Tax Transparency and Social Welfare:
The Role of Government Commitment

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Abstract

Although transparency has long been held as the key principle of taxation, recent behavioral public finance theory has shown that it may *reduce* social welfare as inattention can alleviate behavioral distortions. This paper extends this analysis by modeling inattention as a noise in the tax rate signal received by Bayesian citizens. In equilibrium, we find that transparency will *improve* social welfare by ensuring the government's ability to commit to a fairly low tax rate that is socially optimal. Moreover, this model yields a new sufficient statistics formula. Based on typical estimates of attention parameters and marginal cost of public funds, this formula suggests that ensuring tax transparency is worth incurring approximately 10 percent of the revenue currently estimated for the U.S. tax system.

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> "The tax which each individual is bound to pay ... ought to be clear and plain to the contributor, and to every other person."
>
> – Adam Smith (1776)

> "If all taxes were direct, taxation would be much more perceived than at present; and there would be a security which now there is not, for economy in the public expenditure."
>
> – John Stuart Mill (1848)

1 Introduction

There is a critical disagreement between policy practices and optimal tax theory regarding the welfare implications of tax transparency. Since *The Wealth of Nations* (Smith, 1776) and *Principles of Political Economy* (Mill, 1848), clarity of tax rates has been regarded as a key principle for a good system of taxation. Today, tax authorities implement various programs to inform citizens of tax liabilities, and simplicity has been at the center of controversial tax debates, such as whether to introduce universal basic income instead of complex income transfers (See, e.g. Slemrod and Bakija, 2017.) However, prominent models in behavioral public finance (e.g. Chetty et al., 2009; Mullainathan et al., 2012; Taubinsky and Rees-Jones, 2018; Farhi and Gabaix, 2020) have found that transparency may actually *reduce* welfare in the benchmark with a small income effect. The reason for these surprising results is clear: when inattention reduces the behavioral distortion of taxes, the government can raise more revenue with a smaller excess burden on its citizens. One historical example is 1943 introduction of the salience-reducing Federal income tax withholding system that helped the U.S. government finance its urgent wartime expenditure.

This paper extends this taxation model by using an alternative model of inattention, and shows that tax transparency *improves* social welfare for reasons consistent with the traditional policy discussions. This approach is motivated by evidence: a series of field survey shows, even when the taxpayers are initially uncertain about their taxes due to non-salient or complex schedules, they eventually learn with more experience and information (Finkelstein, 2009; Caldwell et al., 2021). We build on the standard information economics framework to analyze this learning effect. Specifically, we consider Bayesian consumers that face uncertainty regarding their tax rates and receive signals whose noisiness reflects the extent of their inattention (Sims, 2003). Focusing on the equilibrium in which they update their prior beliefs (Harsanyi, 1967; Kreps and Wilson, 1982), we find that the inattention must *reduce* social welfare. Moreover, this effect can be large even when the noise in tax rates is slight. These findings are practically important as the model implies a new sufficient statistics formula that informs the controversial debate over the efficiency of costly policies to ensure the tax salience.

We begin by rederiving the result that transparency may reduce welfare by facilitating behavioral responses in the short-run, notwithstanding the existence of countervailing effects. In the model, Bayesian consumers rely not only on noisy signals but also on their prior beliefs to infer their tax rates. Consequently, their aggregate response to the change in the tax rate will be lower when the signals are noisier because they rely less on them. Let us consider an impulse response of consumption decisions to the introduction of a new tax rate from an initial state of zero tax. In the short-run, when their prior beliefs have not been updated from the initial state, this inattention reduces the behavioral distortions (Chetty et al., 2009; Mullainathan et al., 2012). In addition to this effect, there are also idiosyncratic misperceptions that result in ex-post suboptimal consumption decisions. Nonetheless, so long as the overall noise is small, the former effect will dominate, and inattention will improve short-run welfare.

In the long-run, however, we show that transparency will improve social welfare. An important yet implicit assumption in taxation models is that the government sets the tax rate *before* the citizens make consumption decisions. If the tax rate is set *after* the citizens' choice, then even a benevolent government will choose a very high tax because there would be zero behavioral distortion. In equilibrium, however, rational citizens will then expect the high tax rate and consume less, resulting in the unintended outcome of large excess burden and low total welfare. That is, government needs an ability to *commit* to a low tax rate. Now if the tax rate signal is completely uninformative, then it is as if the government chooses the tax rate *after* the citizens' choice even if they physically choose the tax *before*. In this way, more generally, inattention compromises the government's ability to commit to a low tax rate that is socially optimal.[1] Further, in contrast with the idiosyncratic errors, this long-run

[1]The U.S. tax withholding system has institutionally remained even after the war, and the income tax is now known to be high. Milton Friedman has attributed this increase to the lack of

commitment effet may be discontinuously large even when the noise is slight, and thus, can be important even when citizens strive to pay attention.

This long-run analysis yields a new sufficient statistics formula that informs the controversial debate over the costly programs to improve tax transparency. In the U.S., the widely used estimate of compliance cost to ensure tax transparency is roughly 10 percents of its revenue (Slemrod, 1996). Proposals to reduce this cost and allow non-salient taxes are often opposed by the policy makers as they expect, when the taxes are not transparent, the government will raise them above the socially optimal levels (Finkelstein, 2009). This model provides a simple formula that quantifies this concern in terms of the attention parameters and the marginal cost of public funds. Given typical estimates from the literature, this formula suggests that the costly effort to ensure tax transparency in the U.S. is socially worthwhile.

Overall, this model shows that imperfect attention results in the issues of government commitment that had previously been the focus of tax debates but which are absent in recent models. Traditionally, Smith (1776), Mill (1848) and subsequent research (e.g. Buchanan and Wagner, 1977) suggest that taxes will be *excessively* large unless they are transparent to the citizens.[2] However, recent behavioral public finance models have suggested that inattention will lead to *optimally* high taxes. In this way, this paper shows that reduced-form vs. Bayesian approaches to inattention lead to vastly different equilibrium outcomes[3], analogous to the contrast between taste-based (Becker, 1957) vs. statistical (Arrow, 1973) discrimination. When the Bayesian equilibrium analysis is applied, the traditional argument of government commitment re-emerges as the principal concern.

The remainder of this paper is organized as follows: Section 2 presents the model overview, the set-up, and derives its equilibrium implications for belief updating and behavioral responses; Section 3 analyzes the model's short-run and long-run implications; and Section 4 discusses the key assumptions of the model and concludes.

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transparency (Friedman and Friedman, 1998), critizing the tax level to be too high.

[2]There are taxation models that embed commitment concerns through political economy constraints of electoral processes (e.g. Acemoglu et al. 2008; Farhi et al. 2012; Scheuer and Wolitzky 2016 among others). In contrast, this paper shows that, the commitment concerns emerges under inattention even without explicitly modeling future elections.

[3]This equilibrium analysis is related to Spiegler (2015) that shows that the explicit modeling of consumer bias can alter the welfare implications. While Spiegler (2015) focuses mostly on the effect of market competition, this analysis focuses on the role of prior belief.

2 Model

This Section provides an overview of the key argument, introduces the taxation model with Bayesian inattention, and presents its interpretation for the short-run and long-run analyses.

