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Demographic Aging and Macroeconomy in Japan

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Females, the Elderly, and Also Males: Demographic Aging and Macroeconomy in Japan *

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Abstract

The speed and magnitude of ongoing demographic aging in Japan are unprecedented. A rapid decline in the labor force and a rising fiscal burden to finance social security expenditures could hamper growth over a prolonged period. We build a dynamic general equilibrium model populated by overlapping generations of males and females who differ in employment type and labor productivity as well as life expectancy. We study how changes in the labor market over the coming decades will affect the transition path of the economy and fiscal situation of Japan. We find that a rise in the labor supply of females and the elderly of both genders in an extensive margin and in labor productivity can significantly mitigate effects of demographic aging on the macroeconomy and reduce fiscal pressures, despite a decline in wage during the transition. We also quantify effects of alternative demographic scenarios and fiscal policies. The study suggests that a combination of policies that remove obstacles hindering labor supply and that enhance a more efficient allocation of male and female workers of all age groups will be critical to keeping government deficit under control and raising income across the nation.

Keywords: Japanese Economy, Demographic Trends, Female and Elderly Labor Force Participation, Overlapping Generation Model.

JEL Classification: E62, J11, J21, H55.

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1 Introduction

Japan is experiencing rapid and massive demographic aging. Despite the imminent urgency to deal with macroeconomic and fiscal concerns, how Japan is going to handle a dramatic shift in its demographic structure and rising expenditures remains to be seen.

In this paper, we build a general equilibrium model of overlapping generations of male and female individuals and quantify effects of demographic trends affecting the future path of individual behaviors, macroeconomic variables and fiscal situations in Japan.

The Japanese government is keen on encouraging labor force participation of females and nowadays also of the elderly. The employment rate for prime-age males is above 90% and there is little room to further increase their participation. Females and the elderly are two groups of individuals whose participation is much lower than that of prime-age males and who could contribute, thereby slowing down the expected decline in aggregate labor supply and potentially improving the fiscal situation. What is unknown, however, is whether and how a major rise in participation of females and the elderly affects the Japanese economy including the transition path of output and factor prices as well as fiscal tensions associated with rising social security expenditures. We quantify effects of changes in the labor supply, treated exogenously in the paper, of individuals of different genders, age as well as employment types and productivity.

We find that a rise in participation of females and the elderly, for example, as projected by the Japan Institute for Labour Policy and Training (JILPT) over the coming decades until 2040, will mitigate a massive decline in the labor force expected during the coming decades. Quantitative effects, however, are most significant when not only labor force participation rates but also distribution of employment types as well as their productivity measured in terms of their wages also grow, for example, to the levels of males.

Many female workers in their late 20s and 30s leave the labor force as they marry and have children. A large number of them return to work after several years, yielding a so-called "M-shaped" profile of participation rates over the life-cycle. When they return, however, their employment types often differ from the ones prior to their departure and most of them are engaged in a contingent job rather than a regular job. The former typically pays significantly less than the latter. There exists also a major gap between male and female wages even after controlling for age and employment type. Our study suggests various efforts to narrow the gap between males and females in the dimensions of participation rate, employment type and wage, would go far in making up for the decline in the labor force and filling the gap between rising government expenditures and shrinking revenues.

Although much focus in the policy discussion is on females and the elderly and this paper is not an exception, there are some concerning trends in the labor supply of males during recent years. Among prime-age working males, there has been a rise in the fraction of contingent workers across a wide range of age groups in their 20s to 50s. The trend has been observed since the early 2000s, when a breakdown of employment types among workers became available from the Labor Force Survey (LFS). Similar to females, contingent workers have much lower earnings compared to regular workers. We find that aggregate labor supply, as well as capital and output, will be much lower and the fiscal burden in terms of tax rates to finance rising expenditures will be higher, if the trend of a rising share of contingent workers among males continues.

Our study is an extension of general equilibrium models of overlapping generations to study life-cycle allocations of individuals, in the tradition of Auerbach and Kotlikoff (1987). We follow literature that uses such models to study effects of cross-generational redistributional policies, such as De Nardi et al. (1999), Conesa and Krueger (1999), Nishiyama and Smetters (2007), and Kitao (2014). There are papers that use a life-cycle model to analyze effects of aging demographics and fiscal challenge in Japan, such as Hansen and İmrohoroğlu (2016), Braun and Joines (2015), Kitao (2015), Hoshi and Ito (2014), Doi et al. (2011) and Okamoto (2013).

These papers abstract from differences between males and females and assume life-cycle profiles of wages and demographics of either males or an average of the two groups. The two, however, have very different profiles of labor market experience over their life-cycle as mentioned above. Work by İmrohoroğlu et al. (2016) presents a generational accounting model with details of employment types of males and females, as well as institutional structures of the Japanese social security system. The study, however, assumes partial equilibrium and differs from ours in that it explicitly incorporates production and endogenous determination of factor prices driven by the ongoing demographic aging.

This paper, however, does not present a model to investigate why gender gaps in terms of labor market experience of males and females exist, or why females are sorted to particular employment types initially and also later in their life-cycle. While these are important issues, this paper's focus is to quantify the effects of exogenous changes in the pattern of labor supply, extrapolating the trend that we identify from the data in several possible ways, on the future path of the macroeconomy, factor prices and fiscal situations, as implied by rising social security expenditures and falling tax revenues. We aim to identify factors that are critical in projections of these variables in a general equilibrium framework of males and females.

Papers such as Attanasio et al. (2008) and Blundell et al. (2016) build a model with endogenous female labor supply and study their life-cycle behavior. Recent work by Blundell et al. (2019) extends these models and estimates a structural life-cycle model to evaluate roles of formal training and experience for females in mitigating the wage gap between men and women. They show that effects of training can be important especially for women who have fewer years of education. Extending the current paper in such a direction in the context of the Japanese economy will be a challenging but promising direction of future work.

The rest of the paper is organized as follows. We present the model in section 2. Details of the calibration and data sets used are given in section 3. The benchmark results, sensitivity analyses and policy experiments are discussed in section 4. Section 5 concludes.

2 The Model

In this section we describe the model, the problem of individuals and the definition of a competitive equilibrium. Details of the computation of the transition dynamics are provided in Appendix A.

2.1 Demographics

We let t denote time, i age of an individual with a maximum age of I. g denotes the gender, taking one of two values $\{m, f\}$, representing male and female, respectively. The model is populated by overlapping generations of individuals of gender g who enter the economy at age $i = 1$. $s_{i,g,t}$ denotes the probability that an individual of gender g alive at time $t-1$ at age $i-1$ survives until the next period. Unconditional probability of surviving i periods up to time t is given by

$$S_{i,g,t} = \prod_{k=1}^{i} s_{k,g,t+(k-i)}.$$

$S_{1,g,t} = s_{1,g,t} = 1$ and $S_{I+1,g,t} = s_{I+1,g,t} = 0$ for all g and t. We denote by $\mu_{i,g,t}$ the size of the population of age i, gender g and at time t. The size of a new cohort $\mu_{1,g,t}$ grows at rate $n_{g,t}$.

2.2 Technology

Output is produced according to a constant returns to scale (CRS) aggregate production function, $Y_t = F(Z_t, K_t, N_t)$. Output can be used for consumption or investment. Z_t denotes the total factor productivity at time t, N_t is the aggregate labor supply and K_t is the aggregate capital. Capital depreciates at a constant rate δ each period. Firms are competitive and interest rate and wage equal marginal products of capital and labor, respectively. We assume that the level of productivity grows exogenously at rate λ_t.

2.3 Preferences and Endowments

Individuals' instantaneous utility function is given as

$$u(c_{i,g,t}) = \frac{c_{i,g,t}^{1-\theta}}{1-\theta}, \tag{1}$$

where $c_{i,g,t}$ denotes consumption of an individual of age i and gender g at time t. Individuals of gender g born at time t choose the optimal path of consumption and saving in order to maximize their lifetime utility given as

$$U_{g,t} = \sum_{i=1}^{I} \beta^{i-1} S_{i,g,t+i-1} \frac{c_{i,g,t+i-1}^{1-\theta}}{1-\theta}. \tag{2}$$

β denotes a subjective discount factor. Assets that are accidentally left by the deceased are distributed as a lump-sum transfer to all surviving individuals and the amount is denoted as b_t.

Individuals of age i and gender g at time t exogenously supply $\varepsilon_{i,g,t}$ efficiency units of labor in the market. They receive the market wage w_t for each unit of efficiency. Before-tax earnings are given as $\varepsilon_{i,g,t} w_t$.

We let $a_{i,g,t}$ denote assets held by an individual of age i and gender g at time t. We assume that individuals enter the economy with no initial assets, that is, $a_{1,g,t} = 0$. We allow individuals to borrow and impose no borrowing constraint during their lifetime, except that they cannot have negative assets at the end of their last age I, that is, $a_{I+1,g,t} = 0$.

2.4 Government

The government raises revenues through taxes on consumption at rate $\tau_{c,t}$, labor income at $\tau_{w,t}$, and capital income at $\tau_{a,t}$, and issuance of risk-free debt, B_{t+1}. Government borrowing and tax revenues finance a stream of expenditures, G_t, benefits of the PAYG social-security program and payment of debt, $(1 + r_t)B_t$.

