastes which are out of the control of the firm in many circumstances. On the contrary, the firm can invest in its production technology (to reduce costs) and decides the wage rate it offers to the employees. Thus, the firm probably knows more about its costs and the wage rate than its demand shifter. Reasonably, the macro and industry-specific inflation processes are probably exogenous to the firm as well. To some extent, forecasting the industry-specific inflation rate is similar to forecasting the firm's competitors' pricing strategy (in the same industry). We believe that comparing the FE of the macro inflation rate and that of the industry-specific inflation rate is fair, as both targets are out of the control of the focal firm. In total, we think the three target variables that the firm knows imperfectly are more or less out of the

control of the firm.

We assume the firm chooses output (instead of setting the price) in the model, since information frictions matter in such a case. If the firm were to set the price, the markup rule implies that the optimal price is the product of a constant (i.e., the markup) and the firm’s marginal cost (i.e., the wage rate). As the marginal cost is constant (i.e., one) and the firm knows the wage rate, the firm would not need to know macro-, industry- and firm-level information in order to set the price. Therefore, we assume that the firm chooses output.²⁰

3.2 Optimal Forecasting and Elastic Attention

In this subsection, we describe the firm’s optimization problem, which can be divided into two steps. First, we solve the filtering/forecasting problem of the three target variables *given* the channel capacity allocated. Second, we solve for the allocation of attention to each of the three target variables.

3.2.1 Filtering/Forecasting

For macro inflation and industry-specific inflation, the filtering problem is standard. Specifically, we have the following updating rule when firm ω minimizes the variance of the filtering (or forecasting) error:

$$\widehat{\pi}_{t+1}^i = (1 - G_{\pi^i}) \rho_{\pi^i} \widehat{\pi}_t^i + G_{\pi^i} (\pi_{t+1}^i + \eta_{\pi^i, t+1}^\omega),$$

and

$$\widehat{\pi}_{t+1} = (1 - G_\pi) \rho_\pi \widehat{\pi}_t + G_\pi (\pi_{t+1} + \eta_{\pi, t+1}^\omega).$$

²⁰Alternatively, it can be assumed that the marginal cost is non-constant and increases with the production scale. In such a case, the firm needs to know macro-, industry- and firm-level information in order to set the price, as the resulting output affects the marginal cost which in turn affects the optimal price. We do not pursue this direction, as this alternative modeling choice would complicate the model substantially.

$\hat{x}_t = \mathbb{E}_{\omega,t} [x_t]$ (x can be either π^i or π) is the filtered state. The associated Kalman gain is

$$G_x = 1 - \exp(-2\kappa_x), \quad (12)$$

where κ_x ($x \in \{\pi^i, \pi\}$) is the channel capacity. The forecast is simply

$$\mathbb{E}_{\omega,t} [x_{t+1}] = \rho_x \mathbb{E}_{\omega,t} [x_t] = \rho_x \hat{x}_t.$$

As a result, the conditional variance of the FE is

$$\Psi_x \equiv \text{var}_t (x_{t+1} - \mathbb{E}_{\omega,t} [x_{t+1}]) = \rho_x^2 \Sigma_x + \sigma_x^2 = \frac{\exp(2\kappa_x) \sigma_x^2}{\exp(2\kappa_x) - \rho_x^2}, \quad (13)$$

where x can be either π^i or π , $\Sigma_x \equiv \text{var}_t (x_t - \mathbb{E}_{\omega,t} [x_t]) = \sigma_x^2 / (\exp(2\kappa_x) - \rho_x^2)$ is the variance of the filtering error, and σ_x^2 is the variance of the fundamental shock. As the conditional variance of the FE (Ψ_x) is a linear function of the conditional variance of the filtering error (Σ_x), minimizing Ψ_x is equivalent to minimizing the standard mean squared error (MSE).

As the macro and industry-specific inflation rates are changes in the price levels, we assume that firm ω forecasts the change in its demand shifter in the second step, $a_{t+1}(\omega) - a_t(\omega)$ as well. A rationale for this is that the firm wants to know by how much it should adjust the output (and employment) between two adjacent periods. Formally, we have the following forecasting problem for firm ω :

$$\min_{G_a} \text{var}_t [(a_{t+1}(\omega) - a_t(\omega)) - (\mathbb{E}_{\omega,t} [a_{t+1}(\omega)] - \mathbb{E}_{\omega,t} [a_t(\omega)])],$$

where G_a is the associated Kalman gain. Given the AR(1) structure of the demand process, we can rewrite the above problem as

$$\min_{G_a} \text{var}_t [(\rho_a - 1)(a_t(\omega) - \mathbb{E}_{\omega,t} [a_t(\omega)]) + \epsilon_{a(\omega),t+1}] \quad \text{or} \quad \min_{G_a} (1 - \rho_a)^2 \text{var}_t [a_t(\omega) - \mathbb{E}_{\omega,t} [a_t(\omega)]] + \sigma_a^2, \quad (14)$$

where we have used the result that $\mathbb{E}_{\omega,t}[a_{t+1}(\omega)] = \rho_a \mathbb{E}_{\omega,t}[a_t(\omega)]$. The problem in equation (14) is the same as minimizing the variance of the conditional filtering error of $a_t(\omega)$. Therefore, the usual RI techniques apply and we have

$$\Psi_a \equiv \text{var}_t [(a_{t+1}(\omega) - a_t(\omega)) - (\mathbb{E}_{\omega,t}[a_{t+1}(\omega)] - \mathbb{E}_{\omega,t}[a_t(\omega)])] = \frac{(1 - \rho_a)^2 \sigma_a^2}{e^{2\kappa_a} - \rho_a^2} + \sigma_a^2. \quad (15)$$

3.2.2 Attention Allocation

Now, we discuss how the firm allocates its attention optimally in the first stage. To determine the optimal level of attention/capacity devoted to monitoring the three target variables, we make the following assumptions for our model:

Assumption 1 *Individual firms face a constant marginal cost of acquiring and processing information concerning each variable when choosing the channel capacity.*

With a fixed information-processing cost, the agent is allowed to adjust the optimal level of attention in such a way that the marginal cost of information-processing for the problem at hand remains constant. The optimal forecasting problem for the typical firm can thus be written as:

$$\min_{\{\kappa_\pi, \kappa_{\pi^i}, \kappa_a\}} \{(w_\pi \Psi_\pi + \lambda_\pi \kappa_\pi) + (w_{\pi^i} \Psi_{\pi^i} + \lambda_{\pi^i} \kappa_{\pi^i}) + (w_a \Psi_a + \lambda_a \kappa_a)\}.$$