2.1 Basic Model Overview

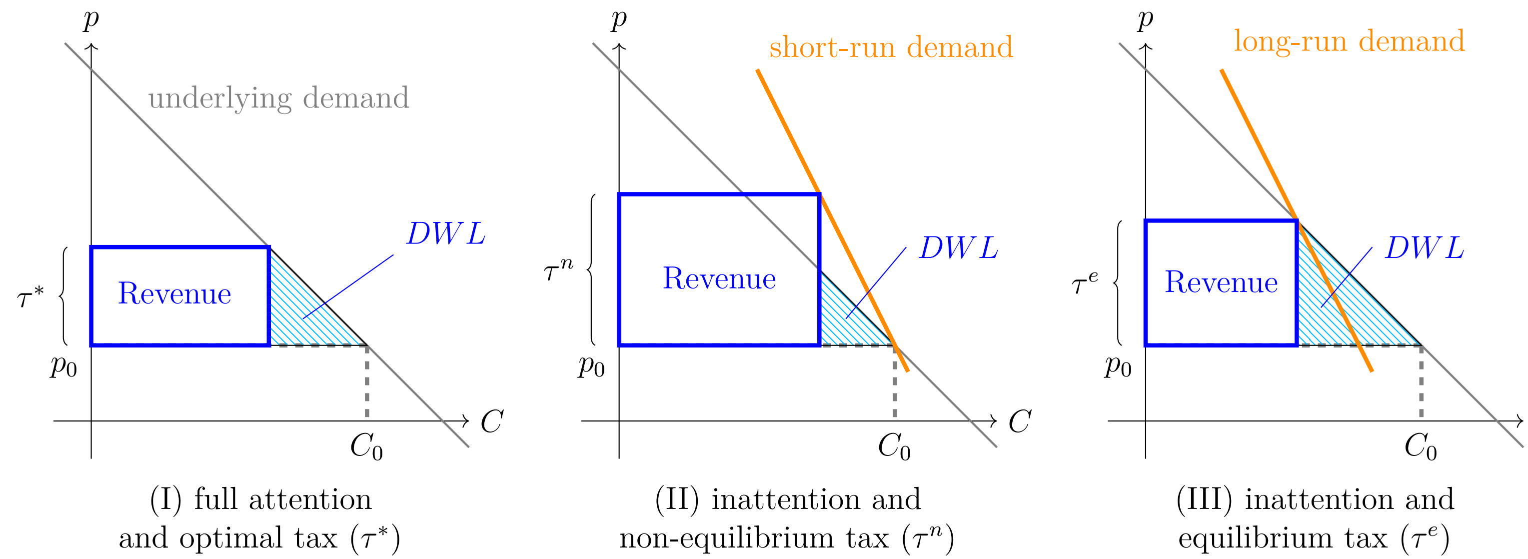
We begin with an overview of the main argument. As illustrated in Figure 1, its essential logic can be summarized by deadweight loss (DWL). In the traditional case when citizens are fully attentive, their consumption decision[4] will coincide with their underlying valuations of the goods (Panel I). The social planner then sets the optimal tax, τ^*, which is fairly low because many attentive citizens with low valuations would otherwise cut their consumption, resulting in a large deadweight loss.

In the short-run, we replicate the findings from behavioral public finance models that inattention improves welfare (Panel II). Here, the only consequence of inattention is a reduction in citizens' behavioral responses, represented by the less elastic demand curve. Social welfare increases under inattention because the social planner can raise the tax rate, τ^n, to increase revenue while achieving a smaller deadweight loss.

In the long-run, however, we find that inattention lowers welfare because the Bayesian citizens' prior beliefs adjust (Panel III). After paying the taxes many times, they will eventually update their prior expectation of the tax rate to the equilibrium level, τ^e. That is, in equilibrium, their demand curve must coincide with the underlying demand *at the average tax level*, rather than at the initial zero tax as was assumed in the short-run. Crucially, the social planner will in equilibrium take citizens' expectations as given, and since demand is nonetheless less elastic, the social planner will increase the tax rate above the full attention benchmark: $\tau^e > \tau^*$. But since that benchmark tax rate τ^* was socially optimal by construction, the resulting welfare under inattention will be sub-optimal. In summary, the low responsiveness to taxes due to consumer inattention compromises the government's ability to commit to the relatively low socially optimal tax rate.

[4]Here, we consider a market of consumption demand to be consistent with the prominent models in behavioral public finance (e.g. Chetty et al. 2009). However, we can also interpret this model as a linear income taxation on labor supply by relabeling the variables. See Section 1.2.

Figure 1: Taxes and Welfare under Full and Imperfect Attention of Bayesian Citizens



Notes: Figure 1 shows that, contrary to the short-run effects, inattention *reduces* long-run welfare by compromising the government's commitment to set the relatively low optimal tax. Panel (I) – (III) illustrate the government revenue (i.e. square box) and deadweight loss (i.e. triangles labeled as DWL) resulting from the taxes in each setting. Here, p denotes price, C denotes total consumption, and $\{p_0, C_0\}$ denotes the zero-tax market equilibrium. Panel (I) shows that the standard optimal tax (τ^*) is fairly low when the citizens are fully attentive. Panel (II) depicts the short-run demand curve that is less elastic (or, steeper inverse demand curve in the Figure) because inattentive citizens are less responsive to taxes. In this setting, the non-equilibrium tax rate (τ^n) is higher, and given the larger revenue and smaller deadweight loss, the implied welfare is also higher. Panel (III) shows that in the long-run, however, the demand curve will intersect the underlying demand at the equilibrium tax rate (τ^e) because the Bayesian consumers will update their prior belief. Since the demand curve is nonetheless less elastic due to their inattention, the social planner will set a tax rate that is higher than the full attention benchmark ($\tau^e > \tau^*$). But since the full attention benchmark (τ^*) was optimal by construction, the welfare under inattention must be strictly lower.

Henceforth, we will build the Bayesian model with additional structures necessary for analytic tractability. First, there will be a continuum of citizens who make binary decisions instead of one representative agent with a continuous decision. Second, the social planner determines the tax rate slightly imperfectly (in a sense defined precisely later) rather than with perfect control. Third, the underlying demand of citizens is assumed to be linear for the entire range of prices, and not just for some neighborhood of the equilibrium. Even though these auxiliary assumptions may appear foreign in the context of optimal taxation, they are based on standard assumptions in information economics to solve otherwise intractable Bayesian models. As illustrated in Figure 1, the essential argument henceforth will nonetheless be based on the familiar deadweight loss minimization.

2.2 Bayesian Model Set-up

Let us now formally introduce the static model of linear taxation. There is a continuum of Bayesian citizens with valuations $v \sim F(v)$ for a good who decide whether to consume it, $c \in \{0, 1\}$. The benevolent social planner chooses the level of the tax rate, $\tau \in \mathbb{R}$, to raise the government revenue.

The citizens consume the good if their own valuation v exceeds the expected post-tax price. In particular, normalizing the pre-tax price p_0 to be 1, the citizen with a type v chooses c to maximize the expected value of the utility,

$$u(c, v|\tau) \equiv c[v - (1 + \tau)]. \tag{1}$$

Each citizen imperfectly observes the tax rate through an idiosyncratic noisy signal,

$$\hat{\tau} = \tau + \varepsilon, \tag{2}$$

where $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is an independently distributed normal noise. Given a posterior perceived tax, $\tilde{\tau}$, the aggregate consumption is thus $C(\tilde{\tau}) = 1 - F(1 + \tilde{\tau})$.

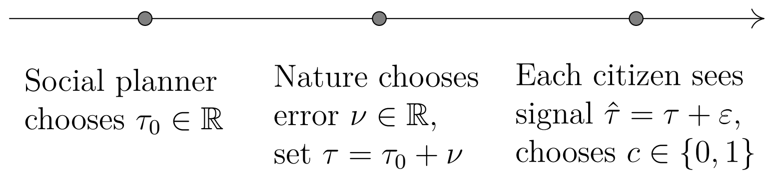
The social planner chooses the *intended* tax, τ_0. Then, the *actual* tax, τ, is determined by

$$\tau = \tau_0 + \nu, \tag{3}$$

where $\nu \sim \mathcal{N}(0, \sigma_\nu^2)$ denotes some disturbances that perturb the tax rate.[5] This

[5] Recent survey evidence shows that the policy uncertainty is large and has substantial welfare

Figure 2: Time line



Notes: Figure 2 describes the time line of this model.

idiosyncratic uncertainty may reflect various determinants that the planner cannot fully control, such as legislative bargaining processes and elections. Faced with the uncertainties over the actual tax τ and the perceived tax $\tilde{\tau}$, the planner chooses the intended tax τ_0 to maximize

$$\mathbb{E}w \equiv \mathbb{E}\left[\lambda\tau\overline{C}\left(\tau\right) - DWL|\tau_0\right], \tag{4}$$

where $\lambda > 0$ is the weight on the revenue[6]. $\overline{C}\left(\tau\right) \equiv \mathbb{E}\left[C\left(\tilde{\tau}\right)|\tau\right]$ denotes the expected aggregate consumption given actual tax τ. Here, the expected deadweight loss is

$$\mathbb{E}\left[DWL|\tau_0\right] \equiv \int \left\{ \underbrace{u\left(c_{v,\tilde{\tau}=0}, v|\tau = 0\right)}_{\text{zero-tax welfare}} - \mathbb{E}\left[\underbrace{u\left(c_{v,\tilde{\tau}}, v|\tau\right)}_{\text{welfare}} + \underbrace{\tau c_{v,\tilde{\tau}}}_{\text{revenue}} |\tau_0 \right] \right\} dF\left(v\right). \tag{5}$$

This is the welfare loss resulting from taxes that does not contribute to government revenue. $c_{v,\tilde{\tau}}$ denotes the individual consumption of type v given the perceived tax $\tilde{\tau}$. While this expression may appear non-standard due to uncertainty and heterogeneity, we will henceforth show that it reflects the area of standard deadweight loss triangles in Figure 1.

