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A Game Theoretic Approach

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AKIHIKO MATSUI

Abstract. This paper takes a game theoretic approach to disability-related issues by constructing a model that studies the case of hereditary deafness on Martha's Vineyard Island, U.S.A. in the past centuries, where the island community adjusted itself to the hereditary deafness and made it "non-disability."

The model of the present paper has two stages. First of all, there are two types of continua of agents, the deaf and the non-deaf. In the first stage, the non-deaf agents become either bilinguals or monolinguals. In the second stage, agents are classified into the deaf people, bilinguals, and monolinguals. They are then randomly matched to form a trio to play a three-person bargaining game with infinite horizon, random proposers, and language constraints. Two bargaining games are considered. The first one is a majority bargaining game where only two out of three can agree to implement a bargaining outcome. The second one is a unanimity bargaining game where all three agents are required to reach an agreement. The majority game exhibits strategic complementarity, while the unanimity game exhibits strategic substitutability.

This paper also takes an inductive approach to examine how prejudice against people with disability may emerge.

Keywords: disability, hereditary deafness, Martha's Vineyard Island, game theory, bargaining, bilingual, inductive game theory.
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> ..., the fact that a society could adjust to disabled individuals, rather than requiring them to do all the adjusting ... raises important questions about the rights of the disabled and the responsibilities of those who are not. ...
> The most important lesson to be learned from Martha's Vineyard is that disabled people can be full and useful members of a community if the

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> community makes an effort to include them. The society must be willing to change slightly to adapt to all. (p.108, [?])

1. Introduction

This paper takes a game theoretic approach to disability-related issues by constructing a model that studies the case of hereditary deafness on Martha's Vineyard Island, U.S.A. in the past centuries, where the island community adjusted itself to the hereditary deafness and made it "non-disability."

The present paper asserts that disability presents not only physical/medical issues, but societal/economic issues, and that the notion of disability is socially relative. It further proposes that game theory is useful for understanding disability-related issues such as prejudice, which in turn call for a new approach in game theory like inductive game theory.[1]

Martha's Vineyard Island, lying off the Massachusetts coast of the United States of America, had a high frequency of hereditary deafness for more than two hundred and fifty years. In many modern societies, those who use sign language are labeled as the one with disability and suffers from social exclusion. On this island, however, due to a significant population of deaf people, many, if not all, of those without hearing impairment used sign language for communication.

In order to study this case by using game theory, we should first identify the key fundamental difference between the group of people with hearing impairment and the group of people without. The merit of using game theory to study disability is that it forces us to clarify the individual, i.e., physical, difference as opposed to societal difference between the two groups of people, and that any societal difference, disadvantage, or disability is explained as an equilibrium phenomenon rather than something that is socially predetermined, implying that disability is often a socially relative concept.

In the case of Martha's Vineyard, we differentiate the two groups according to their selection of communication tool. The only fundamental difference between the two is that the former cannot choose oral language as a communication tool, while the latter can choose oral language, sign language, or both. This difference in individual traits induces two types of equilibria. In the first type of equilibrium, those who cannot choose oral language become people with disability in the sense that they cannot communicate with the majority and thus cannot access the resources shared by the majority. In the second type of equilibrium, although they may still be a minority, they can access the resources.

The model and its analysis of the present paper are roughly described as follows. In the beginning, there are two types of continua of agents, the deaf and the non-deaf. There are

[1]On the other hand, this paper does not present a general framework of disability, nor does it follow disability studies, though the present paper shares some of their motivation. Disability studies, first developed in the United Kingdom in the 1980s, is an academic field of study that regards disability as a social construct rather than something associated with individuals (see, e.g., [?], [?]). It differentiates disability from impairment that is ascribed to individuals' physical and mental traits. Disability studies holds that it is disability, a social construct, rather than impairment, comprising individuals' traits, that induces a variety of social disadvantages to a specific group of people. Also, the term "model" used in the present paper is the one used in game theory as opposed to the term used in disability studies such as a "social model of disability."

two stages. In the first stage, the non-deaf agents become either bilinguals or monolinguals. As we shall see later, this process is expressed as an evolutionary process rather than a rational decision making process.

