The Welfare Implications of Massive Money Injection: The Japanese Experience from 2013 to 2020

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The Welfare Implications of Massive Money Injection: The Japanese Experience from 2013 to 2020

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Abstract

This paper derives a money demand function that explicitly takes the costs of storing money into account. This function is then used to examine the consequences of the large-scale money injection conducted by the Bank of Japan since April 2013. The main findings are as follows. First, the opportunity cost of holding money calculated using 1-year government bond yields has been negative since the fourth quarter of 2014 and most recently (2020:Q2) was -0.2%. Second, the marginal cost of storing money, which was 0.3% in the most recent quarter, exceeds the marginal utility of money, which was 0.1%. Third, the optimum quantity of money, measured by the ratio of M1 to nominal GDP, is 1.2. In contrast, the actual money-income ratio in the most recent quarter was 1.8. The welfare loss relative to the maximum welfare obtained under the optimum quantity of money in the most recent quarter was 0.2% of nominal GDP. The findings imply that the Bank of Japan needs to reduce M1 by more than 30%, for example through measures that impose a penalty on holding money.

JEL Codes: E31; E41; E43; E51; E52
Keywords: deflation; quantitative easing; money demand functions; Friedman rule; optimum quantity of money; satiation level of real money balances; money storage costs; ZLB (zero lower bound)

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1 Introduction

More than seven years have passed since the Bank of Japan (BOJ) welcomed Haruhiko Kuroda as its new Governor and started a new regime of monetary easing, which was nicknamed the "Kuroda bazooka." The policy goal that the BOJ set itself was to overcome deflation. The measure the BOJ chose to escape deflation was to print lots of money. Specifically, in April 2013, the BOJ announced that it would double the monetary base within two years and thereby raise the consumer price index (CPI) inflation rate to 2%. However, currently, CPI inflation remains stuck at 0.3%. The BOJ has not achieved its target of 2%, and there is little prospect that it will be achieved in the near future. This paper seeks to examine the economic effects of the Kuroda bazooka and, by doing so, clarify where the BOJ and the Japanese economy stand now. Further, based on this, it discusses where the BOJ and its monetary policy are headed and where they should be headed.

The remainder of the paper is organized as follows. Section 2 provides a brief overview of the consequences of the BOJ's massive money injection since 2013. Section 3 derives a money demand function that explicitly takes the cost of storing money into account. Section 4 then presents the empirical results. Finally, Section 5 discusses the policy implications.

2 Consequences of Massive Money Injection in 2013-2020

2.1 Things that were expected to happen but did not happen

Why was the Kuroda bazooka unsuccessful? The reason is unforeseen developments – in two senses. First, things expected by the BOJ and academic researchers, including the author, in 2013 did not come to pass.

The first thing that many had expected in 2013 but that did not happen in practice is a rise in household inflation expectations. The BOJ made a major switch in its approach and started to actively communicate to households that it would raise the inflation rate in the future. Its calculation was that this would lower real interest rates and stimulate aggregate demand, so that prices would rise. However, various surveys on inflation expectations, including one conducted by the BOJ itself, indicate that while inflation expectations rose slightly in the early days of the Kuroda bazooka regime, this did not last, and expectations have now returned more or less to the level before the Kuroda bazooka.[1]

The second thing that was expected to happen but did not happen was a change in firms' pricing behavior. At the start of the Kuroda bazooka, many thought that if the money supply was increased, the yen would depreciate, the prices of imported raw materials would rise, and firms would pass on the increased costs to product prices. The Kuroda bazooka was successful in bringing about a large depreciation of the yen. However, the number of firms that passed on increased costs to product prices was limited.

2.2 Things that no one anticipated to happen but that did happen

While what was supposed to happen did not happen, things also happened that no one had anticipated. In this regard, what this paper particularly focuses on is the consequences of the large-scale increase in the quantity of money. In order for the BOJ to increase the money supply, it needs to purchase assets issued by the government and private firms. Potential negative effects of the BOJ's large-scale asset purchases on the economy were extensively discussed from quite an early stage. For example, it was highlighted that the large-scale purchases of government bonds by the BOJ could loosen the government's fiscal discipline. On the other hand, the BOJ's large-scale purchases of exchange traded funds (ETFs) might give rise to distortions in the governance of private companies.

However, interestingly, the debate about the side effects of the Kuroda bazooka focused only on the BOJ's asset purchases, and, to the best of the author's knowledge, there was no debate about the possible side effects of the increase in the money supply itself. However, as detailed in Sections 3 and 4, it is highly likely that the large quantity of money supplied through the Kuroda bazooka itself may have had an adverse effect on the economy.

When the Kuroda bazooka program started, researchers, including the author, had no idea that the large scale money injection itself would adversely affect the economy. It is only natural that the BOJ, which believes that by increasing the money supply it is possible to overcome deflation, thought that the more the money supply is increased, the better this would be for the economy. However, why did researchers – including those critical of the Kuroda bazooka – not realize that the large-scale money injection itself could have an adverse effect on the economy?

The reason is that the academic debate on the liquidity trap and the zero lower bound (ZLB) on nominal interest rates is based on the assumption that there is a finite satiation level for real money balances. At the time, the depiction of Japan's economy that attracted the most attention was that by Krugman (1998), who made the famous argument that deflation could be overcome if it was possible to instill in people the expectation that there would be sufficiently high inflation in the future. However, Krugman (1998) makes important points not only about inflation expectations but also about money. In his model based on the cash-in-advance constraint, when deciding whether to hold money or bonds, households weigh up the convenience of money, which can be used to pay for purchases, and the interest income earned from bonds. When the money supply becomes extremely large, interest rates will drop to zero; at the same time, a situation arises in which holding more money does not provide additional convenience to money holders. That is, a point is reached at which people's demand for money is satiated. Krugman's analysis is based on the assumption that the demand for money is satiated in an economy with zero interest rates, and his policy recommendations crucially depend on this.