The time line is as follows (Figure 2): first, the social planner chooses the intended tax rate $\tau_0 \in \mathbb{R}$; second, Nature chooses the perturbation parameter $\nu \sim \mathcal{N}\left(0, \sigma_\nu^2\right)$, and thereby sets the actual tax rate, $\tau = \tau_0 + \nu$; third, each citizen observes the

consequences in the context of Social Security benefits (Luttmer and Samwick, 2018).

[6]Note that $\lambda = MCPF - 1$, the marginal cost of public fund *minus 1*, to avoid double counting the government revenues included in the deadweight loss as transfers.

noisy signal $\hat{\tau} = \tau + \varepsilon$, $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, updates their perceived tax rate $\tilde{\tau}$, and decides whether to consume, c, by maximizing their own expected utility; and finally, the citizens' payoffs are realized and the revenue is collected.

We can interpret this model both in terms of a consumption tax and an income tax. When c represents a commodity, τ denotes a consumption VAT tax. When the VAT tax is noted on the price tag, for example, it is very salient so that σ_ε^2 is very small (Chetty et al., 2009). When c is interpreted as the return to labor, $-\tau$ denotes an income tax or social insurance premium. When their opacity and complexity confuse the workers' understanding of the tax, the parameter σ_ε^2 is very large (Caldwell et al., 2021). These tax rates are determined as a result of political negotiation and election, whose outcomes can be uncertain for the benevolent policy makers when proposing the tax level τ_0.[7]

2.3 Equilibrium

2.3.1 "Static" Equilibrium Concept and its "Dynamic" Interpretation

This paper will focus on the Perfect Bayesian Nash Equilibrium, the standard equilibrium concept to analyze sequential-move games of incomplete information. Let the citizens' strategy be denoted by $s_c : \mathbb{R} \times \mathbb{R} \mapsto [0, 1]$, a mapping from the space of values v and signals $\hat{\tau}$ onto the probability distribution over consumption c. Let the planner's strategy be denoted by $s_p : \Delta(\mathbb{R})$, a distribution over the tax rate τ. Denote the citizens' prior belief over the government's strategy be denoted by $\mu \equiv \Delta(\Delta(\tau))$.

Definition 1 **Equilibrium** *An* equilibrium *is a tuple of strategies and beliefs* $\{s_c, s_p, \mu\}$ *such that (i) citizens strategies maximize the objective* (1) *in expectation given the strategies of the planner; (ii) planner strategy maximizes* (4) *given strategies of citizens; (iii) beliefs are consistent with the Bayes' rule and off-equilibrium strategies also maximize the objective* (1)*.*

The key new condition, relative to the existing models, is (iii) the prior belief must be consistent with the Bayes' rule (Harsanyi, 1967; Kreps and Wilson, 1982). In the short-run immediately after the new tax rate is introduced, citizens may not be familiar with the new tax rate and thus, this condition (iii) is unlikely to be satisfied.

[7] Formally, this approach is the same as the trembling hand of Selten (1975), which models the possibility of small mistakes by the decision-makers.

However in the long-run when citizens have paid their taxes many times, their beliefs may well-approximate the true distribution so that this condition (iii) is reasonable. Therefore, henceforth, we will refer to the analysis that assumes the mean prior tax rate to be the initial value of 0 as the *short-run* analysis; in turn, we will refer to the analysis that imposes the (iii) prior consistency as the *long-run* analysis.[8]

2.3.2 Inattention as Reduction in Behavioral Response

We show that the average expectation over tax is the sum of the actual tax and the prior weighted by an attention parameter, and thus, the behavioral response to tax is attenuated by that parameter.

Lemma 1. Bayesian Updating with Attention Parameter. *Suppose the social planner sets some intended tax rate whose expectation is $\overline{\tau}_0$. the average perceived tax rate among citizens conditional on realized tax rate τ is*

$$\mathbb{E}\left[\tilde{\tau}|\tau\right] = m\tau + (1-m)\,\overline{\tau}_0, \tag{6}$$

where

$$m \equiv \frac{\sigma_\nu^2}{\sigma_\nu^2 + \sigma_\varepsilon^2} \tag{7}$$

is called the attention parameter.

Proof. Suppose the planner sets a distribution of intended tax rate, $G(\tau_0)$. Given each τ_0, the Bayesian citizens who receive the normal signal $\hat{\tau}$ form the perceived tax rate

$$\tilde{\tau} = m\hat{\tau} + (1-m)\,\tau_0$$

since the disturbances ν is also normally distributed. Thus, if citizens are uncertain about τ_0,

$$\mathbb{E}\left[\tilde{\tau}\right] = m\hat{\tau} + (1-m)\int \tau_0 dG = m\hat{\tau} + (1-m)\,\overline{\tau}_0.$$

by the Law of Iterated Expectations. Further, by linearity and mean zero additive

[8] "Static" equilibrium can be naturally viewed as the steady state of the dynamic adjustment process of myopic players (see, e.g. Fudenberg and Levine (2009) for a survey) For example, in the context of statistical discrimination, the equilibria with and without discrimination is also analyzed with such dynamic interpretation, even when the model is static (e.g. Coate and Loury, 1993).

noise,

$$\mathbb{E}\left[\mathbb{E}\left[\tilde{\tau}\right]|\tau\right] = m\mathbb{E}\left[\hat{\tau}|\tau\right] + (1-m)\,\overline{\tau}_0 = m\tau + (1-m)\,\overline{\tau}_0.$$

□

Let us now introduce an assumption that makes the analysis significantly more transparent and tractable[9].

Assumption A1. Linear Demand Curve. *Given some $\eta \in (0,1)$, $F(v) = \eta v$ for all $v \in \mathbb{R}$.*

Lemma 2. Reduction in Behavioral Response in the Unique Equilibrium. *Suppose Assumption A1. Then, the equilibrium will be unique. Further, inattention attenuates the effect of raising the tax rate τ on the aggregate consumption, $\overline{C}(\tau)$, by the attention parameter m:*

$$\frac{\partial}{\partial\tau}\overline{C}(\tau) = -m\eta. \tag{8}$$

Sketch of Proof. Suppose the social planner chooses some distribution of intended tax rate whose expectation is $\overline{\tau}_0$. By Lemma 1, and by additivity and linearity of aggregate consumption across the entire values of v (Assumption A1),

$$\begin{aligned}\mathbb{E}\left[C(\tilde{\tau})|\tau\right] &= \mathbb{E}\left[1-\eta(1+\tilde{\tau})|\tau\right]\\ &= 1-\eta\left\{1+\mathbb{E}\left[\tilde{\tau}|\tau\right]\right\}\\ &= C_0 - \eta\left[m\tau + (1-m)\,\overline{\tau}_0\right]\end{aligned}$$

for any τ and $\overline{\tau}_0$, where $C_0 \equiv C(0) = 1-\eta$ denotes the zero-tax level of consumption. Since $\overline{C}(\tau)$ is linear in the actual tax rate τ, $\mathbb{E}\left[\overline{C}(\tau)|\tau_0\right]$ is also linear in the intended tax rate τ_0. Thus, the social planner's problem is quadratic in τ_0, and hence globally concave. Thus, the social planner chooses a pure strategy of a tax rate τ_0, and the equilibrium is unique. □

[9]This assumption essentially extends the linear demand curve for negative prices and quantities, as depicted in Figure 1. While this assumption implies that the value distribution is improper, it is commonly applied in the model with Bayesian updating given normal distributions (Morris and Shin, 2002). Morris and Shin (2001) contains a thorough discussion that shows validity of this approach. In the context of Ramsey taxation, this assumption is equivalent to assuming quadratic utility of a representative agent over his aggregate consumption. Without this assumption, one has to work with the truncated normal distributions, which will not allow for decomposition of various effects. While this assumption may not hold literally, the result is approximately valid since normal distribution is thin-tailed.