Note that the constant marginal cost of acquiring and processing information, λ_x where $x \in \{\pi, \pi^i, a\}$, can be different across various target variables. Variables Ψ_π , Ψ_{π^i} , and Ψ_a are defined in (13) and (15), respectively. Additionally, w_π , w_{π^i} , and w_a are the three sensitivity parameters. This minimization problem demonstrates the optimizing firm's trade-off between the uncertainty of the perceived state and the cost attached to reduction in the perceived uncertainty. The following proposition summarizes the solution:

Proposition 1 *At optimum, the individual firm equalizes the marginal benefit of reducing the variance of the ex-post forecast errors and the constant marginal cost of information*

acquisition and processing:

$$\lambda_\pi = w_\pi \left| \frac{\partial \Psi_\pi}{\partial \kappa_\pi} \right|; \quad \lambda_{\pi^i} = w_{\pi^i} \left| \frac{\partial \Psi_{\pi^i}}{\partial \kappa_{\pi^i}} \right|; \quad \lambda_a = w_a \left| \frac{\partial \Psi_a}{\partial \kappa_a} \right|. \quad (16)$$

Proof. The proof is straightforward. ■

It is worth noting that this result is consistent with the concept of “elastic” capacity proposed in Kahneman (1973). In addition, in a dynamic setting, the marginal cost of information-processing might also be constant over time. In contrast, the optimal degree of attention/capacity can be time-varying. For example, Coibion and Gorodnichenko (2015) used the SPF forecast survey data to test the degree of information rigidity due to both noisy information and sticky information and found that the degree of information rigidity decreased with the volatility of macroeconomic conditions. Specifically, they found that the incidence of information rigidity decreased from the late 1960s to the start of the Great Moderation (1983 – 1984) and had continued to decline since then. They argued that one should be wary of treating the degree of information rigidity as a structural parameter because it responds to changes in macroeconomic conditions.

In our model, the sensitivity parameters are different across the target variables because the variances of FEs of the three target variables play different roles in affecting the firm’s payoff. In what follows, we derive the expression for the sensitivity parameters through the lens of our model. First, the optimal output level under full information is

$$\log q_t^{full}(\omega) = \sigma \log \left(\frac{\sigma - 1}{\sigma} \right) + \log C_t + \delta \log P_t + (\sigma - \delta) \log P_{i,t} + a_t(\omega) - \sigma \log (w_t).$$

The (actual) output choice under information rigidities is

$$\log q_t(\omega) = \sigma \log \left(\frac{\sigma - 1}{\sigma} \right) + \sigma \log \left[\mathbb{E}_{\omega,t-1} \left(C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} \right)^{\frac{1}{\sigma}} \right] - \sigma \log (w_t). \quad (17)$$

Calculation shows that the expected loss in firm profits (due to information frictions) is

proportional to²¹

$$\mathbb{E}_{\omega,t-1} \left(\log q_t(\omega) - \log q_t^{full}(\omega) \right)^2 = \sigma^2 \mathbb{E}_{\omega,t-1} \left[\log \left(C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} \right)^{\frac{1}{\sigma}} - \log \left(\mathbb{E}_{\omega,t-1} \left(C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} \right)^{\frac{1}{\sigma}} \right) \right]^2,$$

which is closely related to the variance of filtering/forecasting error of C_t , P_t , $P_{i,t}$ and $a_t(\omega)$.

From equation (17), we have the following expression for the *change* in the optimal output under full information:

$$g_{q,t}^{full}(\omega) = \log \left(\frac{q_t^{full}(\omega)}{q_{t-1}^{full}(\omega)} \right) = g_t^c + \delta \pi_t + (\sigma - \delta) \pi_t^i + \tilde{a}_t(\omega) - \sigma [\log(w_t) - \log(w_{t-1})],$$

where $\tilde{a}_t = a_t(\omega) - a_{t-1}(\omega)$. In reality, firms face not only information rigidities but also adjustment costs of labor. Therefore, they have incentives to minimize the expected loss due to changes in their labor (i.e., output) choices. Specifically, the firms want to minimize the following expected loss due to the change in the output:

$$\begin{aligned} L_{t+1}(\omega) &= \mathbb{E}_{\omega,t} \left[g_{q,t+1}(\omega) - g_{q,t+1}^{full}(\omega) \right]^2 \\ &= \text{var}_t \left(g_{t+1}^c - \mathbb{E}_{\omega,t} [g_{t+1}^c] \right) + \delta^2 \text{var}_t \left(\pi_{t+1} - \mathbb{E}_{\omega,t} [\pi_{t+1}] \right) + (\sigma - \delta)^2 \text{var}_t \left(\pi_{t+1}^i - \mathbb{E}_{\omega,t} [\pi_{t+1}^i] \right) \\ &\quad + \sigma^2 \text{var}_t \left[\left(1 - \frac{1}{\rho_a} \right) \left(\frac{a_{t+1}(\omega)}{\sigma} - \frac{\mathbb{E}_{\omega,t} [a_{t+1}(\omega)]}{\sigma} \right) \right] + \frac{1}{\rho_a} \left(2 - \frac{1}{\rho_a} \right) \sigma_a^2 \\ &= \Psi_g + \delta^2 \Psi_\pi + (\sigma - \delta)^2 \Psi_{\pi^i} + \sigma^2 \left(1 - \frac{1}{\rho_a} \right)^2 \Psi_{\frac{a}{\sigma}} + \frac{1}{\rho_a} \left(2 - \frac{1}{\rho_a} \right) \sigma_a^2, \end{aligned} \quad (18)$$

where $g_q(\omega)$ is the optimal change in output under information rigidities and we have used the results that $a_{t+1}(\omega) = \rho_a a_t(\omega) + \epsilon_{a(\omega),t+1}$ and $\mathbb{E}_{\omega,t} [a_{t+1}(\omega)] = \rho_a \mathbb{E}_{\omega,t} [a_t(\omega)]$.²² Note that the variance of FE of the real GDP growth rate is simple $\sigma_{g_c}^2$, which cannot be reduced by allocating more attention to it (as its persistence is zero). For the remaining three terms in

²¹See Online Appendix 6.3 for the detail.

²²Note that $[\mathbb{E}_{\omega,t} [a_{t+1}(\omega)] - \mathbb{E}_{\omega,t} [a_t(\omega)]] - [a_{t+1}(\omega) - a_t(\omega)] = \left(1 - \frac{1}{\rho_a} \right) (\mathbb{E}_{\omega,t} [a_{t+1}(\omega)] - a_{t+1}(\omega)) - \frac{\epsilon_{a(\omega),t+1}}{\rho_a}$. Thus, we have $\text{var}_t [\mathbb{E}_{\omega,t} [a_{t+1}(\omega)] - \mathbb{E}_{\omega,t} [a_t(\omega)]] = \sigma^2 \text{var}_t \left[\left(1 - \frac{1}{\rho_a} \right) \left(\frac{a_{t+1}(\omega)}{\sigma} - \frac{\mathbb{E}_{\omega,t} [a_{t+1}(\omega)]}{\sigma} \right) \right] + \left[\frac{1}{\rho_a^2} + \frac{2}{\rho_a} \left(1 - \frac{1}{\rho_a} \right) \right] \sigma_a^2$ which leads to the expression in equation (18).

equation (18), we can infer the sensitivity parameters as follows:

$$w_\pi = \delta^2, w_{\pi^i} = (\sigma - \delta)^2, \text{ and } w_a = \sigma^2 \left(1 - \frac{1}{\rho_a}\right)^2, \quad (19)$$

In summary, the sensitivity parameters are related to the elasticity of substitution, both between and within industries, and the persistence of the process of the demand shifter. In Section 4, we obtain values of these three parameters from the literature and by estimating the demand process of the firm.