An important implication of Krugman's (1998) analysis is that a further increase in the money supply has no effect on the economy, since real money balances have reached their satiation level when interest rates are zero. Eggertsson and Woodford (2003) presented this as their "irrelevance proposition." Their message that increasing the money supply beyond this satiation level has no effect on the economy is often interpreted as simply meaning that any further increases in the money supply will have no positive impact such as raising the rate of inflation. However, this can also be interpreted as meaning that even if large quantities of money are supplied, nothing bad will happen. While it is unclear how the irrelevance proposition affected the BOJ's policy making, what can be said for certain is that mainstream researchers, including the author, on the basis of this proposition thought that increases in the money supply beyond the satiation level were neither "medicine" nor "poison" and therefore did not stop the BOJ in turning toward massive money injection. In other words, researchers took the position of being "not dissatisfied" with the Kuroda bazooka.

But what actually happened as a result of the large-scale money injection? To address this question, it is important to focus on the relationship that the marginal utility of money and the opportunity cost of holding money are equal, which comes directly from households' optimization. Money here refers to the money held by households and non-financial firms, specifically M1. The opportunity cost of holding money is equal to the return on interest bearing financial assets less the return on money (for example, interest on money held in savings accounts, which is included in M1). The marginal utility of money decreases as the amount of money held increases, but it never becomes negative, so that the opportunity cost should be non-negative as well. However, when calculating the opportunity cost using Japanese data, it often takes a negative value. For example, the opportunity cost calculated using 1-year government bond yields has been negative since the fourth quarter of 2014 and in the fourth quarter of 2016 fell as low as –0.3%.

Why does the opportunity cost take a negative value? Recent studies on negative interest rate policies, such as Eggertsson et al. (2019), argue that while holding money provides utility, its storage involves costs. Specifically, they suggest that the cost of storing money depends on the amount of money held and that the marginal cost of storage is an increasing function of the amount of money held.[2] Under these circumstances, what people compare the opportunity cost with is the marginal utility of money minus the marginal cost of storing money. The marginal utility of money is non-negative, but the marginal cost of storage is positive, so the difference between the two can be negative.[3] Therefore, the opportunity cost can also be negative.

The presence of non-negligible storage costs of money has important welfare implications for the Kuroda bazooka. As the money supply increases, the marginal utility of money decreases and approaches zero. On the other hand, the marginal cost of storing money increases as the money supply increases. Therefore, the marginal utility and marginal cost coincide at a certain finite level of real money balances. Economic welfare increases as the amount of money increases as long as the amount of money is below the threshold. However, once the quantity of money exceeds the threshold, increases in the quantity of money deteriorate economic welfare.

According to the estimates in Section 4, for Japan, the optimum quantity of money is given by 1.2 in terms of the ratio of M1 to nominal GDP. In contrast, the actual ratio of

M1 to GDP in the second quarter of 2020 was 1.8, meaning that the quantity of money in Japan is excessive. The BOJ made the opportunity cost of holding money too low, thereby creating excessive demand for money. Measuring economic welfare following Bailey (1956), the welfare loss relative to the maximum welfare obtained under the optimum quantity of money currently (as of 2020: Q2) corresponds to 0.2% of nominal GDP.

3 Demand for Money in an Economy with Non-Negligible Storage Costs of Money

This section sets up the theoretical framework for the empirical analysis conducted in Section 4. Specifically, a money demand function explicitly taking the storage costs of holding money into account is derived. Next, Bailey's (1956) measure for the welfare cost of inflation is calculated to discuss how economic welfare depends on the quantity of money held by households and non-financial firms. Finally, Friedman's (1969) optimum quantity of money in an economy with money storage costs is discussed.

3.1 The case of no money storage costs

Let me start with a version of Sidrauski's (1967) model. The representative household maximizes the present value of the sum of utilities,

$$U_t = \sum_{T=t}^{\infty} \beta^{T-t} U(c_T, z_T)$$

where c and z denote consumption and real money balances. Following Lucas (2000), it is assumed that the current period utility function is given by

$$U(c, z) = \frac{1}{1-\sigma} \left[c\varphi\left(\frac{z}{c}\right) \right]^{1-\sigma} \tag{1}$$

where $\sigma > 0$ and $\sigma \neq 1$, and $\varphi(\cdot)$ is a strictly increasing and concave function. $\varphi(\cdot)$ will be specified later. The household faces the following flow budget constraint:

$$M_t + B_t = (1 + i_{t-1}^m) M_{t-1} + (1 + i_{t-1}) B_{t-1} + P_t y_t - P_t c_t \tag{2}$$

where B_t, i_t, i_t^m, P_t, and y_t respectively denote the amount of a one-period risk-free bond held by the household, the nominal interest rate associated with the bond, the nominal interest rate on money (e.g., the interest rate on saving deposits), the price level, and income. The first order conditions for utility maximization imply that the optimal holding of money has to satisfy

$$\frac{U_z}{U_c} = \frac{\varphi'\left(\frac{z}{c}\right)}{\varphi\left(\frac{z}{c}\right) - \frac{z}{c}\varphi'\left(\frac{z}{c}\right)} = x \tag{3}$$

where x is the opportunity cost of holding money, which is defined as $x \equiv i - i^m$.