We assume that individuals are entitled to public pension benefits once they reach the normal retirement age (NRA) I^R. Note that this is the age to become eligible to receive pensions and not necessarily the age to leave the labor force. We let $p_{i,g,t}$ denote public pension benefits that individuals of age i and gender g receive at time t. We assume that social security benefits are determined by the formula

$$p_{i,g,t} = \kappa_t \frac{W_{i,g,t}}{I^R - 1},$$

where κ_t is the replacement ratio of average past earnings. Cumulated past gross earnings $W_{i,g,t}$ are defined recursively as

$$W_{i,g,t} = \begin{cases} \varepsilon_{1,g,t} w_t & \text{if } i = 1 \\ \varepsilon_{i,g,t} w_t + W_{i-1,g,t-1} & \text{if } 1 < i < I^R \\ W_{i-1,g,t-1} & \text{if } i \geq I^R. \end{cases} \tag{3}$$

The government budget constraint reads as

$$
\begin{aligned}
G_t + (1+r_t) B_t \quad &+ \quad \sum_{i=I^R}^{I} \sum_g p_{i,g,t} \mu_{i,g,t} = \tau_{w,t} w_t \sum_{i=1}^{I} \sum_g \mu_{i,g,t} \varepsilon_{i,g,t} \\
&+ \quad \tau_{a,t} r_t \sum_{i=1}^{I} \sum_g \mu_{i,g,t} (a_{i,g,t} + b_t) + \tau_{c,t} \sum_{i=1}^{I} \sum_g \mu_{i,g,t} c_{i,g,t} + B_{t+1}.
\end{aligned} \tag{4}
$$

In the simulations, we let the labor income tax rate $\tau_{w,t}$ in (4) adjust to achieve a budget balance in each period t.[1]

2.5 Individuals' Problem

The problem faced by an individual is to choose a sequence of consumption and saving at each age so that the objective function defined in equation (2) is maximized. We formulate a recursive problem below. An individual's state vector at each time period is given as $\{i, g, a\}$, where i is age, g gender and a assets. The value function $V_t(i, g, a)$ of an individual in state $\{i, g, a\}$ at time t is given as follows.

$$
V_t(i, g, a_t) = \max_{c_t, a_{t+1}} \{u(c_t) + \beta s_{i+1,g,t+1} V_{t+1}(i+1, g, a_{t+1})\}
$$

subject to

$$
(1+\tau_{c,t}) c_t + a_{t+1} = (1-\tau_{w,t}) \varepsilon_{i,g,t} w_t + [1 + (1-\tau_{a,t}) r_t] (a_t + b_t) + p_{i,g,t}. \tag{5}
$$

where $p_{i,g,t}$ denotes pensions and is zero for individuals aged below I^R.

2.6 Competitive Equilibrium

A competitive equilibrium, for a given sequence of demographics, total factor productivity levels $\{Z_t\}_{t=1}^{\infty}$, and fiscal variables $\left\{G_t, B_t, \tau_{a,t}, \tau_{c,t}, \kappa_t, \{p_{i,g,t}\}_{i,g}\right\}_{t=1}^{\infty}$, is a sequence of

- individuals' choices $\left\{\{c_{i,g,t}, a_{i,g,t}\}_{i,g}\right\}_{t=1}^{\infty}$,
- labor income tax rates $\{\tau_{w,t}\}_{t=1}^{\infty}$,
- wage rates $\{w_t\}_{t=1}^{\infty}$, interest rates $\{r_t\}_{t=1}^{\infty}$, and
- aggregate variables $\{K_t, N_t\}_{t=1}^{\infty}$

such that:

[1] We note that the model assumes exogenous labor supply and abstracts from distortionary effects of taxation on work incentives. A rise in labor income taxes controlling for everything else will reduce the amount of disposable life-time income of individuals in the same way as the lump-sum taxation.

1. Individuals choose sequences of consumption and saving, maximizing the objective function in (2) subject to the budget constraint (5).

2. Firms maximize profits by equating the marginal product of each input equal with its price, i.e.

$$w_t = F_N(Z_t, K_t, N_t), \quad (6)$$
$$r_t = F_K(Z_t, K_t, N_t) - \delta. \quad (7)$$

3. The lump-sum transfer of accidental bequests equals the amount of assets left by the deceased, distributed equally to all surviving individuals.

$$b_t = \frac{\sum_{i=1}^{I-1} \sum_g a_{i+1,g,t}(1 - s_{i+1,g,t})\mu_{i,g,t-1}}{\sum_{i=1}^{I} \sum_g \mu_{i,g,t}}$$

4. The labor market clears at wage w_t and aggregate labor supply is given by

$$N_t = \sum_{i=1}^{I} \sum_g \mu_{i,g,t}\varepsilon_{i,g,t}. \quad (8)$$

5. The bond and capital markets clear at interest rate r_t, and the aggregate stock of capital satisfies

$$K_t + B_t = \sum_{i=1}^{I} \sum_g \mu_{i,g,t}(a_{i,g,t} + b_t). \quad (9)$$

6. The tax rate $\tau_{w,t}$ satisfies the government budget constraint (4).

7. The goods market clears.

$$\sum_{i=1}^{I} \sum_g \mu_{i,g,t}c_{i,g,t} + K_{t+1} + G_t = F(Z_t, K_t, N_t) + (1 - \delta)K_t \quad (10)$$

3 Calibration

In this section, we describe parametrization of the model and various data sources that we use to calibrate the model. The transition starts in an economy, which approximates the Japanese economy in 1990. We call this economy the initial steady state. A final steady state is computed, where the demographic transition is complete and all variables, including demographics and micro and macro variables, have become stationary.[2] We then let the economy make a transition between the two steady-states, by imposing the projected path of demographics. The model frequency is annual.

[2]We set the final year of the transition to 2500, which needs to be far enough so that all variables converge to those in the final steady state smoothly.

3.1 Demographics

We let individuals enter the economy at age $i = 1$, which corresponds to 20 years old, and live up to the maximum age of $I = 85$, or 104 years old. We use demographic projections of the National Institute of Population and Social Security Research (IPSS) for age and gender-specific survival rates, $s_{i,g,t}$, and growth rates of a new cohort, $n_{g,t}$.[3] In computation of the baseline transition, we use population projections based on medium scenarios of fertility rates and survival rates. In section 4.2, we consider low and high scenarios of both demographic parameters and study sensitivity of our results to alternative population projections.

3.2 Preferences

A parameter of relative risk aversion θ is set at 2.0. The subjective discount factor β is set at 1.047 so that the model achieves a capital-output ratio of 3.2 in 2015, which is based on the average ratio of the sum of tangible and intangible fixed assets and GDP between 2010 and 2014.[4]

3.3 Technology

We assume a constant returns to scale production function,

$$F(Z_t, K_t, N_t) = Z_t(K_t)^{\alpha}(N_t)^{1-\alpha}.$$

The capital share parameter α is set at 0.40 and the depreciation rate of capital δ at 7% per year, based on the values reported in the Actuarial Valuation of the Ministry of Health, Labour and Welfare (MHLW) in 2014.[5] In the baseline simulation, λ_t, the annual growth rate of the total factor productivity (TFP), which is Z_t, is set to 1.0% throughout the transition periods.[6] We consider alternative scenarios for the growth rate of TFP in section 4.2.

[3]The official demographic projection under various scenarios can be obtained at the website of the IPSS. http://www.ipss.go.jp/pp-zenkoku/e/zenkoku_e2017/pp_zenkoku2017e.asp

[4]The data is from the SNA. https://www.esri.cao.go.jp/jp/sna/data/data_list/kakuhou/files/h26/h26_kaku_top.html (in Japanese).

[5]The average depreciation rate is 7.1% in 2003-2012 (or 7.5% in 1983-2012) and capital ratio 42.8% in 2003-2012 (or 40.8% in 1983-2012), based on https://www.mhlw.go.jp/stf/seisakunitsuite/bunya/0000093204.html (in Japanese, Chapter 3-4).

[6]The average TFP growth rate was 0.9% in 2010-2015, according to the estimates of the Monthly Economic Report of the Cabinet Office. The growth was much higher in 1980s at 2.1%, falling to 1.2% in 1990s and 0.9% in 2000s.

3.4 Employment and Wages

In our model, $\varepsilon_{i,g,t}$ denotes efficiency units supplied to the labor market by a group of individuals of age i, gender g at time t. As such, it is considered as a composite of average efficiency units of workers of age i and gender g that also depends on their labor force participation rate. The former, the average efficiency units, are an average of workers in heterogeneous types of employment.

Figure 1 shows labor force participation rates of males and females by age, based on the Labor Force Survey (LFS) in 2015. In the figure, labor force participation rate is defined as the number of employed workers in the labor market divided by the number of all individuals of each gender and age.[7]

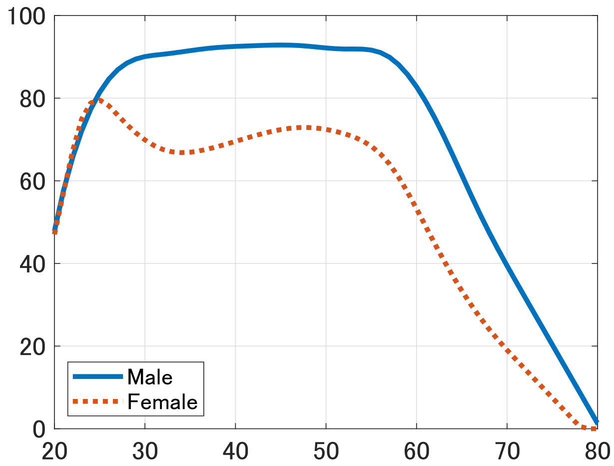


Figure 1: Labor Force Participation Rates in 2015 (%)

Among males, participation rates are high and stay above 90% in their late 20s to mid-50s. Participation rates start to decline thereafter. Note that, however, they do not reach zero until long after they turn 65, the normal retirement age (NRA) of the public pension, or even 70.

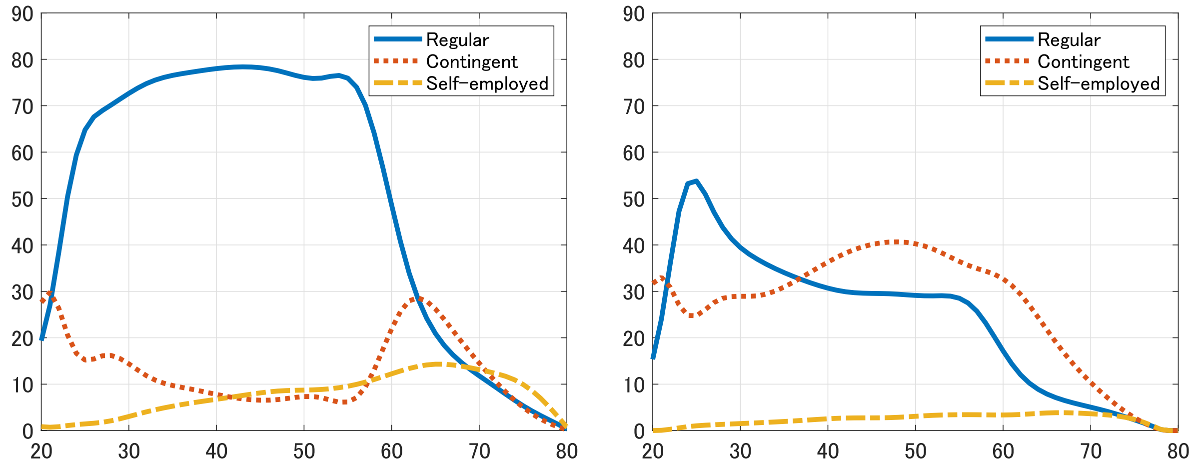
Participation rates are lower among females than males at almost all ages, and the profile shows two humps in their late 20s and more mildly in their late 40s and early 50s, exhibiting a so-called "M-shaped" profile of participation. A large number of female workers leave the labor force when they become married and start raising children and then return to work after a certain number of years.[8]

[7]Note that we do not include unemployed individuals and family workers (*kazoku jugyosha*) in computing labor force participation rates. Family workers are members of a self-employed individual's household, who help the business but do not receive wage. Unemployment rates have been low, with the average of 3.7% between 2010 and 2018, standing at 2.4% in 2018. Consistently, when we simulate alternative scenarios of participation using the projections in section 4, we use those of age and gender specific employment rates, not including family workers and unemployed individuals.