3.3 Testable Implications

In this subsection, we derive the model's testable implications. In particular, we focus on the serial correlation regression of the forecast error at the individual firm level; that is, we regress FE in period t on its one-period lag and a set of fixed effects. As our dataset is at the annual frequency and spans 15 years, we have to exploit cross-sectional variations of the forecasts and FEs. As shown in Ryngaert (2017), the serial correlation regression can be run at the individual level, while the regression of the ex-post FE on ex-ante forecast revision studied in the seminal work of Coibion and Gorodnichenko (2015) *cannot* be run at the individual level (as idiosyncratic noise terms are not canceled out).

We discuss the regression of serial correlation first. Calculation shows that

$$\begin{aligned} \mathbb{F}\mathbb{E}_{\omega,t}^\pi(t+1) &= \pi_{t+1} - \mathbb{E}_{\omega,t}[\pi_{t+1}] \\ &= \epsilon_{\pi,t+1} + \frac{\rho_\pi(1-G_\pi)\epsilon_{\pi,t}}{1-\rho_\pi(1-G_\pi)\cdot L} - \frac{\rho_\pi G_\pi \eta_{\pi,t}^\omega}{1-\rho_\pi(1-G_\pi)\cdot L}. \end{aligned}$$

This yields the following regression equation:

$$\mathbb{F}\mathbb{E}_{\omega,t}^\pi(t+1) = \rho_\pi(1-G_\pi)\mathbb{F}\mathbb{E}_{\omega,t-1}^\pi(t) + error_{\pi,t+1}^\omega, \quad (20)$$

where $error_{\pi,t+1}^\omega = \epsilon_{\pi,t+1} - \rho_\pi G_\pi \eta_{\pi,t}^\omega$ is the error term of the regression which is uncorrelated

to $\mathbb{F}\mathbb{E}_{\omega,t-1}^{\pi}(t)$. Similarly, we have

$$\mathbb{F}\mathbb{E}_{\omega,t}^{\pi^i}(t+1) = \pi_{t+1}^i - \mathbb{E}_{\omega,t}[\pi_{t+1}^i] = \epsilon_{\pi^i,t+1} + \frac{\rho_{\pi^i}(1 - G_{\pi^i})\epsilon_{\pi^i,t}}{1 - \rho_{\pi^i}(1 - G_{\pi^i}) \cdot L} - \frac{\rho_{\pi^i}G_{\pi^i}\eta_{\pi^i,t}^{\omega}}{1 - \rho_{\pi^i}(1 - G_{\pi^i}) \cdot L},$$

and

$$\mathbb{F}\mathbb{E}_{\omega,t}^{\pi^i}(t+1) = \rho_{\pi^i}(1 - G_{\pi^i})\mathbb{F}\mathbb{E}_{\omega,t-1}^{\pi^i}(t) + error_{\pi^i,t+1}^{\omega}, \quad (21)$$

where $error_{\pi^i,t+1}^{\omega} = \epsilon_{\pi^i,t+1} - \rho_{\pi^i}G_{\pi^i}\eta_{\pi^i,t}^{\omega}$.

Although our data only contain the forecast of total sales, we can infer the FE of the demand shifter from the FE of sales. As the firm chooses $q_{\omega,t+1}$ at the beginning of period $t+1$, it knows its output (i.e., quantity) when forecasting its sales at the beginning of period $t+1$. Thus, the (logarithm of) FE of sales equals

$$\mathbb{F}\mathbb{E}_{\omega,t}^{log,sales}(t+1) \equiv \log(R_{\omega,t+1}) - \log(\mathbb{E}_{\omega,t}[R_{\omega,t+1}]) = \log(p_{\omega,t+1}) - \log(\mathbb{E}_{\omega,t}[p_{\omega,t+1}]),$$

where $p_{\omega,t+1}$ is the price charged in period $t+1$. Therefore, we can rewrite the FE of sales as

$$\begin{aligned} \mathbb{F}\mathbb{E}_{\omega,t}^{log,sales}(t+1) &= \frac{\delta}{\sigma} [\log(P_{t+1}) - \mathbb{E}_{\omega,t}[\log(P_{t+1})]] + \left(\frac{\sigma - \delta}{\sigma}\right) [\log(P_{i,t+1}) - \mathbb{E}_{\omega,t}[\log(P_{i,t+1})]] \\ &+ \frac{1}{\sigma} [\log(C_{t+1}) - \mathbb{E}_{\omega,t}[\log(C_{t+1})]] + \frac{1}{\sigma} [a_{t+1}(\omega) - \mathbb{E}_{\omega,t}[a_{t+1}(\omega)]] + const, \end{aligned} \quad (22)$$

where the term *const* includes (subjective) variances of $\log(P_{t+1})$, $\log(P_{i,t+1})$ and $a_{t+1}(\omega)$ which are not stochastic. In order to tease out the component of $\frac{1}{\sigma} [a_{t+1}(\omega) - \mathbb{E}_{\omega,t}[a_{t+1}(\omega)]]$ from the FE of sales, we regress the forecast error of sales on firm-size-bin-industry-year fixed effects and obtain the residual term as the counterpart of $\frac{1}{\sigma} [a_{t+1}(\omega) - \mathbb{E}_{\omega,t}ca_{t+1}(\omega)]$. The logic here is that firms with similar sizes and from the same industry have similar forecast errors of macro-level and industry-level variables such as the inflation rates and the real GDP

growth rate. Therefore, we have

$$\begin{aligned}
& \mathbb{F}\mathbb{E}_{\omega,t}^{res,sales}(t+1) \\
&= \frac{a_{t+1}(\omega) - \mathbb{E}_{\omega,t}[a_{t+1}(\omega)]}{\sigma} - const. \\
&= \frac{1}{\sigma} \left[\epsilon_{a(\omega),t+1} + \frac{\rho_a(1-G_a)\epsilon_{a(\omega),t}}{1-\rho_a(1-G_a)\cdot L} - \frac{\rho_a G_a \eta_{a(\omega),t}}{1-\rho_a(1-G_a)\cdot L} \right] - const,
\end{aligned}$$

and

$$\mathbb{F}\mathbb{E}_{\omega,t}^{res,sales}(t+1) = \rho_a(1-G_a)\mathbb{F}\mathbb{E}_{\omega,t-1}^{res,sales}(t) - const. [1-\rho_a(1-G_a)] + error_{a(\omega),t+1}, \quad (23)$$

where $error_{a(\omega),t+1} = \frac{1}{\sigma} (\epsilon_{a(\omega),t+1} - \rho_a G_a \eta_{a(\omega),t})$ which is uncorrelated with $\mathbb{F}\mathbb{E}_{\omega,t-1}^{res,sales}(t)$. In summary, the coefficient obtained from the serial correlation regression is the product of the persistence parameter and one minus the Kalman gain. When the degree of information rigidity increases, the Kalman gain shrinks, which results in a larger coefficient obtained from the regression.