In the second stage, agents are classified as deaf people, bilinguals, and monolinguals. They are randomly matched to form a trio to play a three-person bargaining game with language constraints. Each trio can produce one unit of good and share it upon agreement, which can be reached only if they can communicate.[2]

We consider two scenarios. The first one is a majority bargaining game in which an agreement can be reached if one responder agrees to a proposal. The second one is a unanimity bargaining game in which both responders must agree in order to reach an agreement. These two bargaining games induce qualitatively different outcomes. The first game exhibits strategic complementarity, while the second exhibits strategic substitutability; the more bilinguals there are, the more (resp. less) incentive a non-deaf agent has to become a bilingual in the majority (resp. unanimity) bargaining game.

We consider these two games *not* because they are the only possible games played in the community. People in the community must have played a variety of games. In language selection games, however, it is often taken for granted that the more people learn a certain language, the more incentive one has to learn this language. We show in the sequel that this assertion needs to be examined.

Following [**?**], this paper discerns three approaches in game theory, deductive, evolutionary, and inductive approaches. The deductive approach is based on the rationality hypothesis, assuming that players deduce their strategies from their knowledge of the game they play. The evolutionary approach typically assumes that players have no knowledge of the game they play. Players may not even "choose" actions but can act only according to some program. Here, the survival of the fittest may apply; the higher the payoff one obtains, the better chance one has of producing offsprings and the likelier one is to take this action in the future. The inductive approach assumes no (sufficient) prior knowledge of the game players play. Instead, like in evolutionary game theory, the players play the game to accumulate experiences. Unlike in evolutionary game theory, however, the players try to construct a model of the game based on their experiences, as described above.

The present paper uses these three approaches in its analysis of hereditary deafness on Martha's Vineyard Island. The second stage bargaining game is analyzed through the deductive approach, as agents often understand the structure of the game they play and make rational decisions in the bargaining process. The three-person bargaining game has been studied extensively (see, e.g., [**?**]). The novelty of the present paper is the incorporation of an additional structure of language selection in its analysis.

The first stage of language selection is analyzed through the evolutionary approach since language acquisition often occurs in childhood and is often affected by the past phenomena, such as the language used by their parents. This paper uses the best response dynamics due to [**?**], though any reasonable dynamics would suffice.[3] This part of the analysis is

[2]Three is the smallest number by which a majority and minority can emerge in the bargaining game.

[3]See also [**?**] for an extensive survey.

related to [?] and [?] where people with different traits interact with each other and adapt to their society in an evolutionary manner.

One of the major focuses of disability studies is prejudice against people with disability. According to the Cambridge Dictionary online, prejudice is "an unfair and unreasonable opinion or feeling, especially when formed without enough thought or knowledge." By nature, prejudice involves some sort of wrong beliefs and limited reasoning. The deductive approach is thus unsuitable, as it begins with "correct" beliefs and "right" reasoning processes. The evolutionary approach is also inappropriate since its main engine of selection is the survival of the fittest rather than contemplation. Wrong beliefs are formed through (often limited) experiences and (again often inadequate) contemplation. This study therefore uses the inductive approach to see how prejudice emerges.