[8]As we discuss in section 4, the M-shape has become less visible in recent years. The Japan Institute

Typically, however, they do not return to the same position when they resume working and earnings are often lower than those before they left the labor force. Many females return to work as contingent workers, rather than regular workers. On a regular job, employment is more stable, typically continuing without an explicit end or even lasting for a lifetime until the retirement age set by a firm. Contingent jobs include part-time jobs or employment with a fixed-term contract and such earnings in general are much lower than those in regular jobs. In addition to regular and contingent jobs, there are also self-employed workers.

Figure 2 shows the population distribution of working individuals of males and females over the life-cycle, by the workers' employment type.[9]



(a) Male

(b) Female

Figure 2: Labor Force Participation Rates by Employment Type (% of total population at each age)

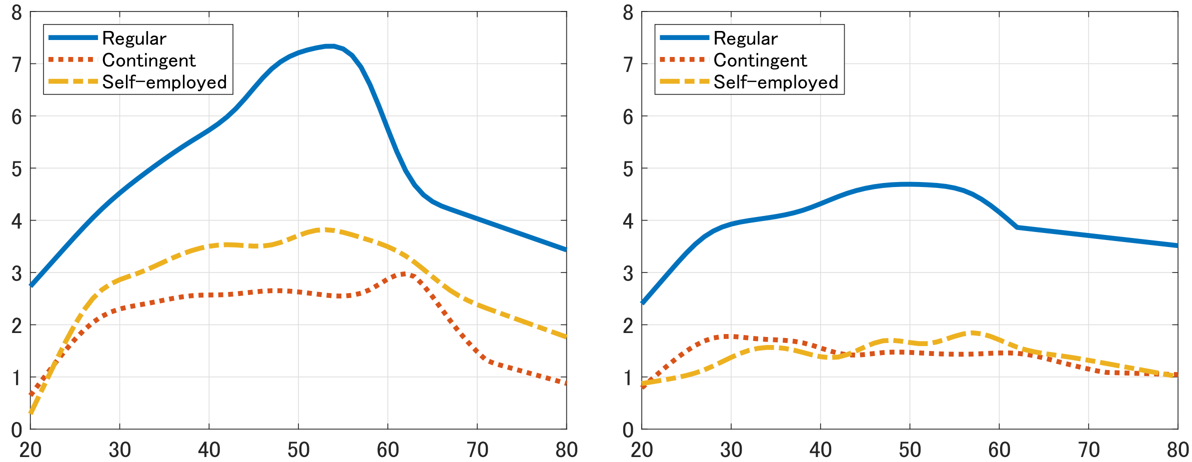
Figure 3 shows earnings profiles of male and female workers of different employment types. For regular and contingent workers, average earnings are computed based on the Basic Survey of Wage Structure (BSWS) data.[10] For self-employed individuals, there are no earnings data in the BSWS and we use data from the Employment Status Survey (ESS).[11]

for Labour Policy and Training (JILPT) projects that the profile will become even flatter over the next 20 years.

[9] "Regular" workers include regular (*seiki*) workers and managers (*yakuin*) and "contingent" contains irregular (*hiseiki*) workers including part-time workers, contracted workers and temporary staff. The LFS data is released by the Statistics Bureau of the Ministry of Internal Affairs and Communications (MIC).

[10] The BSWS data is available at: https://www.mhlw.go.jp/english/database/db-l/wage-structure.html

[11] The ESS is a survey conducted every five years. We use earnings data of 2017 and adjust the level using the CPI.
https://www.e-stat.go.jp/stat-search/file-download?statInfId=000031729381&fileKind=0



(a) Male (b) Female

Figure 3: Earnings by Employment Type (in million JPY)

Using above statistics, efficiency units $\varepsilon_{i,g,t}$ of individuals of age i and gender g are computed as weighted average of earnings:

$$\varepsilon_{i,g,t} = (y^R_{i,g,t}\mu^R_{i,g,t} + y^C_{i,g,t}\mu^C_{i,g,t} + y^S_{i,g,t}\mu^S_{i,g,t})/\mu_{i,g,t}, \tag{11}$$

where $y^R_{i,g,t}$, $y^C_{i,g,t}$ and $y^S_{i,g,t}$ denote before-tax earnings of regular, contingent and self-employed individuals of age i and gender g, respectively. To compute profiles used in the baseline simulations, we use data at time $t = 2015$. $\mu^R_{i,g,t}$, $\mu^C_{i,g,t}$ and $\mu^S_{i,g,t}$ are the number of individuals of each type. Figure 4 shows profiles of average efficiency, $\varepsilon_{i,g,t}$, for males and females. In the baseline simulation, we assume that the age-efficiency profile for each gender is time-invariant. In section 4.2, we consider alternative scenarios about participation rates, employment type distribution and earnings in the future, and evaluate their effects on macroeconomic and fiscal projections.

3.5 Government

Public Pension System: The government operates a pay-as-you-go pension system. The normal retirement age I^R is set at 46 in the baseline simulation, which corresponds to 65 years old. The replacement rate κ_t is determined in equilibrium so that total pension expenditures in 2015 correspond to 9.5% of GDP in that year and the calibrated value is 0.532. Both I^R and κ_t are fixed throughout the transition of the baseline simulation. We simulate alternative paths of these policy parameters in section 4.2.

Debt, Government Expenditures and Taxes: The government debt B_t is set to 156% of GDP, the value of total gross debt of the central and local governments net of

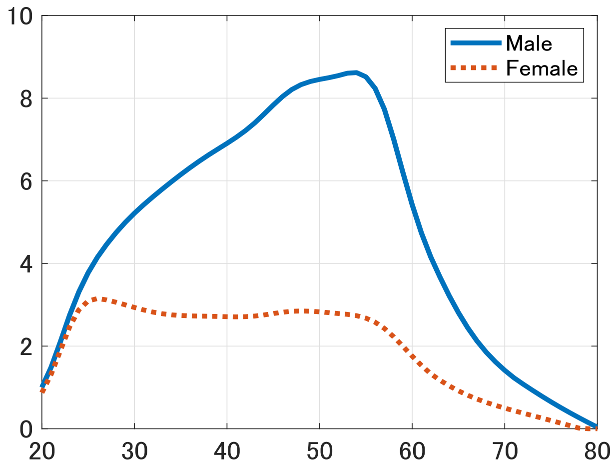


Figure 4: Efficiency Profiles $\varepsilon_{i,g,t}$ (Normalized by the level of males aged 20)

financial assets held by them in 2015. G_t is determined so that the ratio of government expenditures to GDP is 20% as in the data of the central and local governments in 2015.

For the revenue side, we set the tax rate on capital income $\tau_{a,t}$ at 35% in line with the estimates of effective tax rates in the literature.[12] The consumption tax rate $\tau_{c,t}$ follows the actual path in the past, increasing from 8% to 10% in 2019 and staying at that level thereafter. $\tau_{w,t}$ is determined in equilibrium to balance the budget in each year of the transition. In the baseline simulation, the labor income tax rate in equilibrium is 38% in 2015, in line with estimates in the literature.[13] Note that this tax encompasses all taxes imposed on labor income, including social security taxes.

4 Numerical Analysis

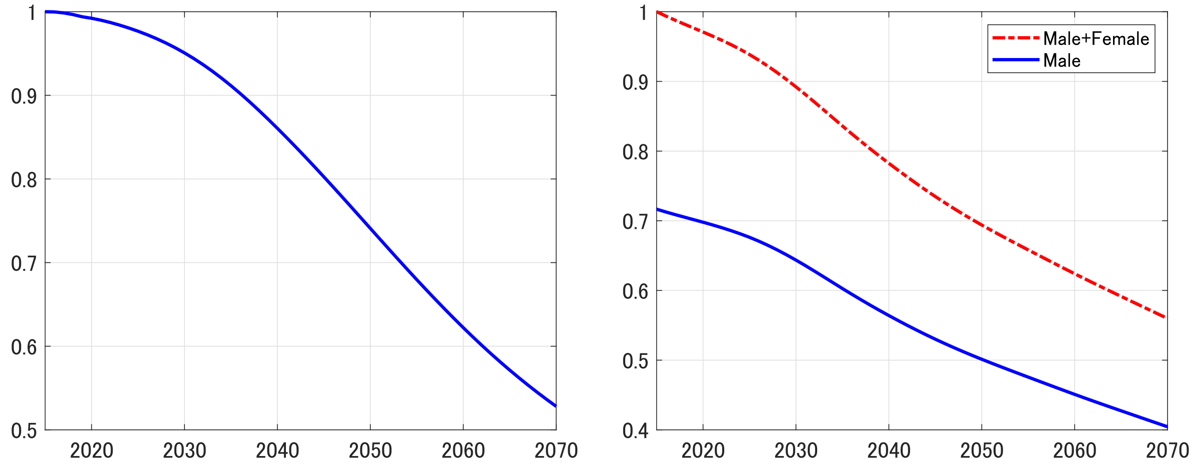
In this section we present numerical results of our baseline simulations and other experiments. Our focus is to analyze effects of changes in the labor supply, especially by females and the elderly (although we show males are also important), on the path of the macroeconomy, as well as the impact of alternative policies and demographic transition. Before we present and discuss these scenarios, we will first examine and characterize the baseline transition, in which we assume that there is no change in labor supply profiles of individuals or government policy (except for the labor income tax rate to balance the government budget in each period) and that demographics follow official projections of the IPSS according to its medium scenarios of fertility rates and survival probabilities.

[12] Hansen and İmrohoroğlu (2016) use the method of Hayashi and Prescott (2002) and estimate the capital income tax rate in 2010 at 35.6%

[13] Gunji and Miyazaki (2011) estimates average marginal tax rates on labor income.