Following Lucas (2000), I consider an endowment economy characterized by a balanced growth equilibrium path on which the money growth rate is constant and maintained by a constant ratio of transfers to income. In this set-up, the money-income ratio, given by $m = z/y$, is also constant. Eq. (3) can then be rewritten as

$$\frac{\varphi'(m)}{\varphi(m) - m\varphi'(m)} = x$$

This implies that, if money demand is of log-log form, i.e.,

$$\begin{aligned} m(x) &= Ax^{\alpha} \qquad \text{for} \quad x > 0 \\ \text{or} \qquad \ln m &= \ln A + \alpha \ln(x) \end{aligned} \tag{4}$$

with $A > 0$ and $\alpha < 0$, the function $\varphi(\cdot)$ solves a differential equation of the form

$$\frac{\varphi'(m)}{\varphi(m)} = \frac{\psi(m)}{1 + m\psi(m)} = \frac{A^{-1/\alpha}m^{1/\alpha}}{1 + mA^{-1/\alpha}m^{1/\alpha}}$$

where $\psi(\cdot)$ is the inverse money demand function (i.e., $(\psi(\cdot) \equiv m^{-1}(\cdot))$. The solution to this differential equation is given by

$$\varphi(m) = \left(1 + A^{-\frac{1}{\alpha}}m^{\frac{1+\alpha}{\alpha}}\right)^{\frac{\alpha}{1+\alpha}}$$

Conversely, if the utility function (1) is specified as

$$U(c,z) = \frac{1}{1-\sigma}\left[c\left(1 + A^{-\frac{1}{\alpha}}\left(\frac{c}{z}\right)^{\frac{1+\alpha}{\alpha}}\right)^{\frac{\alpha}{1+\alpha}}\right]^{1-\sigma} \tag{5}$$

the money demand function derived from utility maximization is of log-log form.

3.2 Linear storage costs

Recent studies on negative interest rate policies such as Eggertsson et al. (2019) and Rognlie (2016) argue that the cost of holding cash is not negligible. Here, the storage cost of money is introduced into Sidrauski's (1967) model closely following Eggertsson et al. (2019). The flow budget constraint now changes to

$$M_t + B_t = (1 + i_{t-1}^m)M_{t-1} + (1 + i_{t-1})B_{t-1} + P_t y_t - P_t c_t - S(M_{t-1}) \tag{6}$$

where $S(M_{t-1})$ denotes the storage cost of money. Note that $S(M_{t-1})$ represents nominal storage costs and depends on nominal (rather than real) money balances. The first order conditions for utility maximization imply

$$\frac{U_z}{U_c} = \frac{\varphi'\left(\frac{z}{c}\right)}{\varphi\left(\frac{z}{c}\right) - \frac{z}{c}\varphi'\left(\frac{z}{c}\right)} = x + S'(M)$$

Following Eggertsson et al. (2019), it is assumed that the marginal storage cost is positive and constant, so that $S'(M) = \theta > 0$.

Starting with the utility function given by Eq. (5) yields a money demand function of the following form:

$$\begin{aligned} m &= A\,(x+\theta)^{\alpha} \qquad \text{for} \quad x > -\theta \\ \text{or} \qquad \ln m &= \ln A + \alpha \ln(x+\theta) \end{aligned} \tag{7}$$

which is close to log-log form but differs from it in that a constant term, θ, is added to x before taking the logarithm. Note that m takes a finite value when $x = 0$, which is an important difference from the case of no storage costs.

3.3 Convex storage costs

The assumption that the marginal cost of storage is constant is a good approximation as long as the quantity of money held is not that large. However, if the opportunity cost of holding money is so low that the demand for money is extremely large, it is inappropriate to assume that the marginal cost of storage is constant irrespective of the quantity of money held. Instead, it would be better to assume that the marginal cost of storing money increases with the quantity of money held (i.e., the cost of storage is convex). In this case, as x decreases, the quantity of money held increases, so that θ in (7) increases. Taking this into consideration, Eq. (7) is modified as follows:

$$\ln m = \ln A + \alpha h(x) \tag{8}$$

where $h(\cdot)$ is the log-like transformation proposed by Ravallion (2017) and is defined as

$$h(x) \equiv \begin{cases} \sinh^{-1}(\mu x) - \ln(2\mu) & x > 0 \\ \sinh(\mu x) - \ln(2\mu) & \text{otherwise} \end{cases} \tag{9}$$

where $\sinh(\cdot)$ is the hyperbolic sine transformation given by $\sinh(k) \equiv 1/2\,[\exp(k) - \exp(-k)]$, and $\sinh^{-1}(\cdot)$ is the inverse hyperbolic sine transformation given by $\sinh^{-1}(k) \equiv \ln(k + \sqrt{k^2+1})$. μ is a positive parameter. Note that, for $k > 0$, this transformation is "log-like" in that $h(k)$ becomes more like $\ln(2k)$ as k rises (i.e., $\lim_{k\to\infty}[h(k) - \ln(2k)] = 0$). However, $h(k)$ deviates from the log when k is close to zero or negative. Also, note that $h'(k) > 0$ and $h''(k) < 0$, so that $h(\cdot)$ is a concave transformation.

The money demand function represented by Eq. (8) has the following properties. First, when x is sufficiently large, $h(\cdot)$ is close to the log, so that the demand for money represented by Eq. (8) behaves like money demand functions of log-log form, such as Eqs. (4) and (7). Second, as $h'(\cdot) > 0$ and $h''(\cdot) < 0$, a marginal decline in the opportunity cost of money leads to a greater increase in the demand for money when the opportunity cost of money is at a lower level, as is the case with money demand functions of log-log and semi-log form. Third, the responsiveness of the demand for money with respect to the opportunity cost of money, namely dm/dx, can never be infinitely large, unlike in money demand functions of log-log form such as Eqs. (4) and (7). An important implication of the second and third properties is that there is no lower bound for the opportunity cost of money. A decline in the opportunity cost of money increases the demand for money, but with strictly convex storage costs, this will be associated with a higher marginal storage cost. As a result, the demand for money never becomes infinite even when the opportunity cost of money is near zero or below zero.