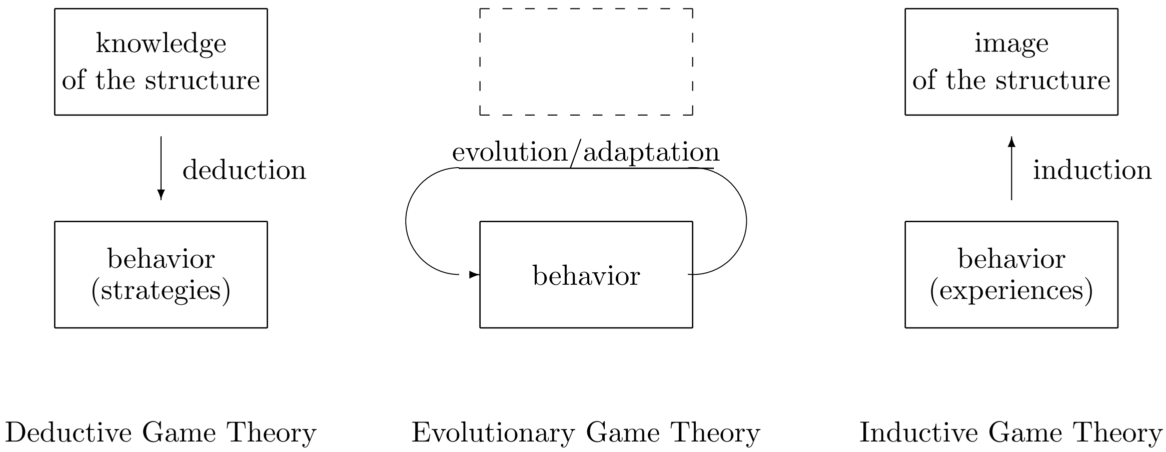

Figure 1. Three Approaches to Game Theory

The rest of this paper is organized as follows. Section 2 presents an example of hereditary deafness in which members of a society adapted to disability in an interesting way that would motivates our study. Section 3 presents the framework of our analysis. Sections 4, 5, and 6 present an analysis of the model based on deduction, evolution, and induction, respectively. Section 7 concludes the paper.

2. Hereditary Deafness in Martha's Vineyard Island

Martha's Vineyard Island, lying off the Massachusetts coast of the United States of America, had a high frequency of hereditary deafness for more than two hundred and fifty years.[4] Hereditary disorders in isolated societies have long been known. Martha's

[4]This section is based on [?].

Vineyard Island is another example of such cases. However, this island was unique in the way it coped with the disorder. In many modern societies, people with disability are often expected to adjust to the lifestyle of people without disability. What is remarkable of this island is that it was the people without disability that adjust to make the hereditary deafness "non-disability".

In the nineteenth century, the frequency of deafness at birth was about 1 in 6,000 in U.S. On the other hand, this number was 1 in 155 in Martha's Vineyard, and 1 in 25 in the town of Chilmark, which is located in the western part of the island. As a result, "Deafness was seen as something that just 'sometimes happened'; anyone could have a deaf child."[5] People's attitudes toward deafness are summarized by [?] as follows (p.51):

- You'd never hardly know they were deaf and dumb. People up there got so used to them that they didn't take hardly any notice of them.[6]
- It was taken pretty much for granted. It was as if somebody had brown eyes and somebody else had blue. Well, not quite so much–but as if, somebody was lame and somebody had trouble with his wrist.
- They were just like anybody else. I wouldn't be overly kind because they, they'd be sensitive to that. I'd just treat them the way I treated anybody.

Being deaf is not a handicap per se. It is social isolation that creates a handicap. In Martha's Vineyard, this isolation hardly occurred as the islanders learned sign language in childhood. They needed to learn sign language "to communicate with deaf adults as well as deaf playmates (ibid, p.54)."

One may casually conclude that on the island, everyone used sign language. It should also be mentioned, however, that some people on the island did not use sign language. Indeed, [?] wrote about an informant who felt uncomfortable but did not learn the sign language. The informant said, "I used to feel chagrined because I couldn't speak the sign language" (p.56).

3. Model

We consider a society that consists of infinitely many agents with either one of two natural traits, deaf (D) and non-deaf (N). The fraction of the deaf agents is exogenously given by $\alpha \in (0,1)$.

There are two stages in the game. In the first stage, N type agents simultaneously choose N_b or N_m, where "b" stands for "bilingual", and "m" for "monolingual". To become an N_b agent, an N agent incurs cost $d > 0$.

In the second stage, the agents are randomly matched to play a three-person bargaining game. Each agent in this stage has one of three statuses, D, N_b, and N_m. The deaf agents can use a sign language, while the non-deaf agents can use a spoken language. Among N type agents, those who become an N_b agent can use both sign language and spoken language, while those who become an N_m agent can use only spoken language.