4.1 Baseline Simulations

Figure 5 shows the paths of aggregate capital and labor supply in the baseline transition between 2015 and 2070.[14] Labor supply will decline immediately and sharply as the size of the working-age population falls by more than 20% by 2040, 30% by 2050 and 40% by 2070. We also note and emphasize that female contribution to aggregate labor, as measured by total efficiency units, is significantly smaller than that of males. In 2015, more than 70% of total labor is supplied by males and less than 30% by females. It implies that if females in an equal number as males were to supply the same efficiency units as males, total labor supply could rise by about 40%, though of course, how wage would adjust in response to the more abundant labor supply remains to be seen. In section 4.2, we consider various experiments to explore channels through which the gap between male and female experience in the labor market is narrowed and we analyze how other variables will respond to changes in the labor market.



(a) Capital

(b) Labor

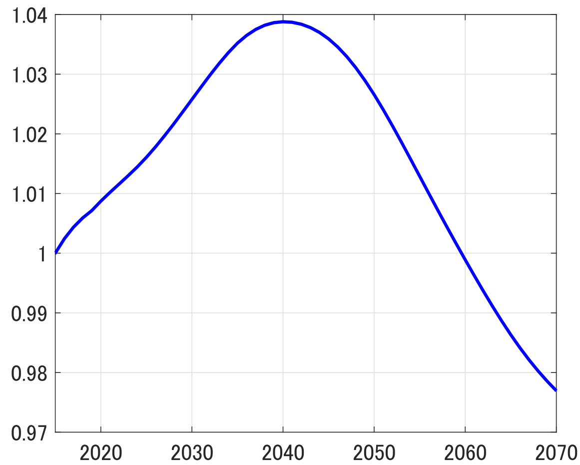
Figure 5: Baseline Transition: Aggregate Variables. Levels are normalized by the aggregate value in 2015.

Aggregate capital does not decline as sharply as labor supply initially since a rise in longevity will give more incentives to save and accumulate wealth for a longer period of retirement, which offsets the decline driven by a fall in the number of savers. The latter effect, however, starts to dominate quickly and aggregate capital falls by approximately 15% by 2040 and close to 50% by 2070.

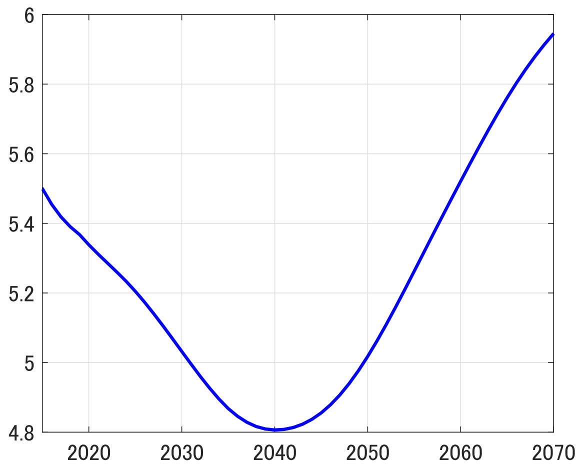
As a result of the unsynchronized decline in aggregate capital and labor, the capital

[14]For the rest of the paper, reported variables are stationarized by the long-run growth rate. More precisely, aggregate capital, output, consumption and wage rates are stationarized by the growth rate of $\lambda/(1-\alpha)$. There is no adjustment for aggregate labor supply since we assume that the population growth rate will converge from current negative values to zero in the long-run.

labor ratio initially rises until around 2040 and falls thereafter. Figure 6 shows the paths of factor prices, the wage rate and interest rate, between 2015 and 2070. The wage rate will rise initially as labor supply becomes scarcer relative to capital and begin to fall in the early 2040s. The interest rate will decline by about 0.7 percentage points between 2015 and 2040 and rise thereafter.



(a) Wage



(b) Interest Rate (%)

Figure 6: Baseline Transition: Factor Prices. Wages are normalized by the value in 2015.

As shown in Figure 7, the equilibrium tax rate on labor income shows a discrete fall in 2020, after the consumption tax is raised from 8% to 10%, but it will increase rapidly to finance rising expenditures in response to demographic aging. It will have increased by about 14 percentage points by 2070 in the baseline transition.

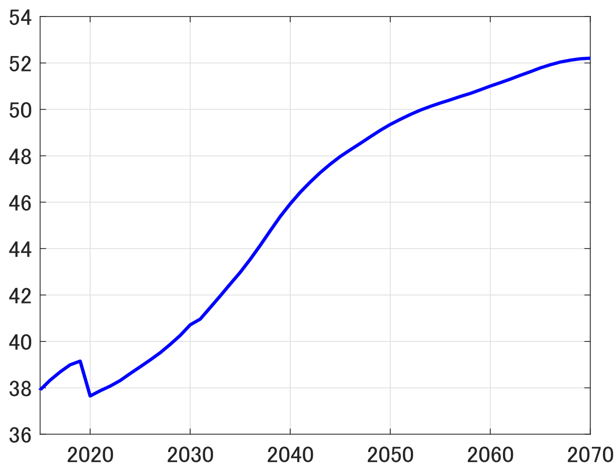


Figure 7: Baseline Transition: Labor Income Tax Rate (%)

4.2 Labor Force Participation, Policy Experiments and Alternative Scenarios

In this section, we simulate the economy under alternative scenarios about various economic assumptions we imposed along the baseline transition path.

4.2.1 Labor Force Participation (Males, Females, and the Elderly)

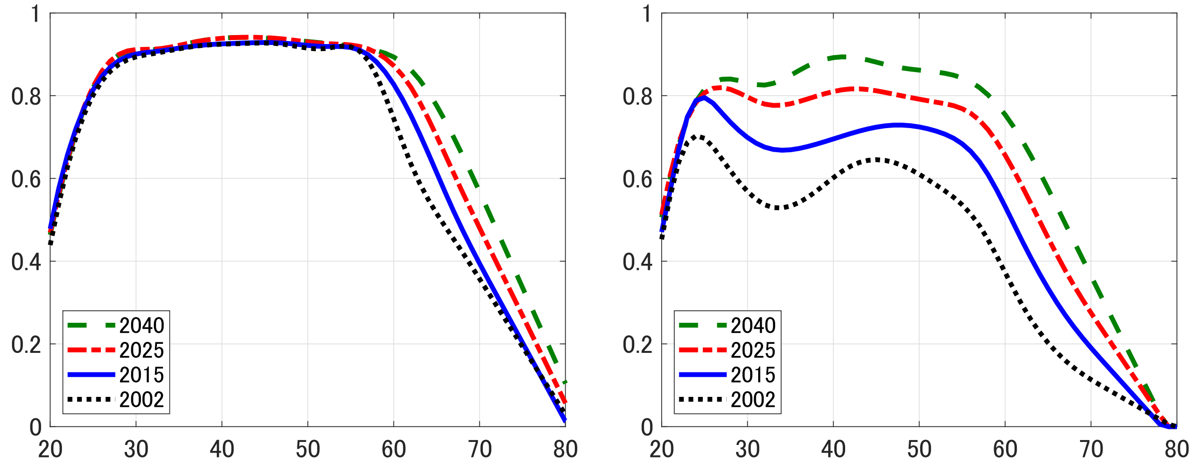
We consider scenarios about labor force participation of individuals of different characteristics. The government is keen on encouraging more participation, especially that of females as well as the elderly. These are two groups of individuals whose participation is much lower than that of prime-age males and who could contribute to a rise in the aggregate labor supply and potentially improve fiscal situations.

Figure 8 shows projections of labor force participation rates by the Japan Institute for Labour Policy and Training (JILPT) in 2025 and 2040, together with the data of the Labor Force Survey (LFS) in 2012 and 2015.[15] The projections are often used as underlying parameters in official economic and fiscal forecasts in Japan.[16] The overall participation rate has been rising, mainly driven by an increase in the participation of females and the elderly during the past decades. The projections assume that this trend will continue for the next few decades.

As shown in Figure 9, which plots the difference between projections and data in 2015 to highlight growth, the projections assume a rapid rise in participation of both male and female elderly above age 65 as well as a major increase in labor supply of females across essentially all ages, but, in particular, during their child-bearing ages of 30s to 40s. We will use these projections of the JILPT in the analysis of alternative scenarios about labor force participation in the future. Details of various scenarios about labor force participation are summarized in Table 1.

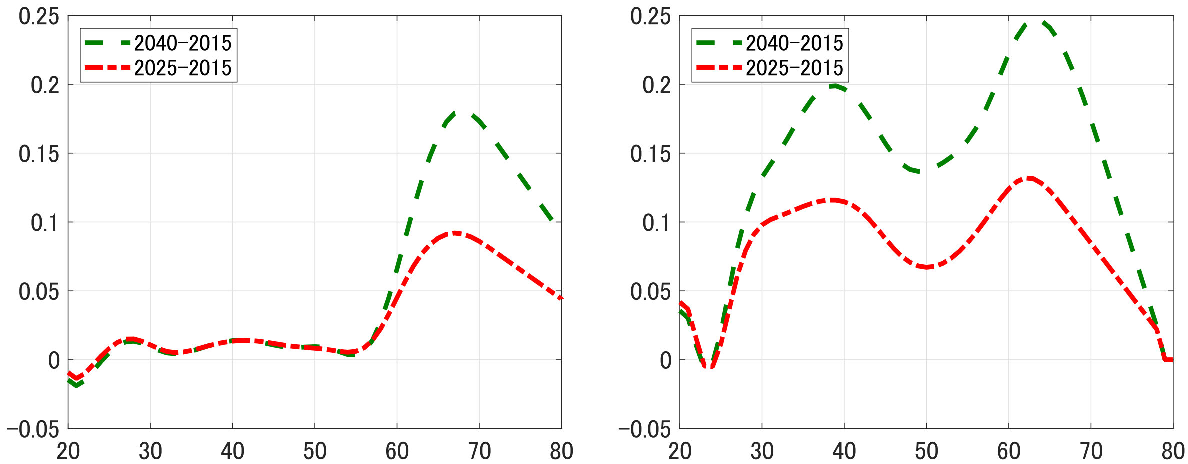
[15]The LFS started in the late 1940s and provides labor force data for long periods. As discussed below, 2002 is the first year that the survey includes information about employment types so we can compare the transition of participation rates and distribution by employment types consistently.