3.4 Bailey's (1956) measure for the welfare loss

Bailey (1956) defines the welfare cost of inflation in terms of how much lower welfare is in an economy with a positive opportunity cost of money than in an economy with a zero opportunity cost.A high opportunity of money typically arises from a high nominal interest rate on bonds, which is normally accompanied by a high inflation rate. High inflation, if not anticipated, erodes the real value of money, so that it can be regarded as a tax on money. Bailey (1956) shows that this inflation tax is costly since it creates a dead-weight loss like in the case of an excise tax on a commodity.

When the money demand function is given by Eq. (7), Bailey's (1956) measure for the welfare gain achieved by lowering the opportunity cost from x to zero is given by

$$\begin{aligned} w(x) &= \int_0^x m(k)dk - xm(x) \\ &= \frac{A}{1+\alpha}\left[(x+\theta)^{1+\alpha} - \theta^{1+\alpha}\right] - xA(x+\theta)^{\alpha} \end{aligned} \tag{10}$$

implying that $w'(x) > 0$ for $x > 0$ and $w'(x) < 0$ for $-\theta < x < 0$, and that $w'(0) = 0$ and $w''(0) > 0$. An important difference from the case without storage costs is that the optimum quantity of money takes a finite value even for the log-log money demand function, at which the marginal utility of money coincides with the marginal storage cost of money. Most importantly, any deviation from $x = 0$ (i.e., the Friedman rule), whether $x > 0$ or $x < 0$, results in a suboptimal outcome.[4]

3.5 Numerical example

Figure 1 shows the relationship between the opportunity cost of money and the money-income ratio (i.e., the money demand function) in the upper panel, the relationship between the money-income ratio and Bailey's measure for the welfare loss in the middle panel, and the relationship between the opportunity cost and Bailey's measure in the bottom panel. The blue dashed line in each panel corresponds to the case of no storage costs, while the red dotted and the green solid lines correspond respectively to linear and convex storage costs. The parameters are set as follows: $\alpha = -0.5$; $A = 1$; $\theta = 0.01$; and $\mu = 100$.

Let us start with the case of no storage costs. The money demand function presented in the upper panel shows that m monotonically increases as x declines, and that m takes an infinitely large value when x is zero. Put differently, the marginal utility of money decreases as m increases, but it is always strictly positive and never reaches zero, meaning that money satiation does not occur at a finite value of m but occurs only asymptotically. In the case of no storage costs, households hold more and more money balances as the opportunity cost declines, so that their welfare monotonically improves (i.e., Bailey's measure for the welfare cost decreases), which is shown in the middle and bottom panels. Note that, as shown in the bottom panel, the marginal welfare gain of lowering the opportunity cost from x toward zero is positive and *increases* as x comes closer to $x = 0$.

Turning to the case of linear storage costs, the demand for money monotonically increases as the opportunity cost declines, as in the case of no storage costs. An important difference from the previous case is that the demand for money takes a finite value even when the opportunity cost is zero. This means that the marginal utility of money coincides with the marginal storage cost of money at that level. The middle panel shows that welfare deteriorates (i.e., Bailey's measure for the welfare cost increases) as m deviates from the optimum quantity of money. The bottom panel shows that welfare deteriorates as x deviates from zero, regardless of whether $x > 0$ or $x < 0$, but the welfare deterioration is asymmetrically larger when x falls from zero than when it rises from zero.

Finally, the demand for money in the case of convex storage costs is also downward sloping, as in the previous two cases, but unlike in the previous cases, m does not approach a certain value as x declines, so that m continues to be finite even when x takes a large negative value. This means that there is no lower bound on the opportunity cost, unlike in the previous two cases.

The three cases shown in the figure have different implications for the conduct of monetary policy near the ZLB. If money storage costs are negligible, money satiation never occurs, so that central banks need not worry about the possibility of injecting too much money into the economy. All they should worry about is a shortage of money supply rather than excess supply of money. However, in the case of non-negligible storage costs, be they linear or convex, the marginal utility of money and the marginal storage cost of money can coincide at a finite level of real money balances, so that central banks need to pay attention to the risk of injecting too much money.[5]

4 Empirical Results

4.1 Money demand functions

Figure 2 shows developments in the money-income ratio and the opportunity cost of holding money over the last 40 years. The money-income ratio is defined as M1 divided by nominal GDP. The opportunity cost of money is defined as the difference between 1-year JGB yields and the interest rate on saving deposits. More specifically, M1, in addition to cash, contains saving deposits, i.e., interest-bearing deposits held for settlement purposes by households and firms (especially small firms) at commercial banks, which make up about 64% of M1. The opportunity cost is calculated by subtracting the interest rate on saving deposits multiplied by their share in M1 from 1-year JGB yields.

The opportunity cost thus calculated shows a rapid decline in the first half of the 1990s and reaches somewhere around zero in the mid-1990s. It has essentially remained near zero since then. However, this does not necessarily mean that there were no significant fluctuations in the opportunity cost. In fact, there were important changes in the opportunity cost even during this period. For example, the opportunity cost rose in the second quarter of 2006, deviating from zero. The BOJ ended quantitative easing in March 2006 and started to raise the policy rate in July 2006, resulting in an increase in 1-year JGB yields. Another episode during which the opportunity cost deviated from zero occurred in 2014:4Q, when it became negative. The opportunity cost has been below zero since then. This can be regarded as the result of the large-scale money injection through the Kuroda bazooka program, as well as the negative interest rate policy introduced in January 2016.

One might say that these fluctuations in the opportunity cost since the mid-1990s still simply represent tiny changes around zero, as shown in the figure. However, the money demand functions derived in Section 3, such as Eqs. (4), (7), and (8), indicate that what matters is not the level of the opportunity cost but its log. Small fluctuations in x, especially those around zero, may not necessarily be that small in $\ln x$.