Prior to the bargaining stage, there is an additional substage where type N_b agents can decide which language to use in the bargaining game, sign language only, oral language

[5] pp.50-51, [?].

[6] The term "deaf and dumb" now has pejorative connotations, but [?] retains this term as it is not pejorative in Martha's Vineyard. Therefore, I retained it in all quotes.

only, or both languages. An N_b agent who decides to use the both languages has to make the same offer in the both languages.

Each bargaining game is played by three agents: call them 1, 2, and 3. A bargaining game is infinitely repeated until an agreement is reached. In each period, one of the three agents is randomly chosen to be a proposer. Suppose Agent 1 is chosen as a proposer. Agent 1's proposal is denoted by $x = (x_1, x_2, x_3)$ with $x_1 + x_2 + x_3 = 1$. After x is proposed, the other two agents choose A ("accept") or R ("reject") on condition that they understand the language used by the proposer. Similarly, we denote by y (resp. z) the offer proposed by agent 2 (resp. 3).

If a responder does not use the language of the proposer, then the responder has to choose R. The actual game tree, therefore, depends upon the profile of the agents' statuses. We write $s = (s_1, s_2, s_3)$ ($s_i \in \{D, N_b, N_m\}$, $i = 1, 2, 3$) to denote the status configuration of the bargaining game. Also, we sometimes write like (D_1, D_2, N_b) when we would like to pay attention to the identity of agents therein.

The game ends if they reach an agreement. Suppose that x is offered, and the game ends in the tth period. Then Agent i's payoff is given by $\delta^{t-1}x_i$, where $\delta \in (0, 1)$ is a common discount factor. If no agreement is reached in any period, then the payoff of each agent will be zero.

This paper considers two classes of bargaining games, the majority bargaining games and the unanimity bargaining games with language constraints.

3.1. **The Majority Bargaining Games.** In the majority bargaining game with language constraints, at least two agents (i.e., majority) need to reach an agreement. To be precise, if at least one responder chooses "A", then the game ends, and x is realized as the outcome of the bargaining game. The bargaining stage at which everyone understands the language used in the bargaining stage is shown in Figure 2.

If the type profile is different, we have a different game tree. The rule for constructing a corresponding game is as follows. If an agent does not understand the language of the proposer, then this agent's response has to be R, and to express this on the tree, we remove from Figure 2 the alternative labelled "A" of this agent to the corresponding proposal.

Suppose, for example, that the type profile is (D, N_b, N_m), and that the second agent, a bilingual, uses sign language, which only D understands. Then the game tree is given by 3.[7]

3.2. **The unanimity bargaining games.** A unanimity bargaining game requires that two responders have to take A to reach an agreement. The rest is exactly the same as in the majority bargaining game. Therefore, if everyone understands everyone else, then the game is given by Figure 6.

4. Analysis of the Second Stage: A Deductive Approach

The analysis of this section is based on deductive game theory where we assume that the agents are aware of the structure of the bargaining games and look for subgame perfect equilibria of the game. We often call it simply "equilibrium" in the sequel.

[7]Although the tree corresponding to this case can be simplified, we leave it this way in order to clarify the rule for modifying the game when some agent does not understand the language used by the proposer.