[16]The projections are available on the JILPT website: https://www.jil.go.jp/english/index.html (English) and https://www.jil.go.jp/kokunai/statistics/rouju.html (Japanese). Labor force participation rates for 2015 are based on the data from the Labor Force Survey (LFS). In computing participation rates for 2015, we take the ratio of the sum of regular, contingent and self-employed workers to the population. We exclude unpaid "family workers" (*kazoku jugyosha*) from the definition of participation since by definition they do not receive wages in the same way as other individuals who supply labor domestically without being paid. We assume the same for these projections (which implicitly include those family workers) and subtract the same fraction of "family workers" as in 2015 from the estimates to derive consistent figures of labor force participation rates. The numbers are relatively small and such treatment does not affect projections in a significant way.



(a) Male (b) Female

Figure 8: Labor Force Participation Rate Projections (2025 and 2040, JILPT) and Data (2002 and 2015, LFS)



(a) Male (b) Female

Figure 9: Labor Force Participation Rate Projections: Change from 2015

Female Labor Force Participation (LFP): As described in section 3, there are significant differences between males and females in their labor market experience. First, they differ in participation rates over the life-cycle. Second, distribution of employment types is different. Third, earnings are much higher for males, even controlling for age and employment type.

We simulate scenarios in which these differences are mitigated and the gap eventually disappears during the transition. According to the projections by the JILPT, female participation rates will rise monotonically over the next few decades and reach levels that are still below but closer to those of males, as shown in Figure 8.

In the first experiment, **LFP-1**, we assume that female participation rates by age will rise as projected by the JILPT. We assume that efficiency units provided by females of different ages will be the same on average, that is, the composition of employment type and efficiency units of participating females remains unchanged.

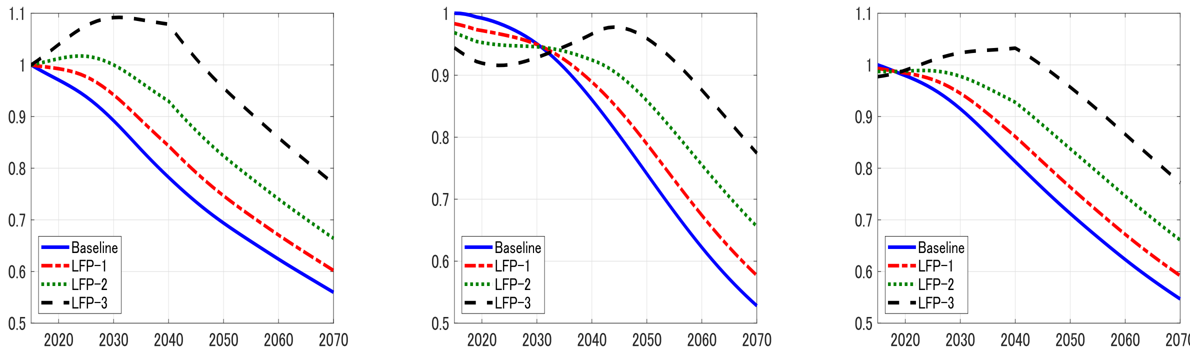
In the second experiment, **LFP-2**, we assume the same path of rising participation rates as in the first experiment, but we let the distribution of employment types gradually converge toward that of males by 2040. In the third experiment, **LFP-3**, we also assume that labor efficiency units by age and employment type will also converge toward those of males by 2040.

Figure 10 shows the paths of aggregate labor supply, capital and output under the three scenarios as well as those of the baseline transition. They clearly show that more participation of females will be not only qualitatively but also quantitatively effective in increasing labor inputs. What would, however, generate a more significant impact is not simply a rise in labor force participation, but also changes in how they participate and contribute to production. As shown in Figure 10a, the labor supply will be much higher if there is also convergence in employment type and efficiency.

Figure 10c shows that the economy would produce more with a larger number of females participating in the labor force, and the effects are larger under the **LFP-2** and **3** scenarios. Given a projected rise in income due to more participation, savings will initially decline to smooth the path of consumption, which is reflected in the initial decline in aggregate capital in Figure 10b. Aggregate capital, however, will eventually be higher than in the baseline transition as the economy becomes more productive and individuals start to save more with additional resources.

Figure 11 shows the paths of factor prices in the three scenarios of female labor supply and the baseline scenario. With a rise in female participation, labor will become more abundant relative to capital, and wages will be lower than in the baseline and even decline after 2015. The effect is intensified by an initial decline in capital. Gradually, however, capital will rise and labor will become scarcer. In the long-run, wages are higher under the three scenarios than in the baseline.

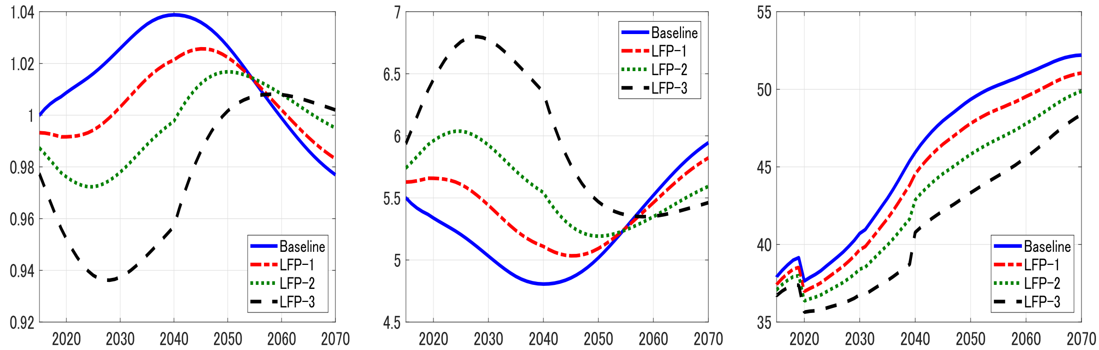
Figure 11c shows more participation by females will significantly reduce the fiscal



(a) Labor (b) Capital (c) Output

Figure 10: Female Labor Supply: Aggregate Variables

burden. By 2045, equilibrium tax rates will be lower by 1.6, 3.5 and 5.9 percentage points lower than in the baseline simulations. Although more participation will lower wage rates during the transition and could potentially lower the tax base, higher labor supply dominates in the net effect. In addition, aggregate output will be higher (and also aggregate capital after the 2030s) during the transition and tax revenues from other sources will rise and help reduce fiscal costs of demographic aging.



(a) Wage (b) Interest rate (%) (c) Tax rate (%)

Figure 11: Female Labor Supply: Wage and Tax Rate

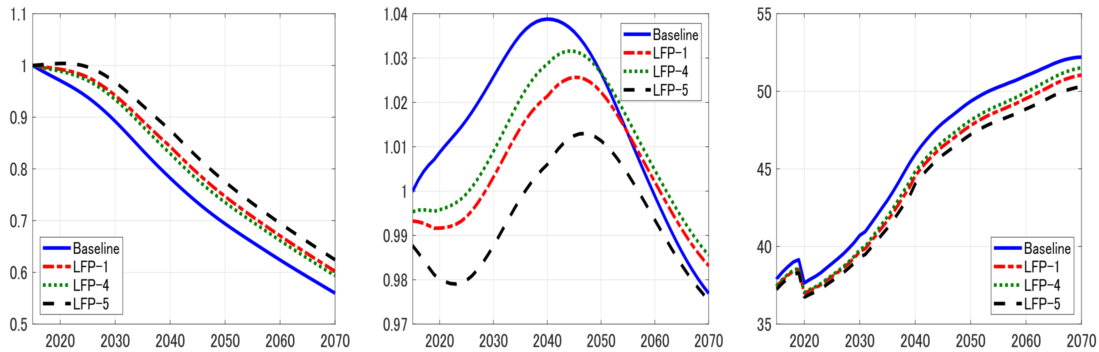
Labor Force Participation of the Elderly and Males: As shown in Figure 8, female labor force participation is projected to rise among both the young and the elderly. In order to isolate effects of rising participation of the elderly, we consider a scenario, **LFP-4**, in which we let female participation rates increase as projected by the JILPT, just as in scenario **LFP-1**, but the rise is restricted to those aged 65 and below. Changes between the baseline and **LFP-4** will highlight the effects of rising participation among younger females. The difference between **LFP-1** and **LFP-4** will indicate effects of rising participation of elderly females.

For males, although there is not much room for a further increase in the participation rate of males in their prime age, a major increase is projected among elderly males as highlighted in Figure 9. We simulate a scenario, **LFP-5**, in which we assume participation rates of both males and females will follow the path as projected by the JILPT. The difference between **LFP-1** and **LFP-5** will highlight effects of rising participation of (mostly elderly) males.

Figure 12 shows the paths of aggregate labor, wage and equilibrium tax rate under the two scenarios of **LFP-4** and **LFP-5** and the baseline. Paths of **LFP-1** are also included in the figures to facilitate comparison.

Difference in aggregate labor between **LFP-1** and **LFP-4** in Figure 12a is the contribution resulting from the rise in participation among elderly females aged 65 and above. Although their participation rates are much higher by 2040, effects on labor supply are not as large (see Table 2 for percentage changes in selected years across scenarios). This is because a large fraction of elderly females in the labor force are contingent workers, whose labor supply in terms of efficiency units is significantly lower than that of regular workers. Much larger effects are expected if the distribution of employment types as well as their efficiency units also change as we saw in scenarios **LFP-2** and **3**. As a result, changes in wages and tax rates are also smaller as shown in Figure 12b and 12c.

A rise in the male participation rate will increase aggregate supply more significantly, which can be seen by comparing the path of labor supply under **LFP-1** and **LFP-5** in Figure 12a. Effects on wage and tax rate are also larger as shown in Figure 12b.



(a) Labor (b) Wage (c) Tax rate (%)

Figure 12: Labor Supply: Males and the Elderly

Employment Types of Males - Regular vs Contingent: Although labor force participation rates of prime-age males are high and there is little room for participation rates to rise further, there is a somewhat concerning trend in employment type among participating males.

Figure 13 shows shares of regular, contingent and self-employed male workers out of all male workers at each age in 2002, the first year when breakdown by employment type is available in the Labor Force Survey (LFS), and in 2015. A fraction of contingent workers increased among prime-age individuals aged mid-20s to 50s at the same time as a fraction of regular workers declined. The trend implies lower average efficiency supplied by male workers of these age groups and lower earnings. A rise in the number of male workers engaged in contingent jobs, which typically provide a stream of less stable and lower income, has been debated as a concerning trend, which may also imply insufficient accumulation of wealth during the working period and for retirement and, at the same time, insufficient entitlement to public pension benefits (see, for example, Kitao and Yamada 2019).