These two results are consistent with the two key assumptions imposed in the canonical models of inattention over taxes, both incorporated in the general model of Farhi and Gabaix (2020). Lemma 1 is consistent with the model in Finkelstein (2009), which assumes that the perceived tax rate is a linear combination of the actual tax and its expectation. Lemma 2 is consistent with the models in Chetty et al. (2009), which formulates the attenuation of behavioral responses by the attention parameter[10]. As forcefully argued previously (Mullainathan et al., 2012; Chetty et al., 2009), the inattention due to noisy signals in Bayes' rule also generates a reduction in behavioral responses similar to other forms of adjustment frictions. While these canonical models assumed these results exogenously, this model has derived them endogenously based on Bayes' rule.

3 Analyses

Section 2 has laid out the Bayesian model of taxation with inattention. Here, we show that this model suggests that the inattention reduces social welfare in equilibrium, as illustrated in the 2.1 Basic Model Overview. Henceforth, the detailed proofs are relegated to the Appendix.

3.1 Benchmark Analysis with Full Attention ($m = 1$)

Let us begin with the benchmark analysis with a full attention that implies the optimal tax formula with an inverse elasticity rule.

Proposition 1. Optimal Tax Rate. *Suppose Assumption A1. Suppose the tax is observed without noise, $\sigma_\varepsilon^2 = 0$. Then, the optimal intended tax rate, τ_0^*, is*

$$\tau_0^* = \frac{C_0}{\eta}\frac{\lambda}{2\lambda + 1}. \tag{9}$$

The optimal expected welfare, $\mathbb{E}w^*$, is

$$\mathbb{E}w^* = \frac{C_0^2}{2\eta}\frac{\lambda^2}{2\lambda + 1} - \eta\left[\lambda + \frac{1}{2}\right]\sigma_\nu^2 \tag{10}$$

[10] The relative weight on the tax signal is called an *attention parameter*, following the terminology of the behavioral public finance literature. Its notation, m, follows Farhi and Gabaix (2020), and corresponds to θ in Chetty et al. (2009) and $\delta_1(\theta)$ in Finkelstein (2009).

Sketch of Proof. As illustrated by Figure 1 (I), the aggregate consumption is consistent with the underlying demand curve because the perceived tax rate equals the actual tax rate, $\tilde{\tau} = \tau$. That is, $m = 1$, and by Lemma 2,

$$\overline{C}(\tau) = C_0 - \eta\tau.$$

The deadweight loss conditional on the actual tax τ is thus the triangle,

$$DWL(\tau) = \eta\frac{\tau^2}{2}.$$

The social planner chooses the intended tax rate τ_0, and the first order condition implies the tax rate (9). Since there is an idiosyncratic deviation of the actual tax τ from the intended tax τ_0, there will be an additional welfare loss, $-\eta\left[\lambda + 1/2\right]\sigma_\nu^2$. Nonetheless, as shown in Appendix Proposition 1, this variance cost is separable due to the Assumption A1, and thus, the optimal intended tax rate (9) is still the standard inverse elasticity formula. □

Proposition 1 shows that the set-up with full attention replicates the classical result of the inverse elasticity rule (Ramsey, 1927). Further, the model yields a closed-form expression for the attained welfare because the demand function is linear by Assumption A1. Henceforth, we will use these results as the benchmark to assess the implications of Bayesian inattention.

3.2 Short-run Analysis with Inattention ($m < 1, \overline{\tau}_0 = 0$)

Let us next consider the effect of inattention in the short-run, when the consumers' prior belief equals the initial tax rate of zero. The results are broadly consistent with the existing models of taxation with inattention: inattention increases the tax rate, and improves the welfare so long as the overall misperception is small.

Proposition 2. Non-equilibrium Tax Rate. *Suppose Assumption A1. Suppose the tax is observed with noise, $\sigma_\varepsilon^2 > 0$. Then, the non-equilibrium intended tax rate, τ_0^n, is*

$$\tau_0^n = \frac{C_0}{m\eta}\frac{\lambda}{2\lambda + m}. \tag{11}$$

The non-equilibrium expected welfare, $\mathbb{E}w^n$, is

$$\mathbb{E}w^n = \frac{C_0^2}{2m\eta}\frac{\lambda^2}{2\lambda+m} - \eta\left[\lambda + \frac{1}{2}\right]\sigma_\nu^2 - m\eta\frac{\sigma_\varepsilon^2}{2}. \tag{12}$$

Thus, the tax rate level is higher than the full attention optimum, $\tau_0^n > \tau_0^*$. Moreover, whenever the uncertainty is sufficiently low, i.e. $\sigma_\varepsilon^2 < \overline{\sigma}_\varepsilon^2$ for some $\overline{\sigma}_\varepsilon > 0$, the attained welfare level is also higher than the full attention setting, $\mathbb{E}w^n > \mathbb{E}w^*$.

Sketch of Proof. As illustrated by the slope of short-run demand curve in Figure 1 (II), inattention reduces the behavioral response to tax rate. By Lemma 2, the aggregate consumption is

$$\overline{C}(\tau) = C_0 - m\eta\tau,$$

where $m < 1$.

The deadweight loss given the actual tax τ consists of the smaller triangle, as well as the errors due to idiosyncratic misperceptions:

$$DWL(\tau) = \eta\frac{(m\tau)^2}{2} + \eta\frac{m^2\sigma_\varepsilon^2}{2}. \tag{13}$$

While there are idiosyncratic errors in both actual tax and its perception, as shown in Appendix Proposition 1, they are separable by the Assumption A1. Thus, the social planner chooses the intended tax rate (11) by the first order condition, and the welfare expression (12) is derived by substitution.

The non-equilibrium tax rate is higher than the optimal tax rate because, for any $m \in [0,1]$ and $\lambda > 0$,

$$\frac{1}{2\lambda m + m^2} > \frac{1}{2\lambda + 1}.$$

However, so long as the uncertainty in perceived tax rate, σ_ε^2, is sufficiently small, the non-equilibrium welfare is higher than the full attention benchmark. $\square$

Proposition 2 confirms that, when the priors have not been updated, the Bayesian model replicates the results of models with reduced-form inattention (Chetty et al., 2009). Specifically, the social planner chooses the tax rate higher than the full attention benchmark because higher revenue can be collected with a smaller deadweight loss. Note that there will also be a welfare loss from consumers who misperceive the tax to be too low, but since their consumption results in the government revenue, it does not increase the deadweight loss. The formula (11) also shows that this effect is in orders of m^2, suggesting that the tax rate increases "relatively fast" when inattention increases slightly from the benchmark of full attention, as noted by Farhi and Gabaix (2020).

However, the analysis also shows that the welfare implications of inattention become more nuanced due to the noise in their tax perceptions[11]. That is, citizens' perceived tax not only systematically deviate but differ idiosyncratically from the actual tax. Even though this effect does not alter the aggregate consumption and cannot be visualized in Figure 1 (II), it still reduces the total consumer welfare. Nonetheless, so long as this uncertainties, σ_ε, is small, this effect will also be negligible. Thus, in the "neighborhood" of the full attention model, inattention still improves social welfare in the short-run.