Following the literature (e.g., Coibion and Gorodnichenko (2012), Coibion and Gorodnichenko (2015) and Andrade and Le Bihan (2013)), we also use the standard deviation of the FEs to measure the degree of forecaster disagreement. Using equations (10) and (11), it is straightforward to show that

$$\mathbb{F}\mathbb{D}_t^x(t+1) = \text{std}(\mathbb{F}\mathbb{E}_t^x(t+1)) = \frac{(1-G_x)\rho_x^2}{1-(1-G_x)\rho_x^2} \sigma_x^2 + \sigma_x^2,$$

where x ($x = \pi, \pi^i$, or $a(\omega)$), and we use the fact that $\text{var}_t(x_t - \hat{x}_t) = (1-G_x)\sigma_x^2/[1-(1-G_x)\rho_x^2]$.

4 Empirical Results

4.1 Estimations of Shock Processes

We estimate the AR(1) processes of the three variables first. For the process of the (annual) macro inflation rate and that of the industry-specific inflation rate, the estimation results are reported in Table 5. Note that we use the data for 2004-2017 to implement the estimation, as the sample period of realized inflation rates and firm sales in our dataset is 2004-2017. In addition, we include both the year and industry fixed effects into the AR(1) regression of the industry-specific inflation rate. We do so because we want to allow for different long-run average inflation rates across industries and to tease out the general impact of macro inflation on the industry-specific inflation rate. The estimated persistence is 0.643 for the macro inflation rate and 0.455 for the industry-specific inflation rate. These numbers are translated to quarterly-level persistence of roughly 0.90 and 0.82.²³ The estimated standard deviation of innovations (to the process of the inflation) is 0.91 percentage points for macro inflation and 2.76 percentage points for industry-specific inflation, which implies that the macro inflation process is less volatile than the industry-specific inflation process.

Table 5: Processes of macro and industry-level inflation

Dep.Var:	π_t	π_t^i
π_{t-1}	0.643*** (0.161)	
π_{t-1}^i		0.455*** (0.047)
Constant	-0.001 (0.003)	0.0176** (0.007)
Year fixed effects	No	Yes
Industry fixed effects	No	Yes
N	14	320
R^2	0.427	0.506

Notes: We regress the macro-level inflation rate (and the industry-specific inflation rate) on its (one-period) lagged term using annual data for 2004-2017. We include both the year and industry fixed effects into the AR(1) regression of the industry-specific inflation rate. Robust standard errors are reported in the parenthesis. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

²³We have also estimated the persistence of the real GDP growth rate at the annual level, which turns out to be extremely low (0.075) and statistically insignificant.

Now, we turn to the estimation of the firm’s demand shifter. Although we only have the information on sales in the dataset, the structure of our model enables us to back out the process of the demand shifter by utilizing the information on the expected sales and output price changes. Specifically, firm ω ’s sales in period $t + 1$ equals

$$R_{\omega,t+1} = p_{\omega,t+1}q_{\omega,t+1}.$$

In our model, the firm chooses and therefore knows its output level $q_{\omega,t+1}$ when forecasting its sales. Thus, we have

$$\log(q_{\omega,t+1}) - \log(q_{\omega,t}) \approx \frac{q_{\omega,t+1} - q_{\omega,t}}{q_{\omega,t}} \approx \frac{\mathbb{E}_{\omega,t}[R_{\omega,t+1}] - R_{\omega,t}}{R_{\omega,t}} - \frac{\mathbb{E}_{\omega,t}[p_{\omega,t+1}] - p_{\omega,t}}{p_{\omega,t}},$$

where \approx means “approximately equal.” As both the expected sales growth and the expected (average) change of output prices are reported in the data, we are able to construct the (percentage and logarithm of) realized output growth from period t to period $t + 1$. Next, the logarithm of realized sales in period $t + 1$ can be stated as

$$\log R_{\omega,t+1} = \frac{\sigma - 1}{\sigma} \log(q_t(\omega)) + \frac{1}{\sigma} \log(C_t) + \frac{\delta}{\sigma} \log(P_t) + \frac{\sigma - \delta}{\sigma} \log(P_{i,t}) + \frac{1}{\sigma} a_t(\omega).$$

Note that C_t , P_t and $P_{i,t}$ are year or industry-year specific. Therefore, we can regress $\log(R_{\omega,t+1})$ on $\log(R_{\omega,t})$ and control for the industry-year fixed effects and the change in output, $\log(q_{\omega,t+1}) - \log(q_{\omega,t})$. This regression yields the estimates of the persistence as well as the standard deviation of innovations for the process of $\frac{1}{\sigma} a_t(\omega)$. As different firms likely have different long-run average demand shifters (i.e., different intercepts for the AR(1) process of the firm’s demand shifter), we calculate the difference between the realized sales and its over-time mean for a given firm. Then, we use the demeaned sales to run the AR(1) regression.