Turning to the money-income ratio, this was stable and remained somewhere between 0.3 and 0.4 up until the mid-1990s but has been on a significant upward trend since then. The flip side of this is that investments by households in interest bearing assets, such as bonds, have been declining since the mid-1990s. Watanabe and Yabu (2019) examine this by decomposing the decline in households' investment in interest bearing assets into the extensive margin (i.e., the fraction of households with interest-bearing assets) and the intensive margin (i.e., the amount of interest-bearing assets per household for households with interest-bearing assets). They show that about two-thirds of the decline in interest-bearing assets per household during this period is accounted for by changes in the extensive margin. In response to the secular decline in interest rates during this period, the number of households with no financial assets outside M1 (i.e., cash and saving deposits) has been increasing, which has contributed to the surge in the money-income ratio. Note that the outbreak of the COVID-19 pandemic accelerated the increase in the demand for M1, so that as of 2020:2Q the money-income ratio had risen to 1.76.

Next, to examine how the demand for money responded to changes in the opportunity cost, money demand functions are estimated using data from 2006:Q2 onward. Specifically, based on Eq. (7), the log of the money-income ratio is regressed on the log of the opportunity cost plus θ. For example, when θ is set to $\theta = 0.005$, the following result is obtained:

$$\ln m = -1.5382 - 0.3162 \times \ln(x + 0.005) \tag{11}$$

The coefficient on $\ln(x + \theta)$ represents the responsiveness of m with respect to x, although it cannot be interpreted as the elasticity unless θ is zero. The estimated coefficient, which is -0.3, is slightly smaller than but close to the estimate of the interest elasticity of -0.5 obtained by Lucas (2000) employing US data containing positive interest rate observations only. Using Japanese data with positive interest rate observations only, Watanabe and Yabu (2019) show that the interest elasticity is somewhere around -0.1. Another important thing implied by the regression result is that the estimate for Friedman's (1969) optimum quantity of money, which is obtained by substituting $x = 0$ into (11), is 1.147. Given that the money-income ratio in 2020:2Q was 1.764, the current level exceeds the optimum quantity of money by more than 50%. This issue will be discussed in more detail in the next subsection.

Similar regressions are conducted using different values for θ ($\theta = 0.01, 0.02$). Also, the log of the money-income ratio is regressed on $h(x)$ with μ set to $\mu = 100$ to obtain the following:

$$\ln m = -2.9417 - 0.5863 \times h(x) \tag{12}$$

The optimum quantity of money implied from this result is 1.179, which is quite close to the estimate obtained earlier. Figure 3 shows the fitted lines obtained for the four different specifications, indicating that the linear storage specification with $\theta = 0.005$ fits better than the other specifications. However, the most recent observation (i.e., the observation for 2020:2Q) lies far from even the specification with $\theta = 0.005$, suggesting that something very different from the past has occurred in the demand for money due to the COVID-19 pandemic.

4.2 Bailey's welfare measure

This subsection conducts welfare analysis regarding the quantity of money making use of the money demand functions estimated in the previous subsection. The first exercise consists of estimating the marginal utility of money. As shown in Sections 3.1 and 3.2, the marginal utility of money, U_z/U_c, is a function of m, so that the marginal utility for a particular value of m can be calculated using the estimates for A and α.[6] Moreover, Sections 3.1 and 3.2 showed that the marginal utility of money equals the sum of the opportunity cost of holding money and the marginal cost of storing money (i.e., $U_z/U_c = x + \theta$). Using this relationship, and given the estimate for the marginal utility for a particular quarter and the actual value for the opportunity cost in that quarter, the marginal cost of storing money for that quarter is estimated.

Figure 4 shows the estimation results for the marginal utility and the marginal cost. The figure shows that the marginal utility follows a downward trend during the entire observation period. The reason is the secular increase in m during that period, resulting from the monetary easing conducted by the BOJ since 2009:Q1, especially from the Kuroda bazooka since 2013. In contrast, the estimated marginal storage cost of money has remained almost constant over time, albeit with some temporary ups and downs. The lack of a secular increase in the marginal cost suggests that money storage cost is not a convex but a linear function of m.

The second exercise based on the money demand functions estimated consists of calculating the extent to which the money-income ratio deviates from the optimum quantity of money. The results are presented in the upper panel of Table 1. Comparing the money-income ratio with the optimum quantity of money shows that, in all four specifications, the money-income ratio in the most recent quarter exceeds the optimum quantity of money by more than 50%. Even before the COVID-19 crisis (i.e., 2019:Q4), it exceeded the optimal level by 30%.

The third exercise consists of calculating Bailey's welfare loss. Specifically, Bailey's welfare loss is calculated using Eq. (10) for the case of linear storage costs, with three different values used for θ ($\theta = 0.005, 0.01, 0.02$). For the case of convex storage costs, the corresponding integral value is numerically calculated using Eq. (8). The lower panel of Table 1 presents the results. It shows that the estimated welfare loss ranges from 0.20% to 0.23% of nominal GDP. Even before the outbreak of the COVID-19 crisis (i.e., 2019:4Q), the welfare loss was not negligible but much smaller, ranging from 0.07% to 0.08%.

5 Summary, Policy Implications, and Future Directions for Research

Using a model that explicitly considers the cost of storing money, this paper examined the effect of the "Kuroda bazooka" launched by the BOJ in 2013 under the new Governor Haruhiko Kuroda. The main results of this paper can be summarized as follows. First, the opportunity cost of holding money, calculated using 1-year government bond yields and the interest rate on money held in savings accounts, has been negative since the fourth quarter of 2014 and most recently (2020:Q2) was –0.2%. Second, the marginal utility of money in the second quarter of 2020 was 0.4%, while the marginal cost of holding money was 0.6%. The difference between the two corresponds to the opportunity cost of holding money, which was –0.2% in the same quarter. Third, the optimum quantity of money, at which the marginal utility of money equals the marginal cost of storing money, is 1.2. In contrast, the actual ratio of M1 to GDP in the second quarter of 2020 was 1.8 , meaning that the quantity of money in Japan is excessive. Fourth, measuring welfare following Bailey (1956) as the sum of the consumer surplus and seigniorage, the welfare loss relative to the maximum welfare obtained under the optimum quantity of money currently (2020:Q2) corresponds to 0.2% of nominal GDP.