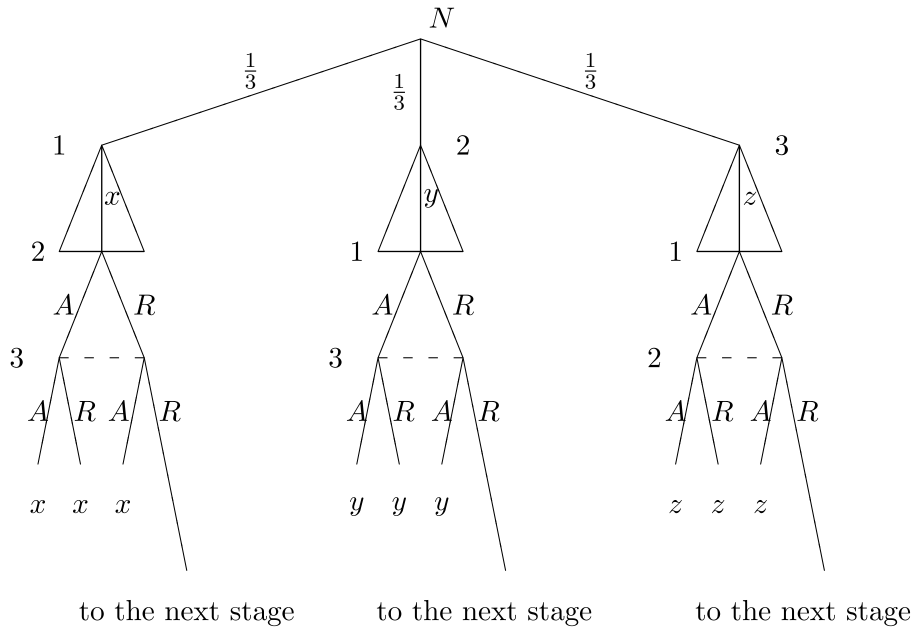

FIGURE 2. A majority bargaining stage when everyone understands the language used in the bargaring

We look for a subgame perfect equilibrium with stationary strategies. A stationary strategy is the strategy according to which one proposes the same (mixed) outcome whenever he/she becomes a proposer, and for any outcome w, if one takes A for w at some decision node (information set), then he/she takes A to w at other decision nodes (information sets).

4.1. **The Majority Bargaining Games.** We divide the analysis into four cases.

4.1.1. *The case of a single language.* This case occurs if the status profile is either one of (D, D, D) and (N_m, N_m, N_m). It is also verified that the profiles (D, D, N_b), (N_m, N_m, N_b), and (N_b, N_b, N_b) essentially correspond to this case.

Let V_i $(i = 1, 2, 3)$ be the continuation value of Agent i at the beginning of the stage game, i.e., at the node of Nature before choosing a proposer. Let x, y, and z be the

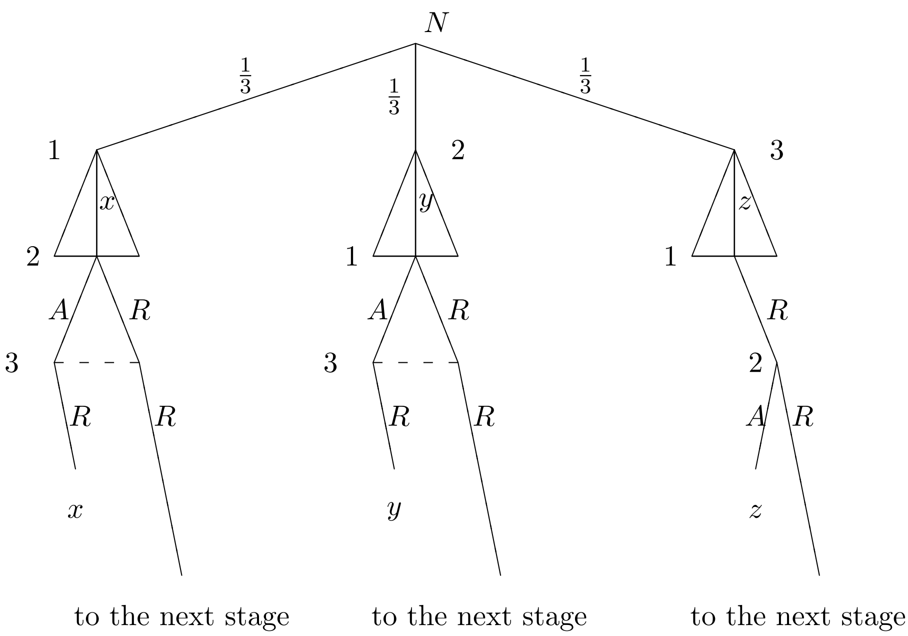

FIGURE 3. A majority bargaining stage with (D, N_b, N_m) and N_b using sign language, i.e., the situation where 1 and 2 communicate with each other, 1 and 3 cannot communicate, and 2 understands 3, but 3 does not understand 2.