Table 6 represents the estimation results. We have tried to include different sets of

fixed effects, and the estimated persistence is robustly around 0.76, which is higher than the persistence of the two inflation processes. This translates to a quarterly persistence of 0.94, which is close to the value used in the literature.²⁴ As the theory predicts, a one percent increase in the output from period t to $t + 1$ (i.e., $\Delta q_{\omega,t-1,t} = 1$) results in a roughly 0.44% – 0.52% increase in sales. Finally, the estimated standard deviation of innovations to the demand shifter is between 0.081 to 0.104, which is much higher than the volatility of macro- and industry-level innovations. For future use, we choose the specification using the industry-year, size-year and region-year fixed effects as our main specification for the estimation. Under this specification, ρ_a is estimated to be 0.765, and the estimated standard deviation of innovations to the process of $\frac{a_t(\omega)}{\sigma}$ is 0.096. Table 10 in Online Appendix 6.1 reports the regression results using the alternative sample, which are similar to the regression results presented here.²⁵

Table 6: Processes of the demand shifter

Dep.Var:	$\log(R)_{\omega,t}^{demeaned}$			
$\log(R)_{\omega,t-1}^{demeaned}$	0.752*** (0.019)	0.764*** (0.018)	0.765*** (0.019)	0.754*** (0.025)
$\Delta q_{\omega,t-1,t}$	0.440*** (0.029)	0.431*** (0.028)	0.430*** (0.030)	0.518*** (0.033)
Constant	0.002 (0.002)	0.002 (0.002)	0.002 (0.002)	-0.001 (0.001)
industry fixed effects	Yes	No	No	No
region fixed effects	Yes	Yes	No	No
size-year fixed effects	Yes	Yes	Yes	Yes
industry-year fixed effects	No	Yes	Yes	Yes
region-year fixed effects	No	No	Yes	Yes
firm fixed effects	No	No	No	Yes
N	4249	4231	4118	3880
R^2	0.624	0.659	0.679	0.773

Notes: We regress the logarithm of demeaned sales in period t on its one-period lag and the percentage change in output (i.e., quantity) produced from period $t - 1$ to period t . Standard errors are clustered at the firm level and reported in parentheses. Top and bottom one percent of the logarithm of firm sales are trimmed. $\log(R)_{\omega,t}^{demeaned}$ is the logarithm of firms i 's sales in period t , while $\Delta q_{\omega,t-1,t}$ is the percentage change output produced from period $t - 1$ to period t (i.e., $\Delta q_{\omega,t-1,t} = 1$ means 1%). Note that each firm belongs to one of the four size-based bins in the data. The regression controls for various fixed effects such as the industry-year, size-year, and region-year fixed effects. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

²⁴Bloom et al. (2018) used 0.95 as the quarterly-level persistence of the demand/productivity shock.

²⁵The estimated persistence is 0.686, and the standard deviation of innovations is 0.089.

4.2 Serial Correlations

Next, we present the results of the serial correlation regressions for the three target variables in Table 7. We use the residual FE of the industry-specific inflation rate to run the AR(1) regression, as we want to exclude the component of the FE (of the industry-specific inflation rate) that comes from the mis-forecast of the macro inflation. Similarly, we use the residual FE of sales to run the serial correlation regression, as we want to exclude the component of the FE of sales that comes from the mis-forecast of the macro-level and industry-level variables.²⁶ We observe that the serial correlations of FEs of all three variables are positively significant, substantiating the existence of information rigidity. Table 11 in Online Appendix 6.1 reports regression results using the alternative sample.

We performed several robustness checks to confirm the estimated serial correlation coefficients presented in Table 7. Specifically, we tried various sets of fixed effects to estimate the serial correlation regressions. The estimated serial correlation is around 0.34 for the FE of macro inflation and around 0.2 for the FE of industry-specific inflation. For the serial correlation of FE of sales, the estimated coefficient is around 0.14 (percentage sales FE) and 0.12 (logarithm of sales FE).²⁷ One potential concern for the regression of FE of the industry-specific inflation rate is that there are several industries that have only a few observations of industry-specific inflation forecasts each year (see Table 4). Thus, the law of large numbers might not hold when we average out the firm-specific noise terms in a given year. In Table 12 of Online Appendix 6.2, we exclude industries where the number of (industry-specific) inflation expectations is too small (e.g., 150 or 225 over 15 years) and rerun the serial correlation regression of FEs of the industry-specific inflation rate. The estimated coefficients are very similar to the one reported in Table 7.

Based on equations (20), (21), (23), and estimates presented in Tables 5-7, we derive the

²⁶For the FE of the industry-specific inflation rate, size-bin-year fixed effects are teased out from the original FE. For the FE of sales, size-bin-industry-year fixed effects are teased out from the original FE.

²⁷Results are available upon request.

Table 7: Serial correlation of forecast errors

Dep.Var:	$\mathbb{FE}_{\omega,t}^{\pi}$	$\mathbb{FE}_{\omega,t}^{\pi^i}$	$\mathbb{FE}_{\omega,t}^{pct,sales}$	$\mathbb{FE}_{\omega,t-1}^{log,sales}$
$\mathbb{FE}_{\omega,t-1}^{\pi}$	0.349*** (0.012)			
$\mathbb{FE}_{\omega,t-1}^{\pi^i}$		0.197*** (0.021)		
$\mathbb{FE}_{\omega,t-1}^{res\ pct,sales}$			0.143*** (0.021)	
$\mathbb{FE}_{\omega,t-1}^{res\ log,sales}$				0.119*** (0.026)
$log(sales)_{\omega,t-1}$	-0.011 (0.008)	0.033 (0.025)	0.002 (0.001)	-0.000 (0.001)
Constant	0.071 (0.207)	-0.701 (0.613)	-0.062* (0.034)	0.008 (0.033)
year fixed effects	No	Yes	No	No
industry fixed effects	Yes	Yes	No	No
region fixed effects	Yes	Yes	Yes	Yes
industry-year fixed effects	No	No	Yes	Yes
N	4689	4061	3828	3844
R^2	0.154	0.246	0.253	0.137

Notes: Standard errors are clustered at the firm level and reported in parentheses. $\mathbb{FE}_{\omega,t}^{\pi}$ is the forecast error of the macro inflation rate. Forecast errors of the industry-specific inflation rate are residual forecast errors. That is, size-bin-year fixed effects are teased out from the original forecast errors. As a result, $\mathbb{FE}_{\omega,t}^{\pi^i}$ is the residual forecast error of the industry-specific inflation rate. Forecast errors of sales are residual forecast errors. That is, size-bin-industry-year fixed effects are teased out from the original forecast errors. $\mathbb{FE}_{\omega,t}^{pct,sales}$ is the residual forecast error of firm sales in percentage term. $\mathbb{FE}_{\omega,t}^{res\ log,sales}$ is the residual logarithm of the forecast error of firm sales. The top and bottom 1% of the FEs are trimmed. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Kalman gains for the three target variables as follows:

$$G_\pi = 0.457; G_{\pi^i} = 0.567; \text{ and } G_a = 0.844.$$

Using (12), we infer the corresponding values of the channel capacity as follows:

$$\kappa_\pi = 0.305; \kappa_{\pi^i} = 0.419; \text{ and } \kappa_a = 0.930. \quad (24)$$

Therefore, the firm allocates most attention to forecasting the demand shifter and least attention to forecasting the (macro) inflation rate. This is consistent with our estimates of the three processes, as the process of the demand shifter is more volatile than the two inflation processes.