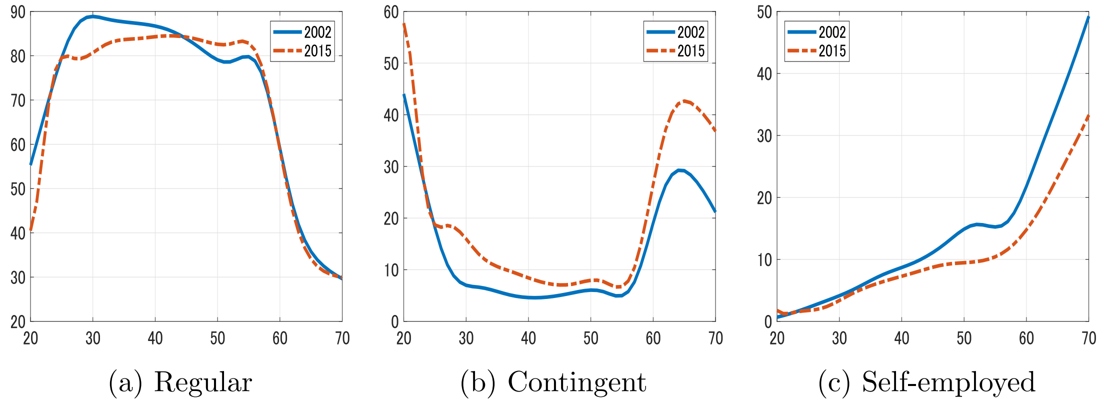


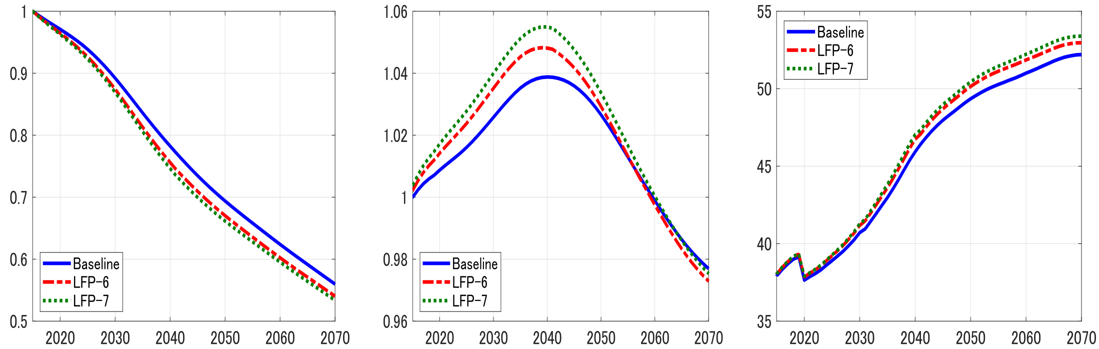
Figure 13: Employment types of male workers in 2002 and 2015 (% of all employed). Labor Force Survey (LFS).

We simulate a scenario, **LFP-6**, in which this trend of rising contingent male workers continues until 2041. More precisely, we let an average rise in the fraction of contingent workers aged 65 and below, observed during the 13 years between 2002 and 2015, continue for another 26 (=13×2) years until 2041 and assume that regular workers are replaced by the same fraction. In **LFP-7**, we also consider a case where the same shift occurs across all age groups, including those aged above 65.[17]

Figure 14 shows the paths of labor supply, wage and equilibrium tax rate under **LFP-6** and **LFP-7**. Aggregate supply will be lower if the trend continues, by more than 3% in **LFP-6** relative to the baseline by 2045 and by almost 5% under **LFP-7**. Wage will be higher with scarcer labor supply, though the effects are relatively small since saving and aggregate capital decline at the same time. Given a lower output and tax base, equilibrium tax rates will be slightly higher.

Summary of labor Force Participation Scenarios: Details of the seven scenarios of labor force participation are summarized in Table 1. Table 2 shows changes in selected

[17]We also simulated other scenarios including one replacing self-employed workers with contingent workers. The effects are not large quantitatively since the difference in earnings of contingent and self-employed individuals is not as great as between these two groups and regular workers. Results are available from the authors upon request.



(a) Labor (b) Wage (c) Tax rate (%)

Figure 14: Labor Supply: Employment Types of Males

aggregate variables relative to levels in the baseline model for each of the experiments.

An increase in participation of not only females but also males will raise labor inputs, but larger effects are identified when employment types of females and their efficiency contributions also change and trend closer to those of males. More participation will lower wage rates compared to the baseline simulation initially, but a rise in savings and aggregate capital will eventually reverse the direction.

More participation, despite a transitional fall in wage rates, will increase economic activities and expand the tax base, which leads to a lower equilibrium tax rate under all five scenarios of rising participation, **LFP-1** to **5**. **LFP-6** and **7** show the opposite effect on tax rate and imply that the ongoing trend of changes in employment type and earnings of male workers is something that we also need pay careful attention to.

Table 1: Description of Labor Force Participation (LFP) Scenarios

Scenario	Assumptions
LFP-1	Female LFP rise from 2015 to 2040, based on JILPT projections.
LFP-2	LFP-1 plus convergence of employment type to males'.
LFP-3	LFP-2 plus convergence of efficiency to males'.
LFP-4	Same as LFP-1 but only aged 65 and below.
LFP-5	Female and male LFP rise from 2015 to 2040, based on the JILPT projections.
LFP-6	A rise of contingent workers among males aged 65 and below. Trend in 2002-2015 extended to 2041.
LFP-7	A rise of contingent workers among males of all ages. Trend in 2002-2015 extended to 2041

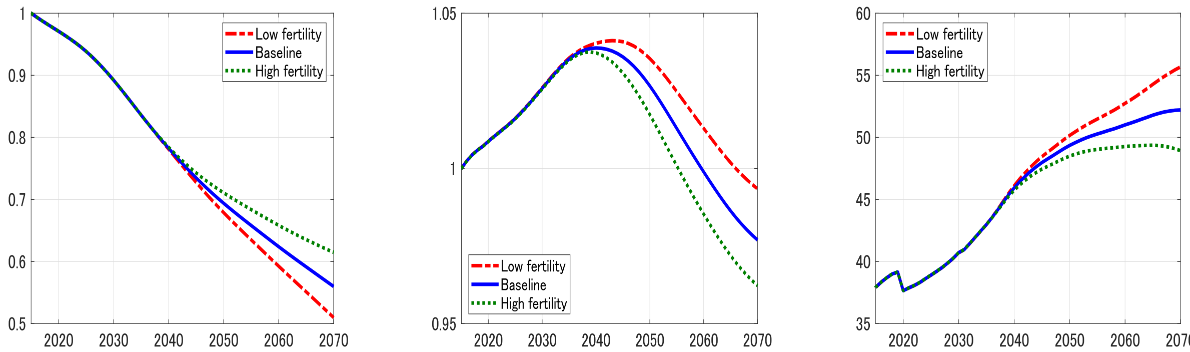
Table 2: Labor Force Participation (LFP) Scenarios

Year	LFP-1	LFP-2	LFP-3	LFP-4	LFP-5	LFP-6	LFP-7
Labor (% change relative to baseline in each year)							
2015	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2030	5.61	12.10	22.36	4.70	8.46	-2.00	-2.58
2045	7.75	18.80	37.73	5.99	11.92	-3.44	-4.77
2060	7.48	18.65	37.67	6.01	11.37	-3.47	-4.59
Capital (% diff from baseline in each year)							
2015	-1.68	-3.14	-5.58	-1.16	-3.06	0.55	0.96
2030	-0.12	-0.51	-2.33	0.45	-1.34	0.25	0.83
2045	5.10	12.12	21.76	4.87	5.70	-2.07	-2.05
2060	8.33	21.41	40.80	7.56	9.87	-3.81	-4.25
Output (% diff from baseline in each year)							
2015	-0.68	-1.27	-2.27	-0.47	-1.23	0.22	0.38
2030	3.28	6.87	11.81	2.98	4.43	-1.11	-1.23
2045	6.68	16.08	31.11	5.54	9.39	-2.90	-3.69
2060	7.82	19.74	38.91	6.63	10.77	-3.61	-4.45
Wage (% diff from baseline in each year)							
2015	-0.68	-1.27	-2.27	-0.47	-1.23	0.22	0.38
2030	-2.21	-4.66	-8.62	-1.64	-3.71	0.91	1.38
2045	-0.99	-2.29	-4.81	-0.42	-2.26	0.57	1.13
2060	0.32	0.92	0.90	0.58	-0.54	-0.14	0.14
Interest rate (ppt diff from baseline in each year)							
2015	0.13	0.24	0.44	0.09	0.24	-0.04	-0.07
2030	0.41	0.89	1.74	0.30	0.70	-0.16	-0.25
2045	0.18	0.42	0.91	0.08	0.41	-0.10	-0.20
2060	-0.06	-0.17	-0.17	-0.11	0.10	0.03	-0.03
Labor tax rate (ppt diff from baseline in each year)							
2015	-0.49	-0.83	-1.20	-0.44	-0.67	0.12	0.14
2030	-1.05	-2.31	-3.96	-0.92	-1.40	0.46	0.57
2045	-1.51	-3.41	-5.81	-1.20	-2.07	0.73	1.01
2060	-1.47	-3.21	-5.38	-1.05	-2.14	0.85	1.22

4.2.2 Demographic Projections

In the baseline simulations, we used demographic projections of the IPSS under the medium scenario. They also report projections based on high and low scenarios for fertility rates and survival rates, respectively. To assess sensitivity of our results to demographic assumptions, we compute the transition based on the four alternative scenarios of low and high cases of fertility and survival rates.

Figure 15 shows simulation results under three scenarios for fertility rates: low, medium (baseline) and high. Even if fertility rates start to rise or fall immediately, a change in labor supply is not visible until around 2040, when a larger (or smaller) number of newborns start to participate in the labor market. Higher fertility implies more abundant labor and lower wages, though the difference is relatively small even in 2070. A larger labor force with higher fertility also implies less fiscal burden while additional newborns are in the labor force and the equilibrium tax rate is lower by about 1 percentage point in 2050 and 3 percentage points in 2070.