Considering the BOJ's future policies based on these results, the BOJ should not continue to increase the supply of money but should start reducing it instead. Specifically, the ratio of M1 to nominal GDP needs to be reduced by more than 30% from the current 1.8 to 1.2.

To do so, the opportunity cost of holding money needs to be raised. One conceivable way to achieve this would be to boost CPI inflation by about 0.2 percentage points and raise nominal interest rates by the same margin, thereby reducing the relative attractiveness of money. In practice, however, the experience with monetary easing since 2013 shows that boosting CPI inflation would not be that easy. One possible alternative therefore would be to penalize the holding of money to make it less attractive. Specifically, as proposed by Agarwal and Kimball (2019) and others, it is worth considering the introduction of a mechanism that allows applying negative interest to money, such as the introduction of central bank digital currency, and setting the interest rate on money to about –0.3%.

Finally, it is worth considering future directions for research related to the present study. First, the analysis in this paper is based on the assumption that the utility function is given by Eq. (1). While a utility function of this special form made it possible to derive a money demand function of log-log form, this was not without costs. Specifically, with a utility function of a less restrictive form, an increase in real money balances held by households due to quantitative easing could potentially increase the marginal utility of goods substantially, leading to a substantial increase in the aggregate demand for goods. The welfare improvement resulting from this might outweigh the higher marginal cost of holding money, especially in an economy with nominal rigidities. However, even if this channel of quantitative easing were taken into account, the main results of this paper would remain qualitatively unchanged, although they would differ quantitatively. An important task for the future therefore is to estimate the optimum quantity of money for Japan taking this channel into account and to examine whether the current level of money exceeds the optimum quantity.

Second, this paper assumes that there exists only a single interest rate, which is associated with one-period risk-free bonds. However, in reality, there are many different interest rates in the economy, which respond differently to central bank money injection, making the effects of central bank money injection much more complicated. As discussed in previous studies on negative interest rate policy such as Brunnermeier and Koby (2018), Eggertsson et al. (2019), and Ulate (2021), bank lending rates decline more than deposit rates, so that bank profits deteriorate in response to the introduction of negative interest rate policy. Specifically, Brunnermeier and Koby (2018) show that, on the one hand, a reduction in the central bank policy rate increases banks' net worth through capital gains on their holdings of fixed-income assets, while on the other it decreases banks' net worth through a worsening of net interest margins. Based on this, they show that there exists a threshold in the policy rate (reversal rate) such that the former effect outweighs the latter if the policy rate is above the threshold and vice versa if it is below the threshold. Most importantly, their analysis implies that excessive monetary easing (i.e., lowering the policy rate below the reversal rate) leads to a deterioration in welfare, which is essentially the same as the main implication of the present study. Note that in the model in this study the threshold is defined in terms of real money balances (i.e., the optimum quantity of money), while in Brunnermeier and Koby's model it is defined in terms of the policy rate. However, this paper differs from Brunnermeier and Koby's (2018) study in some important respects. First, the quantity of money does not play an important role in their model. For example, as they recognize, the reversal rate could be positive, so that real money balances associated with the reversal rate do not necessarily have to be that large, implying that their analysis is not directly relevant for large scale money injections such as the one Japan has experienced in recent decades. Second, their threshold (i.e., reversal rate) exists only for banks that meet certain conditions. Repullo (2020), for example, shows that the reversal rate exists only for banks with net holdings of debt securities. In contrast, the threshold in this paper exists independently of banks' characteristics. Therefore, another important task for the future is to estimate the two thresholds for Japan, and to examine which of the two thresholds the BOJ has actually passed with its policies.

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Endnotes

1. Ito (2021) provides an overview of the BOJ's monetary easing policies since 2013 as well as a more detailed discussion of the BOJ's failure to raise inflation expectations.
2. Thornton (1999) pointed out that currency carries the risk of being stolen or lost, and that physically moving currency is expensive, so that the costs of holding currency are not negligible. Focusing on 13 countries in the euro zone, Schmiedel et al. (2012) examined how much banks and retailers spent on various payment instruments and showed that for cash, the costs came to 2.3 cents per euro. These costs include costs such as those related to machinery and equipment to support the use of cash – such as automatic teller machines provided by banks and point of sale terminals at retailers – as well as labor costs. Since the overall costs include the costs borne by banks and retailers, not all of the costs fall on households that use cash. However, when households use cash, they may potentially be charged by banks and retailers for these costs in the form of cash handling fees.