3.3 Long-run Analysis with Inattention ($m < 1, \tau_0 = \tau_0^e$)

When the tax rate is raised, the inattentive Bayesian consumers will not notice in the short-run. However, in the long-run, they will learn that the tax rate is high, even without observing precise signals. We will now find that this equilibrium effect alters the results found in the short-run analysis.

Proposition 3. Equilibrium Tax Rate. *Suppose Assumption A1. Suppose the tax is observed with noise, $\sigma_\varepsilon^2 > 0$. Then, in the unique equilibrium, the equilibrium intended tax rate, τ_0^e, is*

$$\tau_0^e = \frac{C_0}{\eta} \frac{\lambda}{\lambda (1+m) + m}. \tag{14}$$

The equilibrium expected welfare, $\mathbb{E}w^e$, is

$$\mathbb{E}w^e = \frac{C_0^2}{2\eta} \frac{\lambda^2}{2\lambda+1} \underbrace{\left\{ 1 - \left[\frac{(\lambda+1)(1-m)}{\lambda(1+m)+m} \right]^2 \right\}}_{= \text{value of commitment}} - \eta \left[\lambda + \frac{1}{2} \right] \sigma_\nu^2 - m\eta \frac{\sigma_\varepsilon^2}{2} \tag{15}$$

Thus, the increase in tax rate due to the inattention is mitigated, $\tau_0^e \in (\tau_0^n, \tau_0^*)$, and the resulting welfare is lower than that under the optimal tax rate, $\mathbb{E}w^e < \mathbb{E}w^*$.

Sketch of Proof. As illustrated by Figure 1 (III) and 3, the long-run demand curve is not only less elastic with respect to the actual tax rate τ, but also is shifted

[11] This effect is related to but differs from the attention variation effects in Taubinsky and Rees-Jones (2018) since here the attention level m is fixed but the citizens perceptions vary.

in parallel to intersect with the underlying demand curve at the prior tax rate $\overline{\tau}_0$. That is, by Lemma 1, the aggregate consumption is

$$\overline{C}(\tau) = C_0 - \eta\left[m\tau + (1-m)\overline{\tau}_0\right]. \tag{16}$$

Given the actual tax rate τ, the deadweight loss becomes

$$DWL(\tau) = \frac{\eta}{2}\left[m\tau + (1-m)\overline{\tau}_0\right]^2 + \eta\frac{m^2\sigma_\varepsilon^2}{2}. \tag{17}$$

The key is that the social planner takes the citizens expectation $\overline{\tau}_0$ as given, and chooses the intended tax rate τ_0 to solve

$$\max_{\tau_0} \mathbb{E}\left[\lambda\tau\overline{C}(\tau) - DWL(\tau)\,|\tau_0\right]. \tag{18}$$

That is, by the Appendix Proposition 1, the first order condition with respect to τ_0 implies

$$\lambda\left\{C_0 - \eta\left[2m\tau_0 + (1-m)\overline{\tau}_0\right]\right\} = m\eta\left[m\tau_0 + (1-m)\overline{\tau}_0\right]. \tag{19}$$

Nonetheless, in equilibrium, the prior must coincide with the intended tax rate by the Bayes' rule:

$$\overline{\tau}_0 = \tau_0. \tag{20}$$

We substitute this consistency condition (20) into (19) to derive the equilibrium tax formula (14). The expression (15) for the resulting welfare is obtained by substitution.

Note that $\tau_0^o < \tau_0^e < \tau_0^n$ because, by $m \in (0,1)$,

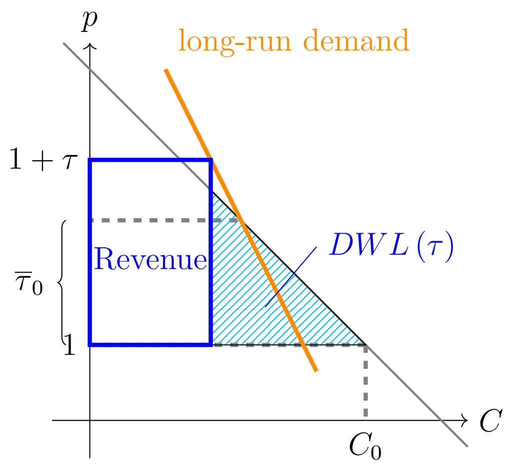
$$\frac{1}{2\lambda+1} < \frac{1}{\lambda(1+m)+m} < \frac{1}{2\lambda m + m^2}.$$

The attained welfare becomes lower than the full attention benchmark because the value of commitment is less than 1 and idiosyncratic misperceptions further reduce the welfare. □

Proposition 3 shows that the equilibrium tax rate (14) will be higher than the optimal rate (9) but not as high as the non-equilibrium tax rate (11).[12] Here, the

[12] It is informative to interpret the limit cases of full attention ($m = 1$) and no attention ($m = 0$). For both non-equilibrium and equilibrium tax formula, when $m = 1$, the optimal tax formula is

Figure 3: Revenue and Deadweight Loss under Inattention in the Long-run



Notes: Figure 3 illustrates the optimal tax problem under inattention in the long-run. The long-run demand curve with inattention (the thick orange solid line) is less elastic than the underlying demand curve (the thin black straight line) because the consumers are inattentive to taxes. Nonetheless, they interact at the expected tax rate $\overline{\tau}_0$ because the citizens' prior belief will adjust in the long-run. Crucially, the social planner considers the welfare effects of tax rate τ taking the citizens' prior as given.

increase in tax rate will be moderated because the aggregate consumption, and consequently the marginal benefit taxation, will be low due to the updated prior belief. Unlike the short-run analysis, the effect of inattention is no longer quadratic, m^2, but linear, m, in the attention parameter. More concretely, a comparative static analysis shows, starting from the benchmark of full attention, the effect of inattention on the implied tax rates will be halved compared to the short-run analysis: for any λ and η,

$$\frac{\partial \tau_0^e}{\partial m} \mid_{m=1} = \frac{1}{2} \frac{\partial \tau_0^n}{\partial m} \mid_{m=1} . \tag{21}$$

The derivation is contained in the Appendix A3.

More importantly, in addition, the formula (15) shows that the inattention *reduces* the resulting equilibrium welfare. That is, the equilibrium tax rate is not optimally

recovered. For the equilibrium tax rate, as the attention decreases to $m = 0$ in equilibrium, the aggregate price $1 + \tau^e$ equals $1/\eta$, the intercept of the demand curve. That is, when there is no attention at all, the government will raise tax maximally so that the aggregate consumption will be 0 on average.

but excessively high because inattention compromises the government's ability to commit to the optimal tax rate. In any optimal tax models, if the government is allowed to choose the tax rate *after* the citizens' consumption decision, then it will choose a very high tax rate since there is zero behavioral distortion. But anticipating such high taxes, the citizens will not consume, and the government's enhanced ability to set tax rates after the consumption perversely reduces welfare. Because inattention essentially leads the consumers to make decisions without precise observation of tax rates, its effect is analogous to allowing the government to change taxes afterwards. That is, even though inattention may reduce the behavioral responses in the short-run, government will have to incur its cost in the long-run as the consumers' priors adjust. Together with the welfare reduction in idiosyncratic errors, the analysis shows that the long-run welfare must be reduced by citizens' inattention; conversely, tax transparency will unambiguously improve the social welfare in this model.

3.4 Implications for Sufficient Statistics Formulas

The long-run analysis of taxation under inattention leads to new empirical implications for the policies of tax transparency and tax rates.

3.4.1 Efficiency of Costly Tax Transparency Policy

We can use the closed-form welfare expressions to examine whether a costly policy to increase tax transparency is welfare-improving. Suppose a tax transparency program increases the citizens' attention from $m < 1$ to 1 at some marginal cost $\kappa \leq \lambda$. Fortunately, a conservative condition to assess whether this program is welfare-improving depends only on the commonly estimated parameters, even though the full model has introduced several other parameters. Specifically, this sufficient statistics formula depends only on two parameters: attention parameter, m, and marginal cost of public fund, $\lambda + 1$.