Finally, we back out the unobservable marginal cost of acquiring and processing information of the three target variables using equation (16) and the three sensitivity parameters. Equation (19) reveals that we need to know values of δ , σ , and ρ_a in order to back out the sensitivity parameters. Following the literature (e.g., Bernard et al. (2003)), we set the elasticity of substitution between firms within an industry, σ , to four. This leads to a mark-up rate of 33%. For the elasticity of substitution between industries, δ , we think it should be close to one as the industries in our data are broad industries.²⁸ Thus, we set its value to either 1.5 or 1.2. Since the estimated ρ is 0.765, we have the following two sets of values for the sensitivity parameters:

$$w_\pi = 2.25, w_{\pi^i} = 6.25, \text{ and } w_a = 1.51, \quad (25)$$

and

$$w_\pi = 1.44, w_{\pi^i} = 7.84, \text{ and } w_a = 1.51. \quad (26)$$

²⁸In the structural change literature, the elasticity of substitution between (three) big sectors is assumed to be less than one.

Based on equation (16) and estimated Kalman gains in equation (24), we can back out the three marginal costs of acquiring and processing information as:

$$\lambda_{\pi} = 0.972 \cdot 10^{-4} \ ; \ \lambda_{\pi^i} = 7.18 \cdot 10^{-4} \ ; \ \text{and} \ \lambda_a = 2.04 \cdot 10^{-4},$$

under the parameter values specified in equation (25) and

$$\lambda_{\pi} = 0.622 \cdot 10^{-4} \ ; \ \lambda_{\pi^i} = 9.01 \cdot 10^{-4} \ ; \ \text{and} \ \lambda_a = 2.04 \cdot 10^{-4},$$

under the parameter values specified in equation (26).

The interesting result is that the marginal cost of acquiring and processing information is highest for the industry-specific inflation process and lowest for the process of macro inflation. Note that the degree of information rigidity associated with forecasting the industry-specific inflation process is revealed to be quite high, while its importance in the firm's payoff function (i.e., $(\sigma - \delta)^2$) and volatility are quite high as well. The latter two findings usually imply a very low degree of information rigidity (conditioning on the information cost). The only way to rationalize these three findings is that the marginal cost of acquiring and processing industry-level information is so high that the firm lacks sufficient incentive to allocate substantial attention to industry-level information. Turning to macro inflation, we know that innovations to this process are extremely non-volatile and the importance of forecasting it (correctly) is low, which imply a small marginal benefit of reducing perceived uncertainty. As the marginal cost and benefit are equalized when the firm chooses channel capacity, the cost of acquiring and processing information concerning the macro inflation process must be very low. Finally, the cost of acquiring and processing information is relatively small for the process of firm-specific demand, as the revealed information rigidity is low and the importance of forecasting it (correctly) is low as well.

We believe that it makes sense for the cost of collecting and analyzing industry-level information to be the highest among the three target variables, as the firm in our dataset

is large (i.e., publicly traded) and thus basically analyzes its competitors' pricing behavior when it comes to forecasting the industry-level inflation. Analyzing macro information is not as costly as analyzing industry-level information, as the firm analyzes the whole economy, which is more stable than each industry. For firm-specific demand, analyzing it is probably less costly than analyzing industry-level information, as the firm has enough internal data for this process (while it does not have its competitors' internal data). In summary, we uncover substantial heterogeneity with respect to the cost of acquiring and processing information concerning different target variables, which is new to the literature. Moreover, we show that the information cost for analyzing industry-level information is extremely high. Therefore, improving firms' ability to analyze this type of information is likely to increase the firm's payoff substantially, which we will discuss in Section 4.4.

4.3 Standard Deviation of FEs

In this subsection, we evaluate how effectively the standard deviations of various FEs implied by the model match with their counterparts in the data. Equations (13) and (15) imply that the standard deviations implied by the model are

$$\Psi_{\pi}^{theory} = 1.0\%; \Psi_{\pi^i}^{theory} = 2.89\%; \text{ and } \Psi_a^{theory} = 0.101,$$

where Ψ denotes the standard deviation. Summary statistics in Table 3 reveal that

$$\Psi_{\pi}^{data} = 1.0\%; \Psi_{\pi^i}^{data} = 2.83\%; \text{ and } \Psi_a^{data} = 0.090.$$

In total, the standard deviations of FEs implied by our calibrated model are quite close to those calculated from the data, although we do not target these moments.

4.4 Gain from Removing Information Rigidities

In this subsection, we implement a simple back-of-the-envelope calculation of the payoff gains when we remove information rigidities (i.e., setting the information cost to zero). Based on equation (18) and the variance of various FEs implied by our calibrated model, the overall gain from removing information frictions is

$$Gain = Gain_{\pi} + Gain_{\pi^i} + Gain_a = 2.25 * (1\%)^2 + 6.25 * (2.89\%)^2 + 1.51 * 0.101^2 \approx 2.08\%,$$

under the parameter values specified in equation (25) and

$$Gain = Gain_{\pi} + Gain_{\pi^i} + Gain_a = 1.44 * (1\%)^2 + 7.84 * (2.89\%)^2 + 1.51 * 0.101^2 \approx 2.21\%,$$

under the parameter values specified in equation (26). We refrain from emphasizing the magnitudes of the gains, as our model is a stylized and partial-equilibrium model. Rather, we want to emphasize the contribution made by removing the information rigidity associated with forecasting each target variable to the overall gain. The contribution depends on two factors positively: the importance of forecasting the target variable (correctly) for the firm's payoff and the variance of the FE of this variable. We find that removing information frictions concerning forecasting the macro inflation only increases the firm's payoff slightly (less than 2% of the overall gain), as the importance of forecasting this variable (correctly) is low and the variance of its FE is small. In fact, the payoff gain from *only* removing information frictions associated with forecasting the macro inflation in our model ranges between 0.0144% and 0.0225%, which is consistent with the finding of a small welfare gain from the literature (e.g., Luo (2008) and Maćkowiak and Wiederholt (2015)). On the contrary, removing information frictions associated with forecasting the industry-specific inflation and firm-specific demand accounts for most of the overall gains, as the importance of forecasting the former variable (correctly) is high, and the variance of the FE of the latter variable is by far the largest

among the three target variables. In summary, we find that helping firms collect and digest industry- and firm-level information is at least as important as helping them collect and digest macro information if the firm's payoff function is the objective function.

5 Conclusion

In this study, we utilize a novel Japanese firm-level panel dataset that contains quantitative forecasts of the macro inflation rate, the industry-specific inflation rate, and firm sales to infer the corresponding degrees of information rigidities at the macro-, industry- and firm-levels. We find that the degree of information rigidity concerning forecasting the macro target is higher than the one associated with forecasting industry inflation and firm's demand. This is consistent with the predictions of the RI model with elastic attention proposed in Sims (2010) and supports the argument in favor of the existence of state-dependent information rigidities. Moreover, we also use our model and data moments to back out the unobservable marginal cost of acquiring and processing information for each of the three target variables. It is shown that the information cost concerning collecting and digesting industry-level information is the highest among the three variables. In addition, we find that removing information rigidities associated with forecasting the industry-specific inflation rate and firm-specific demand would increase the firm's payoff substantially more than removing the information rigidity associated with forecasting the macro inflation.