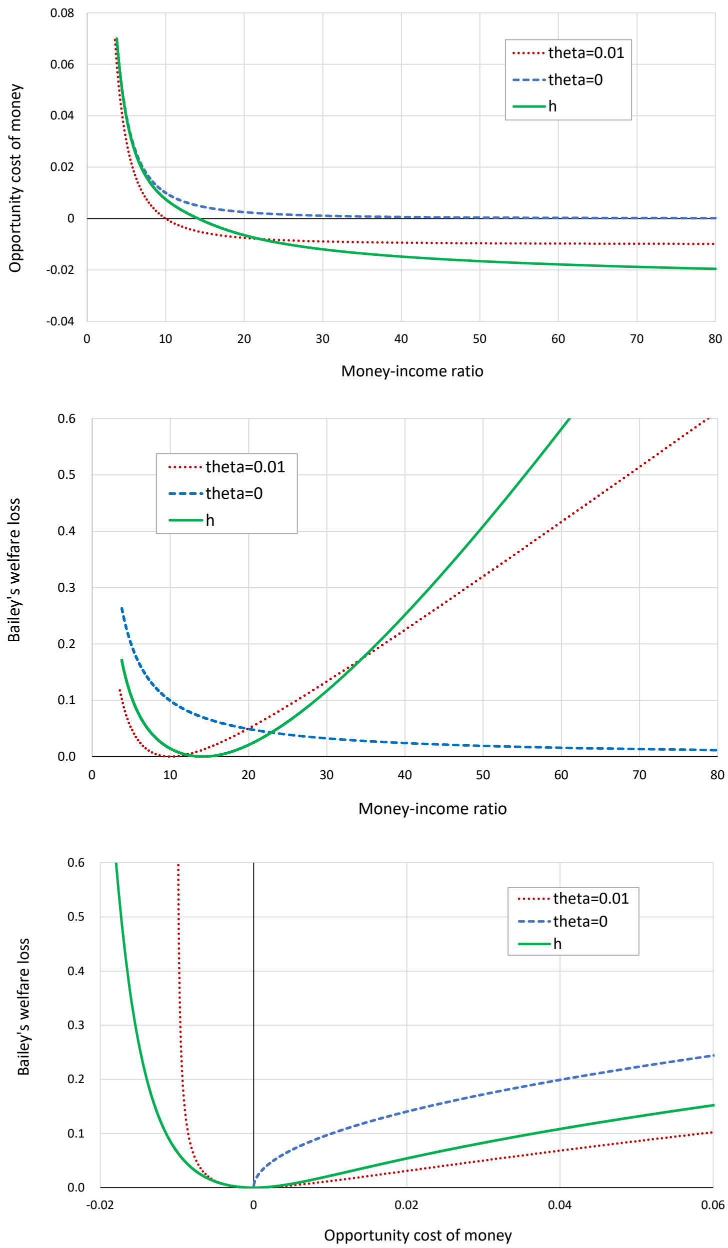
3. Hicks (1937: 154-155) described Keynes's liquidity trap as follows: "If the costs of holding money can be neglected, it will always profitable to hold money rather than lend it out, if the rate of interest is not greater than zero. Consequently, the rate of interest must always be positive." However, based on Japan's experience under the Kuroda bazooka, the assumption that "the costs of holding money can be neglected" is not an appropriate approximation of reality.
4. Along similar lines, Brunnermeier and Koby (2018) suggest that there exists a threshold in the central bank policy rate, which they called the "reversal rate," such that monetary easing improves welfare if the policy rate is above the threshold but it deteriorates welfare if the policy rate is below the threshold. While in this paper the threshold is defined in terms of real money balances (i.e., the optimum quantity of money), Brunnermeier and Koby (2018) define it in terms of the policy rate. A detailed comparison with Brunnermeier and Koby's (2018) study is provided in Section 5.
5. Using a semi-log money demand function, Rognlie (2016) shows that money satiation occurs at a finite level of real money balances. Specifically, if the demand for money is given by $\ln m = \ln B + \beta r$, m takes a finite value even when $x = 0$. An important difference from the result here is that in Rognlie (2016) it is the functional form of the money demand function that yields money satiation at a finite level. In contrast, the analysis here starts from a log-log money demand function and shows that, when money storage costs are non-negligible, the marginal utility of money coincides with the marginal storage cost of money at a finite level, although money satiation never occurs at a finite level. As for the functional form of money demand functions, previous studies seem to suggest that the log-log form performs better than the semi-log form. See Watanabe and Yabu (2018, 2019) for more details.

Table 1: Money-Income Ratio Before and During the COVID-19 Crisis

	Linear storage costs			Convex storage costs
	$\theta = 0.005$	$\theta = 0.010$	$\theta = 0.020$	$\mu = 100$
	Deviation from Optimum Quantity of Money			
2019:Q4	30.1%	28.1%	27.2%	26.6%
2020:Q1	33.2%	31.1%	30.2%	29.6%
2020:Q2	53.7%	51.4%	50.3%	49.6%
	Bailey's Measure for the Welfare Loss			
2019:Q4	0.07%	0.07%	0.08%	0.08%
2020:Q1	0.08%	0.08%	0.09%	0.09%
2020:Q2	0.18%	0.20%	0.22%	0.23%