Proposition 4.1. Sufficient Statistics Formula for Tax Transparency Policy. *Suppose Assumption A1. The tax transparency policy improves the equilibrium welfare for any parameters* $\{\eta, C_0, \sigma_\nu^2, \sigma_\varepsilon^2\}$ *if and only if*

$$m \leq \frac{1}{2(\lambda + 1)}. \tag{22}$$

Proof. By Propositions 1 and 3, in general, the tax transparency program is welfare-improving if and only if

$$\frac{C_0^2}{2\eta}\frac{(\lambda-\kappa)^2}{2(\lambda-\kappa)+1} \geq \frac{C_0^2}{2\eta}\frac{\lambda^2}{2\lambda+1}\underbrace{\left\{1-\left[\frac{(\lambda+1)(1-m)}{\lambda(1+m)+m}\right]^2\right\}}_{=\text{ value of commitment}} - m\eta\frac{\sigma_\varepsilon^2}{2}. \tag{23}$$

This condition always holds whenever $(\lambda+1)(1-m) \geq \lambda(1+m)+m$, that is, the value of commitment is negative. We obtain the condition (22) by rearranging this inequality. Heuristically, if $1+\lambda$ is large, the higher deadweight loss can be more tolerated because the implied revenue is higher. □

Back-of-the-envelope calculation: In the U.S., there persists a controversial debate whether its costly tax system to ensure tax transparency is socially worthwhile. The common estimate of the compliance cost is as high as 10 percent, or $\kappa = 0.1$ (Slemrod, 1996). We can examine whether this cost is socially worthwhile based on typical estimates of relevant parameters. Here, we choose $\lambda = 0.3$ as the commonly used estimate in the literature (Ballard et al., 1985; Poterba, 1995; Finkelstein and McKnight, 2008). Then, since $1/(2\times 1.3) \simeq 0.385$, various estimates of attention parameters in the literature, such as $m = 0.35$ in the grocery store experiment of Chetty et al. (2009) and $m = 0.25$ in the online shopping experiment of Taubinsky and Rees-Jones (2018), suggest that this conservative condition (23) is satisfied. [13] As there are also income effects (Chetty et al., 2009) and idiosyncratic errors σ_ε^2 (Taubinsky and Rees-Jones, 2018; Caldwell et al., 2021), these results together suggest the current U.S. policy to incur the seemingly large compliance costs to ensure tax transparency is not socially wasteful.

[13] The original paper of Ballard et al. (1985) suggests the value of λ between 0.17 and 0.56. As Slemrod (1996) was careful to note, the wide range of plausible values of κ may be wide. Nonetheless, since the smallest suggested value of λ is 0.17, the condition $\kappa \leq \lambda$ is plausible. Even when we are conservative and set $\lambda = 0.56$ and $m = 0.35$, the general formula (23) suggests that the program is nonetheless optimal.

$$\frac{(0.56-0.1)^2}{2(0.56-0.1)+1} \geq \frac{0.56^2}{2\times 0.56+1}\left\{1-\left[\frac{(0.56+1)(1-0.35)}{0.56\times(1+0.35)+0.35}\right]^2\right\} \Leftrightarrow .110 \geq .023.$$

3.4.2 Discontinuously Large Role of Small Noise

Theories of rational inattention suggest that citizens will pay more attention when the welfare consequences are high. In contrast with the effect of idiosyncratic errors that become small (Section 3.2), here we show that this long-run commitment effect can be large even when these uncertainties are negligible.

Proposition 4.2 Discontinuity under Small Noise. *Suppose Assumption A1. Fix any $m \in [0,1]$, and set $\sigma_\nu^2 = \left[2\lambda m + m^2\right]\sigma_\varepsilon^2$. Then, the resulting equilibrium will have a discontinuity as $\sigma_\varepsilon^2 \to 0$: even though $\tau_0^e = \tau_0^*$ when $\sigma_\varepsilon^2 = 0$,*

$$\lim_{\sigma_\varepsilon^2 \to 0} \tau_0^e > \tau_0^*. \tag{24}$$

Proof. The equilibrium tax formula (14) shows that the mean tax rate will depend on the attention parameter, m, which reflects the *relative* precision of signal, but not its *absolute* precision. Thus, for every τ_0^e and its corresponding m, there exist sequence of $\{\sigma_\varepsilon^2, \sigma_\nu^2\}$ such that the overall uncertainty, $\sigma_\varepsilon^2 + \sigma_\nu^2$, converges to 0 while maintaining m constant. Thus, even if σ_ε^2 is arbitrarily small, the implied equilibrium tax rate will be higher so long as σ_ν^2 is also proportionately small. □

Besides this interpretation based on the relative precision of signals, we can also interpret this result based on higher-order beliefs (See e.g. Morris and Shin, 2002). To see this, let us consider citizens that receive the signal $\hat{\tau}$ to form an expectation $\mathbb{E}\left[\tau|\hat{\tau}\right]$ over the tax rate. Citizens also know that the planner sets the intended tax rate based on the government's expectation of citizen's expectation,

$$\mathbb{E}\left[\mathbb{E}\left[\tau|\hat{\tau}\right]|\tau_0\right].$$

However, this higher order belief will always be closer to the prior tax rate. That is, even when the tax rate is higher than the expectation, that increase will be only partially perceived by the citizens on average; conversely, even when the planner sets the tax rate level to be lower, that decrease will be only noisily perceived in expectation. In the model, citizens also know the government objective to increase the tax rate when the behavioral responses are mitigated. Thus, this model implies that the citizens will expect a higher tax rate under inattention. This result is inspired by game theory's insight that commitment is fragile under slightly imperfect observability

(Bagwell, 1995). The new element[14] is the insight from global games (Weinstein and Yildiz, 2007) that the equilibrium implication will depend crucially on the relative precision, but not on the absolute precision, of signals.

Standard economic analyses assume full attention of consumers ($\sigma_\varepsilon^2 = 0$) and full control of government over tax rates ($\sigma_\nu^2 = 0$). This modeling choice is made because people are believed to be fairly attentive and the hypothetical planner makes little mistakes, at least for important items of consumption. Usually, implications are approximately valid whenever their assumptions also approximately hold. This analysis shows, however, that such approximate validity of optimal taxation models is not guaranteed. In this sense, the standard taxation models are a "knife-edge" case: there exists a perturbation of the full attention and full control benchmark that changes their implications for taxes and welfare discontinuously. This "fragility" result is important in considering the robustness of the optimal tax analyses (e.g. Lockwood et al., 2020), and in understanding when and why the actual tax chosen deviates from the socially optimal tax rates.

4 Discussions and Conclusion

This Section discusses the key assumptions of the model, and concludes.

4.1 Discussions

The model has derived its result under many assumptions, such as (1) Bayesian updating of citizens, (2) government's social welfare maximization objective, (3) myopic objective in dynamic interpretation, (4) specific distributional assumptions, and (5) their implication of separability of variance. Let us discuss if the main results hold when these assumptions are altered.

1. **Evidence on Bayesian updating:** *Are there evidence that the citizens update their belief over the tax rates?*

   Yes. Evidence from field surveys suggests that even the inattentive citizens learn about their taxes over time. Recently, Caldwell et al. (2021) has elicited

[14] This approach stands in contrast with the subsequent papers (van Damme and Hurkens, 1997; Maggi, 1999) that considered a limit of small noise while keeping the environmental uncertainties constant.

the mean, confidence, and the probablistic beliefs over the tax refunds among low-income households in Boston. They show that, while the tax filers face substantial uncertainties, they update their beliefs with new information in a way consistent with Bayesian updating. Finkelstein (2009) surveys the belief of the drivers who pay the tolls by cash or electronically, and finds that those who are inattentive to the tolls because they are pay electronically tend to expect higher tolls, consistent with the idea that people adjust their priors even though they are inattentive. At one extreme, citizens assume zero tax when they do not observe them. At another extreme, they update their priors fully to expect the average levels. Both behavioral assumptions clearly have their own merits and reflect some aspects of actual human decisions. As existing models have all focused on the former assumption as surveyed by Bernheim and Taubinsky (2018), this paper takes the later approach to derive new implications.