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6 Online Appendix (Not for Publication)

6.1 Empirical Results using an Alternative Dataset

In this subsection, we present empirical results using the alternative dataset. Based on equations (20), (21), (23), and the estimates presented in Tables 5, 10, 11, we derive the Kalman gains for the three target variables as follows:

$$G_\pi = 0.443; G_{\pi^i} = 0.374; \text{ and } G_a = 0.827.$$

The implied channel capacities are

$$\kappa_\pi = 0.293; \kappa_{\pi^i} = 0.234; \text{ and } \kappa_a = 0.877. \quad (27)$$

Therefore, the firm devotes most of its attention to forecasting the process of its demand shifter and least attention to forecasting the macro and industry-specific inflation rates.

Based on equation (16) and estimated Kalman gains in equation (27), we calculate the marginal cost of acquiring and processing information as

$$\lambda_\pi = 1.008 \cdot 10^{-4} ; \lambda_{\pi^i} = 11.34 \cdot 10^{-4} ; \text{ and } \lambda_a = 3.40 \cdot 10^{-4}.$$

under the parameter values specified in equation (25) and

$$\lambda_\pi = 0.645 \cdot 10^{-4} ; \lambda_{\pi^i} = 14.23 \cdot 10^{-4} ; \text{ and } \lambda_a = 3.40 \cdot 10^{-4}.$$

under the parameter values specified in equation (26). The marginal cost of acquiring and processing information is the highest for the industry-specific inflation process and the lowest for the process of macro inflation. The standard deviations of inflation/demand innovations

implied by the model are

$$\psi_{\pi}^{theory} = 1.04\%; \psi_{\pi^i}^{theory} = 2.96\%; \text{ and } \psi_a^{theory} = 0.093.$$

where ψ denotes the standard deviation. Summary statistics in Table 3 reveal that

$$\psi_{\pi}^{data} = 1.01\%; \psi_{\pi^i}^{data} = 2.48\%; \text{ and } \psi_a^{data} = 0.11.$$

In total, the standard deviation of FEs implied by the theory is quite close to those calculated from the data for each of the targeted variables, although we do not target these moments.

Table 8: Summary statistics of the inflation rates (sales forecasts made in Jan.)

	Obs.	mean	std. dev.	median
realized macro-level inflation rate	9405	-0.35%	1.17%	-0.70%
forecasted macro-level inflation rate	8165	-0.07%	0.77%	0.00%
realized industry-specific inflation rate	9405	0.15%	2.76%	0.19%
forecasted industry-specific inflation rate	7109	-0.02%	0.74%	0.00%

Notes: Realized macro-level and industry-specific inflation rates (23 industries) are obtained from the website of the Economic and Social Research Institute (ESRI) within the Cabinet Office and refer to the fiscal year (April to March). To exclude outliers, we trim the top and bottom one percent of observations of the forecasts. Time span: 2004-2017 (fiscal years).

Table 9: Summary statistics of forecast errors (sales forecasts made in Jan.)

	Obs.	mean	std. dev.	median
forecast error of macro-level inflation rate	7414	-0.26%	1.01%	-0.40%
forecast error of industry-specific inflation rate	6497	0.32%	2.48%	0.33%
(percentage) forecast error of sales	2615	-0.30%	10.43%	-0.02%
(logarithm) forecast error of sales	2614	-0.01	0.11	-0.00
residual forecast error of industry-specific inflation rate	6498	-0.02%	2.10%	-0.075%

Notes: Realized macro-level and industry-specific inflation rates (23 industries) are obtained from the website of the Economic and Social Research Institute (ESRI) within the Cabinet Office and refer to the fiscal year (April to March). The forecast error is defined as the difference between the realized value from the forecasted value. To exclude outliers, we trim the top and bottom one percent of observations of the FEs. Time span: 2004-2017 (fiscal years).

Table 10: Processes of the demand shifter (sales forecasts made in Jan.)

Dep.Var:	$\log(R)_{\omega,t}^{demeaned}$			
$\log(R)_{\omega,t-1}^{demeaned}$	0.657*** (0.031)	0.689*** (0.031)	0.686*** (0.034)	0.576*** (0.050)
$\Delta q_{\omega,t-1,t}$	0.330*** (0.043)	0.320*** (0.042)	0.307*** (0.047)	0.367*** (0.064)
Constant	0.006*** (0.002)	0.007*** (0.002)	0.006** (0.002)	0.005*** (0.001)
industry fixed effects	Yes	No	No	No
region fixed effects	Yes	Yes	No	No
size-year fixed effects	Yes	Yes	Yes	Yes
industry-year fixed effects	No	Yes	Yes	Yes
region-year fixed effects	No	No	Yes	Yes
firm fixed effects	No	No	No	Yes
N	2009	1973	1857	1594
R^2	0.527	0.601	0.633	0.752

Notes: We regress the logarithm of demeaned sales in period t on its one-period lag and the percentage change in output (i.e., quantity) produced from period $t - 1$ to period t . Standard errors are clustered at the firm level and reported in parentheses. Top and bottom one percent of the logarithm of firm sales are trimmed. $\log(R)_{\omega,t}^{demeaned}$ is the logarithm of firms i 's sales in period t , while $\Delta q_{\omega,t-1,t}$ is the percentage change output produced from period $t - 1$ to period t (i.e., $\Delta q_{\omega,t-1,t} = 1$ means 1%). Note that each firm belongs to one of the four size-based bins in the data. The regression controls for various fixed effects such as the industry-year, size-year, and region-year fixed effects. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 11: Serial correlation of FEs (sales forecasts made in Jan.)