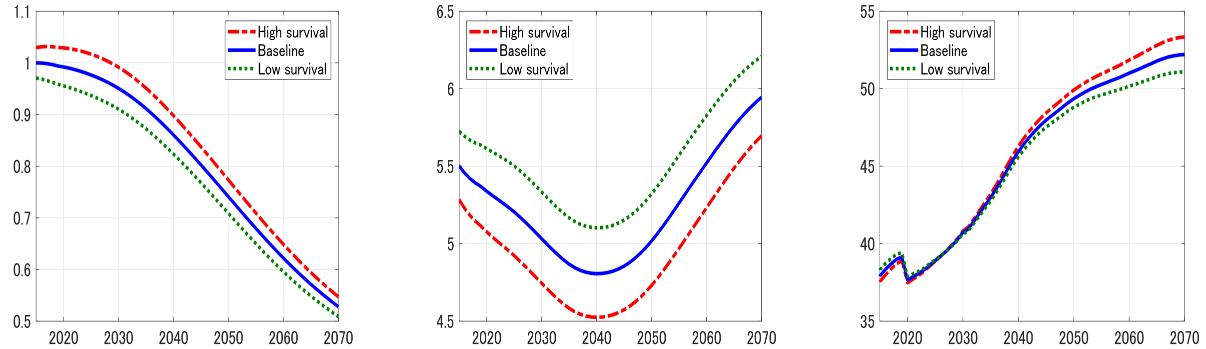
(a) Labor (b) Wage (c) Tax rate (%)

Figure 15: Low and High Fertility Rates

Figure 16 shows simulation results under low, medium (baseline) and high scenarios for survival rate. With a high survival rate, individuals expect to live longer and are incentivized to save more for a longer retirement period, even before they reach retirement age. Figure 16c shows tax rates. Although the equilibrium tax rates are slightly lower than in the benchmark with more years of retirement because of a change in tax revenues from other sources as well as a rise in wage rates, pension expenditures are higher in the long-run and so are equilibrium tax rates.

4.2.3 Public Pension Policy

Replacement Rate: In the baseline simulations, we assume that the replacement rate of the public pension, summarized by the parameter value of κ, remains unchanged during the transition. In this section, we consider a scenario in which the replacement



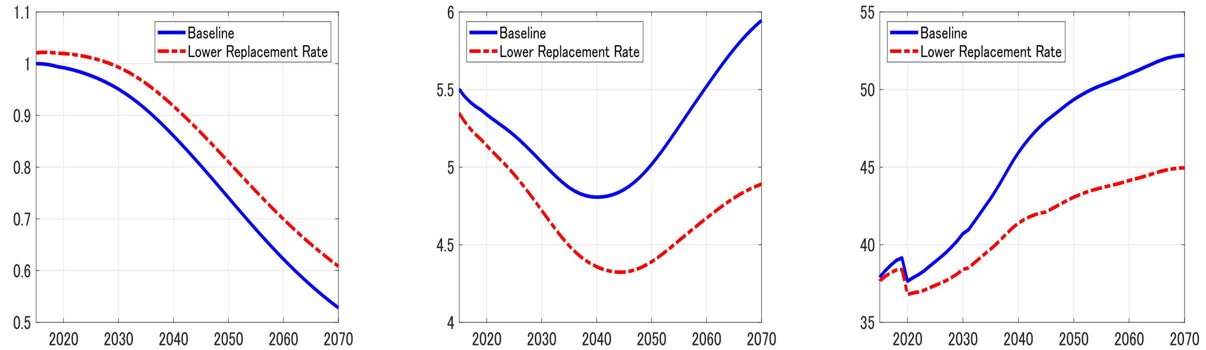
(a) Capital (b) Interest rate (%) (c) Tax rate (%)

Figure 16: Low and High Survival Rates

rate is reduced by 20% gradually over a 30-year period.[18]

Figure 17 compares the paths of capital, interest rate and equilibrium tax rate in the experiment to those of the baseline transition. Expecting lower pension benefits from the government, individuals will save more and aggregate capital will be higher than in the baseline transition. Additional saving will make capital more abundant relative to labor and reduce the cost of capital, as shown in the path of the interest rate in Figure 17b.

Figure 17c shows a change in the tax burden. There is not much difference initially since the replacement rate is reduced gradually, but the fiscal burden eventually will be lower by about 7 percentage points of total labor income.



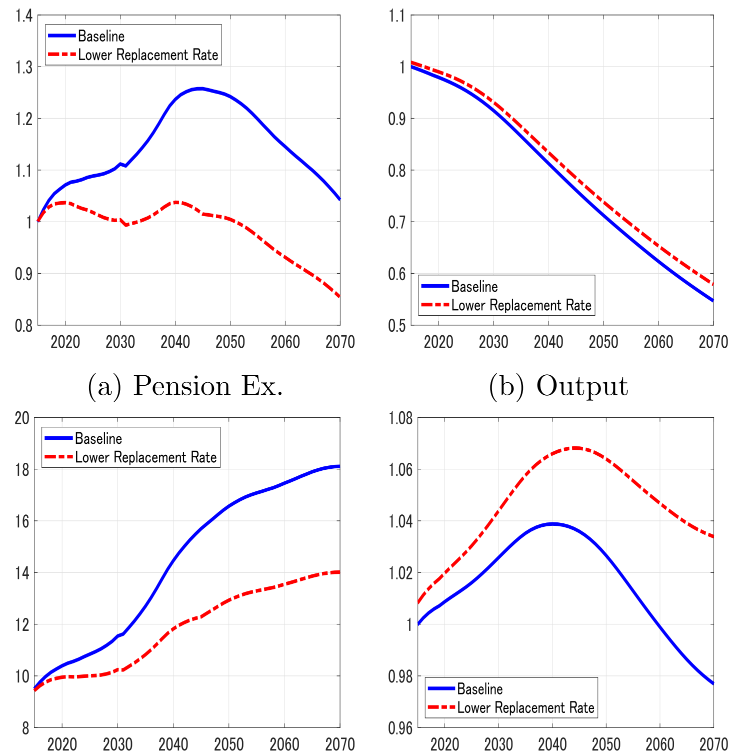
(a) Capital (b) Interest Rate (%) (c) Tax rate (%)

Figure 17: Pension Scenario: 20% Decline in the Replacement Rate (1)

The fiscal burden declines not only because of lower government expenditures on public pensions, but also because of a combination of other factors. Figure 18a shows paths of pension expenditures in the baseline and the experiment and they are eventually lower

[18]The decline of 20% is equivalent to the projected change in the pension benefits under the Actuarial Valuation by the MHLW when the adjustment of benefits under the macroeconomic slide works as expected. See below for details about Actuarial Valuation in 2014. https://www.mhlw.go.jp/file/06-Seisakujouhou-12500000-Nenkinkyoku/2014_Actuarial_Valuatin_3.pdf

in the latter by approximately 20% relative to the baseline. In an economy with lower pensions, output also rises because of more savings and higher capital, and wages increase too, as shown in Figures 18b and 18d.[19] As a result, the ratio of pension benefits relative to GDP, as shown in Figure 18c, falls by more than 20%, contributing to a major decline in tax rates needed to finance the demographic transition.



(a) Pension Ex. (b) Output

(c) Pension Ex./Output (%) (d) Wage

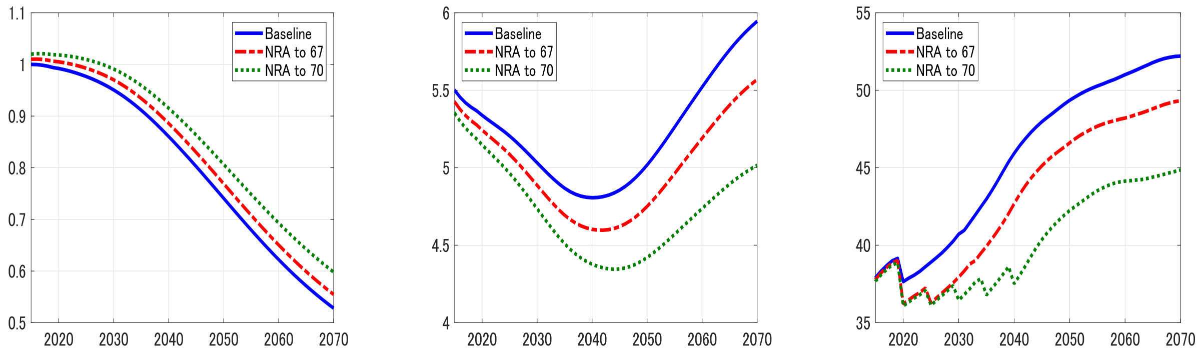
Figure 18: Pension Scenario: 20% Decline in the Replacement Rate (2). Pension expenditures, output and wage are normalized by the level in 2015.

Normal Retirement Age (NRA): We also simulate the transition assuming two scenarios, in which the pension eligibility age is raised from the current NRA of 65 to 67 and 70, respectively. We assume that the retirement age is raised gradually by one year at five-year intervals, starting in 2020.

Figure 19 shows the paths of aggregate capital, interest rate and equilibrium tax rate. As in the experiment lowering the replacement rate, reform will give additional incentives to save for retirement and increase capital. Quantitatively, effects are larger when the NRA is raised to a higher age. With the NRA of 70, the fiscal burden will be significantly reduced and the equilibrium tax rate in the early 2040s will be similar to the current level.

[19]Note that we assume exogenous labor supply, but if labor responds positively to the policy change, effects on output may be larger and our estimates may be considered conservative.

The magnitude of the change in the tax rate under this scenario will be similar to that of lowering benefits by 20% by 2070.



(a) Capital (b) Interest Rate (%) (c) Tax rate (%)

Figure 19: Pension Scenario: Increase in NRA to 67 and 70

4.2.4 Total Factor Productivity Growth

In the baseline economy we assume that total factor productivity (TFP), Z_t, grows at a constant rate of 1%. In this section, we consider alternative scenarios of lower and higher growth rate of TFP.