2. **Time horizon of government objective:** *If the government is far sighted and maximizes the long-run welfare, would the commitment issue be resolved?*

   Yes. The time horizon of government objective can be modeled by whether the behavioral response is measured in either short-term or long-term. As citizens receive more information, the signal precision $(1/\sigma_\varepsilon^2)$ increases over time. Thus, in the short-term, the attention parameter and thus the behavioral response are lower while in the long-term, they are both higher.

3. **Distributional assumptions:** *If the assumptions of uniform distribution and normal distribution were replaced by some other distributions, could the results hold?*

   The quadratic objective, which results from the uniform distribution, and normal type and noise distributions, are a pair of assumptions that is commonly used (e.g. Morris and Shin, 2002; Sims, 2003) to derive closed-form solutions. Most other distributions are often not analytically tractable with the Bayes' rule. Nonetheless, binary distributions have also been used in the models of commitment with imperfect observability (Bagwell, 1995). Since the optimal tax problem yields closed-form solutions when modeled by the quadratic objective (Dupuit, 1844; Ramsey, 1927) the uniform and normal distributions are adopted in this paper. But these distributional assumptions are not the driving force of the main results regarding commitment.

4. **Potential benefit of tax uncertainties:** *Both commitment effect and idiosyncratic errors of tax uncertainty suggest that the tax uncertainty decreases welfare. Are there any reasons that the tax uncertainty may help?*

   Yes. From the subjective perspective of the citizens, the uncertainty over tax rates implies that the realized tax rates will be random. When there is heterogeneity of risk aversion, randomization of tax rates can improve welfare through efficient screening (Atkinson and Stiglitz, 1976; Stiglitz, 1982). In the context of income taxation, if high ability citizens are more risk averse than low ability citizens, then subjective uncertainty over the tax rate given low income can prevent the high ability ones from pretending to be low abilities. More broadly, if citizens are prudent, then future income uncertainty due to random taxes can encourage them to work more. While this paper sets aside this effect by assuming that the risk aversion is constant (i.e. quadratic utility), this effect could exist in the real world.

5. **Other constraints on government objective:** *If the government faces other constraints, such as political constraints, so that it sets the tax rate lower than the social optimum, could inattention improve the social welfare to offset the pre-existing biases?*

   Yes. By the theory of second best (Lipsey and Lancaster, 1956), the inefficiency due to inattention can address other inefficiencies. Mill (1848) discusses the possibility that the "excessively" collected revenues may be used efficiently for valuable public expenditures that do not have political support.

4.2 Conclusion

Robust evidence has shown that short-run behavioral responses to taxes depend on their transparency. Based on this evidence, recent optimal tax theory has modeled inattention as a reduction in behavioral distortion. Yet further evidence shows that citizens may update beliefs over the tax rates in the long-run even when they are inattentive in the short-run. When inattention is thus modeled as a noise of tax rate signals received by Bayesian citizens, this paper has found that inattention reduces the government's commitment ability. That is, without transparency, even benevolent government may set the tax rate above the socially optimal level. This finding is consistent with the traditional discussions on the role of tax salience, and the implied magnitude of welfare consequences suggests transparency to be the principal goal of designing tax systems.

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Appendix

This Appendix contains the formal proofs. A1 shows that the social planner's problem is quadratic in the intended tax rate, providing the basis for the Propositions 1-3 in the main text; A2 derives the attained welfare in the Propositions; and A3 shows the comparative static of the tax rate with respect to the attention parameter.

A1. Social Welfare Formula

In the model set-up, the deadweight was defined as the reduction in welfare due to taxes not included in the revenue. Here, we show that the expression (5) translates into the tractable quadratic formula, as used in the Propositions.

Appendix Proposition 1. *Suppose Assumption A1. Given the expected value of the prior tax rate, $\overline{\tau}_0$, the social welfare (4) is*

$$\mathbb{E}\left[w|\tau_0\right] = -\alpha\tau_0^2 + \beta\tau_0 + \gamma, \tag{25}$$

where

$$\begin{aligned}
\alpha &= m\eta\left[\lambda + \frac{m}{2}\right] \\
\beta &= \lambda C_0 - \eta\left(1-m\right)\left(\lambda + m\right)\overline{\tau}_0 \\
\gamma &= -\eta\left[\lambda + \frac{1}{2}\right]\sigma_\nu^2 - m\eta\frac{\sigma_\varepsilon^2}{2} - \eta\frac{\left(1-m\right)^2\overline{\tau}_0^2}{2}.
\end{aligned}$$

Proof. The proof consists of two steps: first, we derive the individual utilities given the consumption choices, and use the expressions to derive the components of objective; second, we combine them to derive the formula (25).

Step 1. Welfare expressions of each component: the social planner's objective consists of (i) the zero-tax welfare and (ii) the equilibrium welfare and revenue. When we derive each component, we consider definite integrals over a domain $v \in [-\overline{v}, \overline{v}]$ with a uniform density.

(i) pre-tax welfare: since the first-best efficient decision is to consume if and only if $v \geq 1$,

$$\int_{-\overline{v}}^{\overline{v}} \max_{c} u\left(c, v | \tau=0\right) dv = \int_{-\overline{v}}^{\overline{v}} \mathbb{1}\left(v \geq 1\right)\left(v-1\right) dv$$
$$= \int_{1}^{\overline{v}} \left(v-1\right) dv$$
$$= \left[\frac{v^2}{2} - v\right] |_{1}^{\overline{v}}$$
$$= \frac{\overline{v}^2}{2} - \overline{v} + \frac{1}{2}. \tag{26}$$

(ii) equilibrium welfare and revenue: since the equilibrium decision is to consume if and only if $v \geq 1 + \tilde{\tau}$,

$$\int_{-\overline{v}}^{\overline{v}} \mathbb{E}\left[u\left(c_{v,\tilde{\tau}}, v|\tau\right) + \tau c_{v,\tilde{\tau}} \mid \tau\right] dv = \int_{-\overline{v}}^{\overline{v}} \mathbb{P}\left(v \geq 1 + \tilde{\tau} \mid \tau\right)\left(v-1\right) dv$$

Since $\tilde{\tau} = m\hat{\tau} + (1-m)\,\overline{\tau}_0$ by Lemma 1, the variance of the posterior distribution conditional on the actual tax τ is

$$Var\left(\tilde{\tau}|\tau\right) = m^2 Var\left(\hat{\tau}|\tau\right) + \left(1-m\right)^2 Var\left(\overline{\tau}_0|\tau\right)$$
$$= m^2 \sigma_{\varepsilon}^2 \tag{27}$$

Therefore, the fraction of citizens of type v who consume (or, $c^* = 1$) is given by

$$\mathbb{P}\left(v \geq 1 + \tilde{\tau} \mid \tau\right) = \Phi\left(v|\tau\right) \equiv \Phi_0\left(\frac{v - 1 - \left[m\tau + \left(1-m\right)\overline{\tau}_0\right]}{m\sigma_{\varepsilon}}\right),$$

where $\Phi_0\left(\cdot\right)$ is the distribution function of the standard normal distribution. By the integration by parts,

$$\int_{-\overline{v}}^{\overline{v}} \Phi\left(v|\tau\right)\left(v-1\right) dv = \left[\frac{v^2}{2} - v\right] \Phi\left(v|\tau\right) |_{-\overline{v}}^{\overline{v}} - \int_{-\overline{v}}^{\overline{v}} \left[\frac{v^2}{2} - v\right] \phi\left(v|\tau\right) dv$$
$$= \frac{\overline{v}^2}{2}\left[\Phi\left(\overline{v}|\tau\right) - \Phi\left(-\overline{v}|\tau\right)\right] - \overline{v}\left[\Phi\left(\overline{v}|\tau\right) + \Phi\left(-\overline{v}|\tau\right)\right]$$
$$- \frac{1}{2}\mathbb{E}\left[v^2|\tau, |v| \leq \overline{v}\right] + \mathbb{E}\left[v|\tau, |v| \leq \overline{v}\right], \tag{28}$$

where $\phi\left(v|\tau\right)$ is the density, and $\mathbb{E}\left[\cdot\right]$ is the expectation of $\Phi\left(v|\tau\right)$ for the domain $v \in \left[-\overline{v}, \overline{v}\right]$.