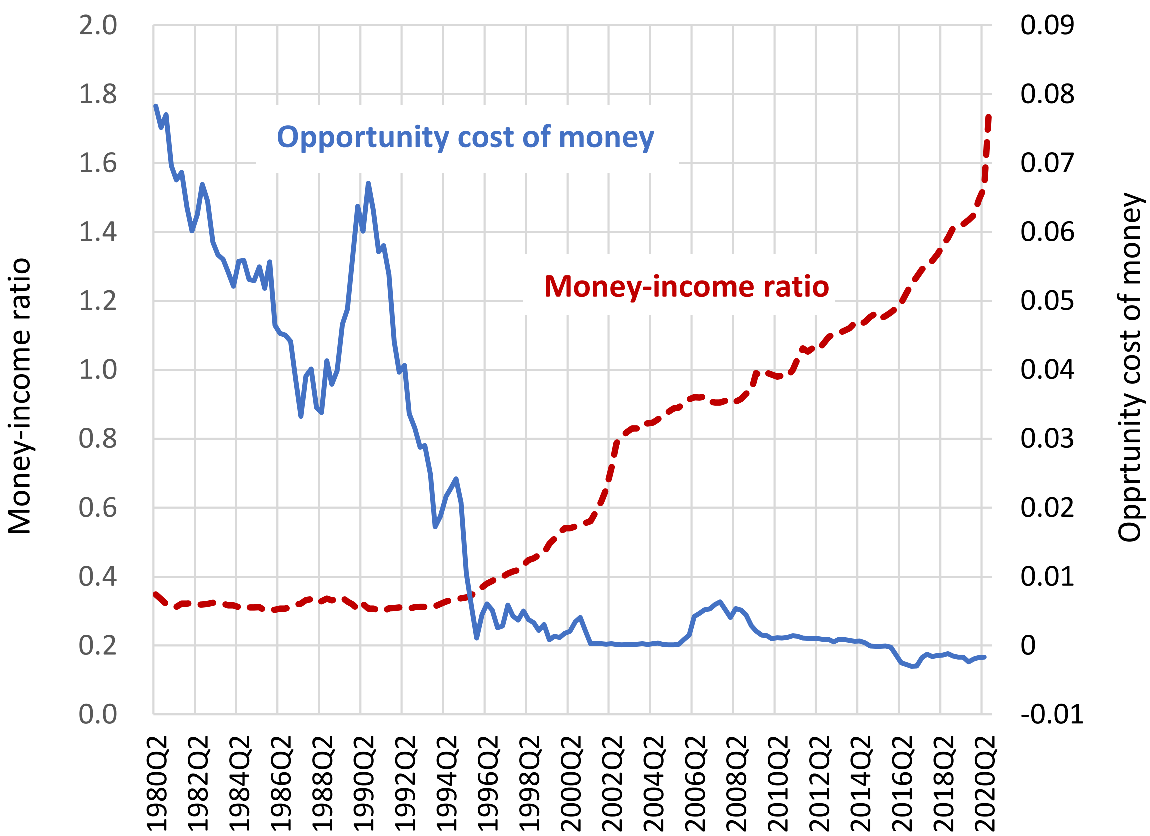
Note: The estimate for the optimum quantity of money is 1.147 for linear storage costs with $\theta = 0.005$, 1.165 for $\theta = 0.01$, 1.173 for $\theta = 0.02$, and 1.179 for convex storage costs with $\mu = 100$. The money-income ratio is 1.493 in 2019:Q4, 1.528 in 2020:Q1, and 1.764 in 2020:Q2.

Figure 1: Numerical Example



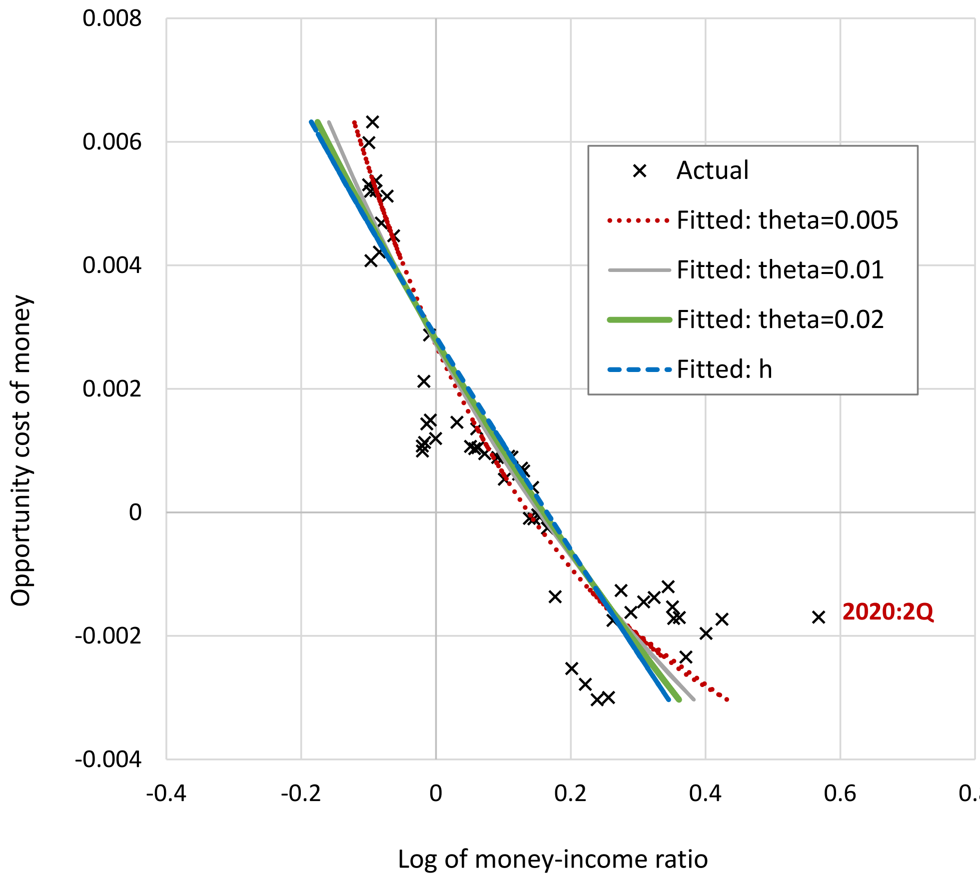
Note: m and $w(x)$ are calculated using Eqs. (4), (7), (9), and (10) with A and α set to $A = 1$ and $\alpha = -0.5$. The blue line corresponds to the case of no storage costs of money. The red line corresponds to the case of linear storage costs with θ set to $\theta = 0.01$. The green line corresponds to the case of convex storage costs with μ set to $\mu = 100$.

Figure 2: M1 and the Opportunity Cost of Holding Money



Note: The money-income ratio is defined as M1 divided by nominal GDP. The opportunity cost of money is defined as the 1-year JGB yield minus the interest rate on saving deposits.

Figure 3: Estimated Money Demand Functions



Note: The observation period is 2006:Q2-2020:Q2. The estimated results are as follows:

$\ln m = -0.3162 \times \ln(x + 0.005) - 1.5382$ for $\theta = 0.005$;

$\ln m = -0.6362 \times \ln(x + 0.01) - 2.7770$ for $\theta = 0.01$;

$\ln m = -1.2217 \times \ln(x + 0.02) - 4.6196$ for $\theta = 0.02$;

$\ln m = -0.5863 \times h(x) - 2.9417$ for $h(\cdot)$ function.

Figure 4: Estimate of the Marginal Utility of Money