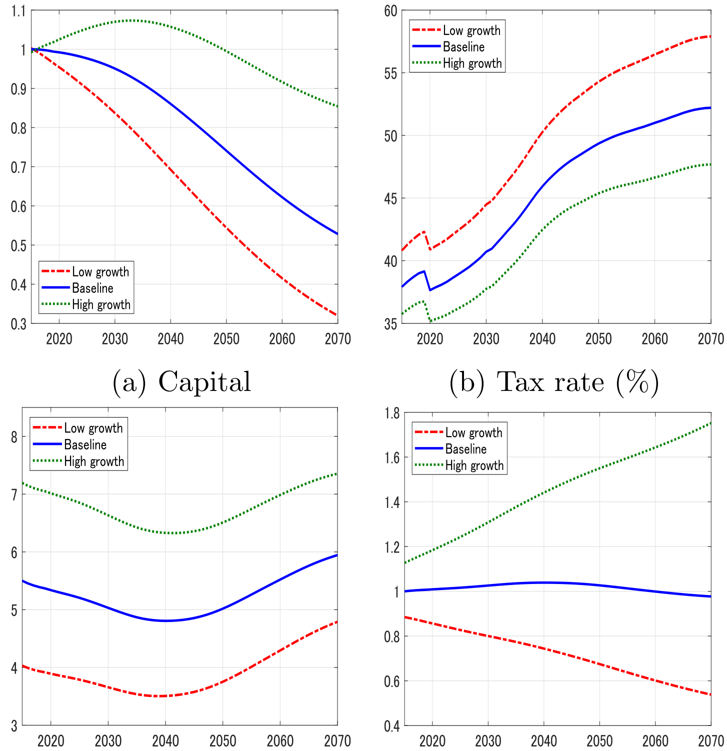
Figure 20 shows the paths of capital, equilibrium tax rate, interest rate and wage when the TFP grows at 0.5%, 1.0% (baseline) and 1.5%. For capital and wage rate, levels are adjusted by the same rate as the growth rate to stationarize the baseline transition so that levels of these variables are comparable in the same figure.

All else being equal, a steeper income profile implies a lower saving rate at the individual level. For aggregate capital, however, such effects are offset by a rise in the level of disposable income. As shown in Figure 20a, aggregate capital will eventually be significantly higher if the economy is able to sustain high growth. Fiscal burden, which is expressed in terms of the equilibrium tax rate, will be lower in a high-growth economy since tax revenues continue to grow at a higher rate.

Capital-labor ratio will be higher when growth is higher. Interest rate, however, is determined by both the level of TFP and capital-labor ratio and stays higher in the experiment in a high growth economy, as shown in Figure 20c. Wages grow faster with higher growth, as shown in Figure 20d.

5 Conclusion

With the unprecedented magnitude and speed of demographic aging, Japan faces an urgent need to present a policy path that makes its fiscal situation and macroeconomy sound and stable. In this paper, we build a general equilibrium model of individuals



(a) Capital

(b) Tax rate (%)

(c) Interest rate

(d) Wage

Figure 20: TFP Growth Rate Scenarios

of different age and gender, who optimally choose a life-cycle path of consumption and saving given a sequence of income, policy and other economic environmental factors. We considered alternative scenarios of the transition, focusing particularly on different assumptions about inclusion of females and the elderly in the labor force, as well as other assumptions about demographic projections, public pension policy and productivity growth.

Females and the elderly are two groups of individuals in Japan whose labor supply could further rise and help mitigate effects of a massive decline in the labor force during the coming decades due to demographic aging. The contribution of female workers was less than 30% of the total labor supply as of 2015, measured in terms of total wage income. We use official projections of the Japan Institute for Labour Policy and Training (JILPT) on labor supply of males and females across different ages and quantify effects on the macroeconomy and fiscal situation in Japan if participation rates change according to projection.

We find that a rise in female labor force participation has large positive effects on aggregate labor supply, output and eventually wages and reduces the fiscal burden and tax rate necessary to finance rising expenditures. Effects are, however, much more significant if not only participation rates but also employment type and productivity of each female worker relative to that of a male worker improve. Many women work on a contingent job rather than a regular job, and the latter provides a significantly higher stream of income than the former. Similarly, a rise in the number of the elderly in the labor force, both males and females, will be effective, but quantitative effects also depend on how they participate in the labor market. Our study suggests that policies to stimulate labor force participation of females and the elderly will be effective and the government also needs to consider ways to remove obstacles for them from choosing jobs that provide more stable and higher earnings.

Although challenges tend to receive more emphasis than bright hopes in analyses of the Japanese macroeconomy and fiscal situation in the future, this paper identifies untapped or under-used potential resources in the labor market and demonstrates their large effects. Our analysis implies that a combination of policies to remove obstacles hindering labor supply and to enhance a more efficient allocation of male and female workers across age groups will be critical to keeping government deficit under control and raising income across the nation. Fiscal reforms to reduce net deficit of the government can significantly mitigate the costs of demographic transitions and they should be implemented without delay as well.

We conclude by noting some directions of research that the current paper implies are highly promising. First, building a model that endogenizes labor supply decisions of both males and females and explains the peculiar pattern of participation of females over a life-cycle will be important. Such a model will enable various normative and positive analysis of policies that affect work incentives of females, including distortionary taxation and those related to social security or human capital formation. Second, given the rapid rise in the number of the elderly during the coming decades, it is important to analyze how their ability and incentives to work interact with policies. Third, participation decisions of females are connected to intra-family decisions such as marriage, child bearing and an allocation of time to home and market production. A model that disentangles such interactions will be essential in considering not only positive effects of female participation on aggregate labor supply and output that we identified in the paper but also tradeoffs involved in terms of within-family activities and externalities to other family members. We leave these interesting and important issues for future research.

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A Computation of the transition dynamics

We compute the transition path following the four steps described below.

Step 1: Compute the initial and final steady-states of the model. The computation of the steady state economy is essentially the same as that described in Step 3 below, without time subscripts since all variables are stationary.

Let the economy make the transition from the initial steady state to the final steady state over T periods. Set T large enough so that all variables converge smoothly to the values in the final steady state.

Step 2: Guess three vectors of the three equilibrium variables, which are aggregate capital, wage tax rate, and accidental bequests, in each period of the transition.

Given the path for aggregate capital, using the properties of constant returns to scale technology and optimization conditions for firms, sequences of interest rates and wages can be derived.

Step 3: Given the paths of factor prices, tax rates and accidental bequests obtained in Step 2, solve individuals' optimization problem. Recall that the budget constraint of an individual at time t is given as follows

$$(1+\tau_{c,t})\, c_{i,g,t} + a_{i+1,g,t+1} = y_{i,g,t} + R_t(a_{i,g,t} + b_t).$$

where $y_{i,g,t}$ denotes total after-tax income of an individual of age i and gender g at time t, which includes after-tax labor income and public pension benefits for those aged at and above I^R. R_t denotes net-of-tax gross interest rate, $R_t \equiv 1+(1-\tau_{a,t})\, r_t$. The Euler equation for asset holdings next period implies the optimal growth rate of consumption between age i and $i+1$ as

$$\frac{c_{i+1,g,t+1}}{c_{i,g,t}} = \left[\beta s_{i+1,g,t+1} \frac{1+\tau_{c,t}}{1+\tau_{c,t+1}} R_{t+1}\right]^{\frac{1}{\theta}} \equiv \gamma^c_{i+1,g,t+1}. \tag{12}$$

$\gamma^c_{i,g,t}$ is the growth rate of consumption of an individual of age i and gender g from time t to $t+1$ (and age $i+1$). Iterating backward over (12), we obtain:

$$c_{i+1,g,t+i} = c_{1,g,t} \prod_{k=1}^{i} \gamma^c_{k+1,g,t+k}.$$

The discounted present value of the total (gross of taxes) lifetime consumption expenditures of an individual of age 1 at time t is given as

$$\bar{c}_{1,g,t} = c_{1,g,t}\left[(1+\tau_{c,t}) + \sum_{i=1}^{I-1}(1+\tau_{c,t+i}) \prod_{k=1}^{i} \frac{s_{k+1,g,t+k}}{R_{t+k}} \gamma^c_{k+1,g,t+k}\right]. \tag{13}$$

The discounted present value of the total (gross of taxes) lifetime consumption expenditures of an individual of age i^* at time t is

$$\bar{c}_{i^*,g,t} = c_{i^*,g,t} \left[(1+\tau_{c,t}) + \sum_{i=i^*}^{I-1} \left(1+\tau_{c,t+(i-i^*+1)}\right) \prod_{k=i^*}^{i} \frac{s_{k+1,g,t+(k-i^*+1)}}{R_{t+(k-i^*+1)}} \gamma^c_{k+1,g,t+(k-i^*+1)} \right]. \tag{14}$$

The discounted present value of the total (net of taxes) lifetime income of an individual of age 1 and gender g at time t is:

$$\bar{y}_{1,g,t} = y_{1,g,t} + \sum_{i=1}^{I-1} \left(\prod_{k=1}^{i} \frac{s_{k+1,g,t+k}}{R_{t+k}} \right) y_{i+1,g,t+i}. \tag{15}$$

The discounted present value of the total (net of taxes) lifetime earnings of an individual of age i^* and gender g at time t is:

$$\bar{y}_{i^*,g,t} = y_{i^*,g,t} + \sum_{i=i^*}^{I-1} \left(\prod_{k=i^*}^{i} \frac{s_{k+1,g,t+k}}{R_{t+(k-i^*+1)}} \right) y_{i+1,g,t+(i-i^*+1)} + R_t a_{i^*,g,t}. \tag{16}$$

Since individual optimization requires $\bar{c}_{i^*,g,t} = \bar{y}_{i^*,g,t}$ for each age i^* and time t, from (14) and (16), we obtain $c_{i^*,g,t}$ as

$$c_{i^*,g,t} = \frac{\bar{y}_{i^*,g,t}}{\left[(1+\tau_{c,t}) + \sum_{i=i^*}^{I-1} \left(1+\tau_{c,t+(i-i^*+1)}\right) \prod_{k=i^*}^{i} \frac{s_{k+1,g,t+(k-i^*+1)}}{R_{t+(k-i^*+1)}} \gamma^c_{k+1,g,t+(k-i^*+1)} \right]}.$$

Note that $a_{i^*,g,t}$ in equation (16) is computed residually from $c_{i^*-1,g,t-1}$ and the budget constraint (5) :

$$a_{i^*,g,t} = y_{i^*-1,g,t-1} + R_{t-1}(a_{i^*-1,g,t-1} + b_{t-1}) - (1+\tau_{c,t-1})\, c_{i^*-1,g,t-1}.$$

Step 4: Aggregating asset holdings of all age groups and genders, we obtain the implied sequence of aggregate capital and accidental bequests. We update a guess for the sequence of aggregate capital and bequests. We use government budget constraints (4) to update our guess for the tax rate. If convergence is not reached, we restart from Step 3 with a new vector of guesses.