Step 2. Expression for total welfare: we now use the expressions in Step 1, (26) and (28), to show that the total welfare (4) is quadratic in the intended tax rate τ_0.

(i) total deadweight loss conditional on τ: the total deadweight loss conditional on the actual tax τ is

$$DWL(\tau) = \lim_{\overline{v}\to\infty} \int_{-\overline{v}}^{\overline{v}} \left[\mathbb{1}(v \geq 1) - \Phi(v|\tau)\right](v-1) f(v)\, dv. \tag{29}$$

Since the density $f(v) = \eta$ for all v by the Assumption A1, we combine and reorganize (26) and (28) to have

$$DWL(\tau)/\eta = \lim_{\overline{v}\to\infty} \frac{\overline{v}^2}{2}\left[1 - \Phi(\overline{v}|\tau) + \Phi(-\overline{v}|\tau)\right] \tag{30}$$

$$+ \lim_{\overline{v}\to\infty} \overline{v}\left[\Phi(\overline{v}|\tau) + \Phi(-\overline{v}|\tau) - 1\right] \tag{31}$$

$$+ \lim_{\overline{v}\to\infty} \left\{\frac{1}{2}\mathbb{E}\left[v^2|\tau, |v| \leq \overline{v}\right] - \mathbb{E}\left[v|\tau, |v| \leq \overline{v}\right] + \frac{1}{2}\right\} \tag{32}$$

Since the distribution function of normal distribution converges to 0 or 1 faster than the linear or quadratic terms, and since the normal distribution is thin-tailed, the expressions (30) and (31) equal zero by the L'Hopital's rule. Since $\mathbb{E}[v^2] = \mathbb{E}[v]^2 + Var(v)$, taking the limit as $\overline{v} \to \infty$, the expression (32) becomes

$$\begin{aligned} \frac{1}{2} + \frac{1}{2}\mathbb{E}\left[v^2|\tau\right] - \mathbb{E}[v|\tau] &= \frac{\{\mathbb{E}[v|\tau] - 1\}^2 + Var(v|\tau)}{2} \\ &= \frac{\left[m\tau + (1-m)\overline{\tau}_0\right]^2}{2} + \frac{m^2\sigma_\varepsilon^2}{2} \end{aligned}$$

by Lemma 1 and (27). Combining these observations, the deadweight loss (29) is

$$DWL(\tau) = \frac{\eta}{2}\left\{\left[m\tau + (1-m)\overline{\tau}_0\right]^2 + m^2\sigma_\varepsilon^2\right\}.$$

(ii) total welfare conditional on τ_0: using Lemma 1 to combine with the effects on revenues, we have

$$
\begin{aligned}
\mathbb{E}\left[w|\tau_0\right] &= \lambda \mathbb{E}\left[\tau\left\{C_0 - \eta\left[m\tau + (1-m)\,\overline{\tau}_0\right]\right\}|\tau_0\right] \\
&\quad - \frac{\eta}{2}\mathbb{E}\left[\left[m\tau + (1-m)\,\tau_0\right]^2|\tau_0\right] - \frac{\eta}{2}m^2\sigma_\varepsilon^2 \\
&= \lambda\tau_0\left\{C_0 - \eta\left[m\tau_0 + (1-m)\,\overline{\tau}_0\right]\right\} - \lambda m\eta\sigma_\nu^2 \\
&\quad - \frac{\eta}{2}\left[m\tau_0 + (1-m)\,\overline{\tau}_0\right]^2 - \frac{\eta}{2}m^2\left(\sigma_\nu^2 + \sigma_\varepsilon^2\right)
\end{aligned}
$$

since $\tau = \tau_0 + \nu$, $\mathbb{E}\nu = 0$, and $\mathbb{E}\nu^2 = \sigma_\nu^2$. Thus, re-organizing the terms, we obtain (25). □

A2. Attained Welfare

Here, we derive the expressions for attained welfare in Propositions 1, 2, and 3. From the Appendix Proposition 1, we can complete the square to obtain

$$
\mathbb{E}\left[w|\tau_0\right] = \alpha\left(\tau_0 - \frac{\beta}{2\alpha}\right)^2 - \frac{\beta^2}{4\alpha} + \gamma
$$

(i) Proposition 1. Optimal Tax under Full Attention: since $\sigma_\varepsilon^2 = 0$ under full attention, we have $m = 1$. Thus,

$$
\begin{aligned}
\alpha &= \eta\left[\lambda + \frac{1}{2}\right] \\
\beta &= \lambda C_0 \\
\gamma &= -\eta\left[\lambda + \frac{1}{2}\right]\sigma_\nu^2.
\end{aligned}
$$

Since this welfare is optimized, we have $\mathbb{E}w^* = -\beta^2/4\alpha + \gamma$ to derive (10).

(ii) Proposition 2. Non-equilibrium Tax under Inattention: while $m < 1$ due to inattention, in the short-run, $\overline{\tau}_0 = 0$. Thus,

$$
\begin{aligned}
\alpha &= m\eta\left[\lambda + \frac{m}{2}\right] \\
\beta &= \lambda C_0 \\
\gamma &= -\eta\left[\lambda + \frac{1}{2}\right]\sigma_\nu^2 - m\eta\frac{\sigma_\varepsilon^2}{2}.
\end{aligned}
$$

Since this welfare is optimized, we again have $\mathbb{E}w^n = -\beta^2/4\alpha + \gamma$ to derive (12).

(iii) Proposition 3. Equilibrium Tax under Inattention: while $m < 1$ due to inattention, the prior belief will be consistent in the long run, $\overline{\tau}_0 = \tau_0^e$. Since we can substitute this into the welfare expression to obtain that the welfare formula is

$$\mathbb{E}w^e = \alpha \left(\tau_0^e - \tau_0^*\right)^2 - \frac{\beta^2}{4\alpha} + \gamma,$$

where

$$\begin{aligned}
\alpha &= \eta \left[\lambda + \frac{1}{2}\right] \\
\beta &= \lambda C_0 \\
\gamma &= -\eta \left[\lambda + \frac{1}{2}\right] \sigma_\nu^2 - m\eta \frac{\sigma_\varepsilon^2}{2}.
\end{aligned}$$

m nnThat is, except the additional term of σ_ε^2, the objectives is identical to the full attention benchmark since $m\tau_0 + (1-m)\,\overline{\tau}_0 = \tau_0$. By the Propositions 1 and 3,

$$\tau_0^e - \tau_0^* = \frac{\lambda C_0}{\eta} \left(\frac{1}{\lambda\,(1+m) + m} - \frac{1}{2\lambda + 1} \right),$$

and we derive (15) by substitution.

A3. Comparative Static of Tax Rate with respect to Attention Parameter

We wish to prove the ratio of comparative static (21) in Section 3.3. Given the formulas (11) and (14), let us denote

$$\begin{aligned}
\Lambda^n\,(m) &= 2\lambda m + m^2 \\
\Lambda^e\,(m) &= \lambda\,(1+m) + m.
\end{aligned}$$

By differentiating the formulas,

$$\frac{\partial \tau_0^n}{\partial m} \mid_{m=1} = \frac{1-\eta}{\eta} \frac{-\lambda}{\Lambda^n(m)^2} \frac{\partial \Lambda^n(m)}{\partial m} \mid_{m=1} = -\frac{1-\eta}{\eta} \frac{\lambda}{(2\lambda+1)^2} (2\lambda+2)$$
$$\frac{\partial \tau_0^e}{\partial m} \mid_{m=1} = \frac{1-\eta}{\eta} \frac{-\lambda}{\Lambda^e(m)^2} \frac{\partial \Lambda^e(m)}{\partial m} \mid_{m=1} = -\frac{1-\eta}{\eta} \frac{\lambda}{(2\lambda+1)^2} (\lambda+1).$$

Thus, the ratio (21) holds.