Dep.Var:	$\mathbb{FE}_{\omega,t}^{\pi}$	$\mathbb{FE}_{\omega,t}^{\pi^i}$	$\mathbb{FE}_{\omega,t}^{pct,sales}$	$\mathbb{FE}_{\omega,t-1}^{log,sales}$
$\mathbb{FE}_{\omega,t-1}^{\pi}$	0.358*** (0.013)			
$\mathbb{FE}_{\omega,t-1}^{\pi^i}$		0.285*** (0.023)		
$\mathbb{FE}_{\omega,t-1}^{res\ pct,sales}$			0.110*** (0.040)	
$\mathbb{FE}_{\omega,t-1}^{res\ log,sales}$				0.119*** (0.044)
$\log(sales)_{\omega,t-1}$	-0.012 (0.010)	-0.006 (0.023)	0.003 (0.003)	0.002 (0.003)
Constant	0.121 (0.207)	0.165 (0.565)	-0.068 (0.66)	-0.043 (0.064)
year fixed effects	No	Yes	No	No
industry fixed effects	Yes	Yes	No	No
region fixed effects	Yes	Yes	Yes	Yes
industry-year fixed effects	No	No	Yes	Yes
N	3963	3471	1297	1306
R^2	0.162	0.243	0.149	0.141

Notes: Standard errors are clustered at the firm level and reported in parentheses. $\mathbb{FE}_{\omega,t}^{\pi}$ is the forecast error of the macro inflation rate. Forecast errors of the industry-specific inflation rate are residual forecast errors. That is, size-bin-year fixed effects are teased out from the original forecast errors. As a result, $\mathbb{FE}_{\omega,t}^{\pi^i}$ is the residual forecast error of the industry-specific inflation rate. Forecast errors of sales are residual forecast errors. That is, size-bin-industry-year fixed effects are teased out from the original forecast errors. $\mathbb{FE}_{\omega,t}^{pct,sales}$ is the residual forecast error of firm sales in percentage term. $\mathbb{FE}_{\omega,t}^{res\ log,sales}$ is the residual logarithm of forecast error of firm sales. The top and bottom 1% of the FEs are trimmed. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

6.2 Regression Results that Exclude Industries with Few Observations

In this subsection, we rerun the serial correlation regression of the FE of industry-specific inflation by excluding industries that have few observations. Regression results are reported in Table 12.

Table 12: Serial correlation of FEs of industry-specific inflation rate

Dep.Var:	$\text{FE}_{\omega,t}^{\pi^i}$			
$\text{FE}_{\omega,t-1}^{\pi^i}$	0.186*** (0.022)	0.190*** (0.023)	0.185*** (0.022)	0.190*** (0.023)
$\log(\text{sales})_{\omega,t-1}$	0.028 (0.026)	0.028 (0.027)	0.020 (0.028)	0.019 (0.029)
Constant	-0.603 (0.635)	-0.601 (0.661)	-0.408 (0.683)	-0.374 (0.718)
year fixed effects	Yes	No	Yes	No
industry fixed effects	Yes	Yes	Yes	Yes
region fixed effects	Yes	No	Yes	No
region-year fixed effects	No	Yes	No	Yes
Sample (obs. of forecasts)	≥ 150	≥ 150	≥ 225	≥ 225
N	3815	3696	3510	3381
R^2	0.241	0.292	0.241	0.292

Notes: Standard errors are clustered at the firm level and reported in parentheses. Forecast errors of the industry-specific inflation rate are residual forecast errors. That is, size-bin-year fixed effects are teased out from the original forecast errors. As a result, $\text{FE}_{\omega,t}^{\pi^i}$ is the residual forecast error of the industry-specific inflation rate. In the first two columns, we exclude industries with observations less than 150. In the last two columns, we exclude industries with observations less than 225. The top and bottom 1% of FEs are trimmed. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

6.3 Theoretical Appendix

In this theoretical appendix, we prove that the loss function due to information rigidities is minimized when the firm minimizes the variance of the forecasting/filtering error of the output. Note that the firm's profit is given by

$$\Pi_t(\omega) = q_t(\omega) \left[\left(C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} \right)^{\frac{1}{\sigma}} q_t(\omega)^{-\frac{1}{\sigma}} - w_t \right],$$

where we have used the result that $p_t(\omega) = (C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)})^{\frac{1}{\sigma}} q_t(\omega)^{-\frac{1}{\sigma}}$. Under full information, the optimal output be written as:

$$q_t^{full}(\omega) = \left(\frac{\sigma - 1}{\sigma} \right)^\sigma \frac{C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)}}{w_t^\sigma}.$$

Taking log on both sides yields:

$$\log q_t^{full}(\omega) = \sigma \log \left(\frac{\sigma - 1}{\sigma} \right) + \log C_t + \delta \log P_t + (\sigma - \delta) \log P_{i,t} + a_t(\omega) - \sigma \log(w_t). \quad (28)$$

Thus, the profit under full information is

$$\Pi_t^{full}(\omega) = \frac{1}{\sigma} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma-1} C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} w_t^{1-\sigma}.$$

Now, we use Taylor expansion (up to the second-order) to approximate the profit function under information rigidities. Specifically, we have

$$\begin{aligned} \Pi_t(\omega) &= q_t(\omega) \left[(C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)})^{\frac{1}{\sigma}} q_t(\omega)^{-\frac{1}{\sigma}} - w_t \right] \\ &\approx \Pi_t^{full}(\omega) - \frac{1}{2\sigma} \frac{w_t^{\sigma+1}}{C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)}} \left(\frac{\sigma - 1}{\sigma} \right)^{-\sigma} \left(q_t(\omega) - q_t^{full}(\omega) \right)^2. \end{aligned}$$

Firm revenue under full information can be written as

$$\begin{aligned} R_t^{full}(\omega) &= \left(q_t^{full}(\omega) \right)^{\frac{\sigma-1}{\sigma}} (C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)})^{\frac{1}{\sigma}} \\ &= \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma-1} C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} w_t^{1-\sigma} \\ &= \sigma \Pi_t^{full}(\omega). \end{aligned}$$

The normalized loss function can thus be written as:

$$\begin{aligned}\frac{\Pi_t(\omega) - \Pi^{full}(\omega)}{\Pi_t^{full}(\omega)} &= -\frac{\sigma - 1}{2\sigma} \left(\frac{q_t(\omega) - q_t^{full}(\omega)}{q_t^{full}(\omega)} \right)^2 \\ &= -\frac{\sigma - 1}{2\sigma} \left(\log q_t(\omega) - \log q_t^{full}(\omega) \right)^2.\end{aligned}$$

Under imperfect information on the target variables, the optimal choice of output can be written as:

$$\log q_t(\omega) = \sigma \log \left(\frac{\sigma - 1}{\sigma} \right) + \sigma \log \left[\mathbb{E}_{\omega, t-1} \left(C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} \right)^{\frac{1}{\sigma}} \right] - \sigma \log(w_t). \quad (29)$$

Therefore, the variance of the output deviation from the full-information scenario can be expressed as

$$\mathbb{E}_{\omega, t-1} \left(\log q_t(\omega) - \log q_t^{full}(\omega) \right)^2 = \sigma^2 \mathbb{E}_{\omega, t-1} \left[\log \left(C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} \right)^{\frac{1}{\sigma}} - \log \left(\mathbb{E}_{\omega, t-1} \left(C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} \right)^{\frac{1}{\sigma}} \right) \right]^2,$$

which is the variance of the forecasting error of $\left(C_t P_t^\delta P_{i,t}^{\sigma-\delta} e^{a_t(\omega)} \right)^{\frac{1}{\sigma}}$. Using equations (28) and (29), we can easily obtain the expression for the expected loss of the output change due to information rigidities.