proposals 1, 2, and 3 make, respectively. It is verified that in an equilibrium, the proposal is accepted right away. Thus, we have the following equations.

$$\begin{aligned} V_1 &= \frac{1}{3}x_1 + \frac{1}{3}y_1 + \frac{1}{3}z_1 \\ V_2 &= \frac{1}{3}x_2 + \frac{1}{3}y_2 + \frac{1}{3}z_2 \\ V_3 &= \frac{1}{3}x_3 + \frac{1}{3}y_3 + \frac{1}{3}z_3. \end{aligned}$$

In an equilibrium, it must be the case that "A" and "R" are indifferent for one of the responders; for if not, the proposer can lower the share of the responder a little, which is

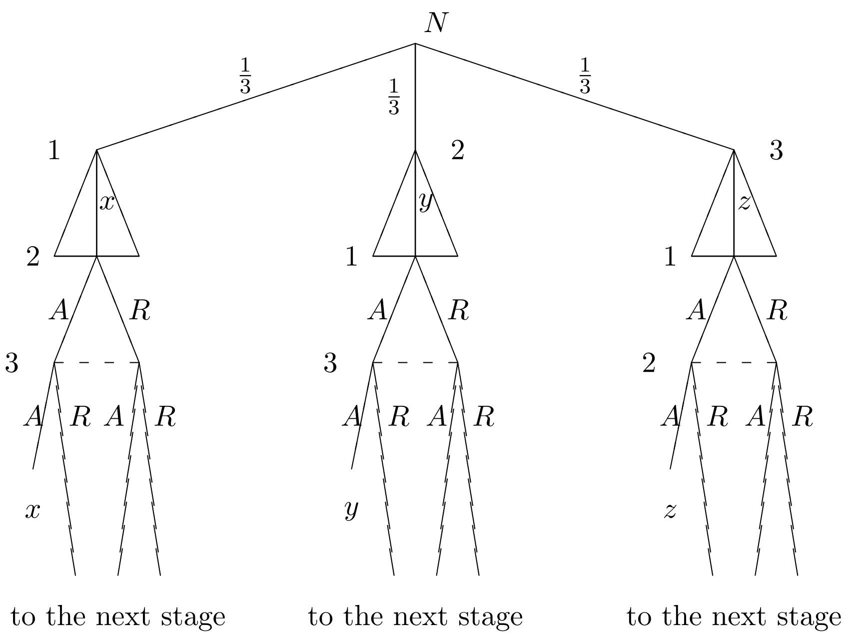

FIGURE 4. A majority bargaining stage when everyone understands the language used in the bargaring

still accepted by the responder, to increase the payoff of the proposer. This observation leads to the following.

$$\begin{aligned} x_2 = \delta V_2 \quad &\text{or} \quad x_3 = \delta V_3 \\ y_1 = \delta V_1 \quad &\text{or} \quad y_3 = \delta V_3 \\ z_1 = \delta V_1 \quad &\text{or} \quad z_2 = \delta V_2. \end{aligned}$$

Once one expects to obtain "A" from one responder, this agent does not need to obtain "A" from the other. Thus, the proposer always offers zero to at least one of the responders.

We have three resource constraints (it is verified that the entire pie is divided among the agents).

$$x_1 + x_2 + x_3 = 1 \tag{4.1}$$
$$y_1 + y_2 + y_3 = 1 \tag{4.2}$$
$$z_1 + z_2 + z_3 = 1. \tag{4.3}$$

It is verified that $V_1 = V_2 = V_3$ hold. Let us check it. Suppose first that $V_1 > V_2 > V_3$. Then Player 1 (resp. 2) tries to obtain "A" from 3 rather than 2 (resp. 1), and we have

$$\begin{aligned} x &= (1-\delta V_3, 0, \delta V_3) \\ y &= (0, 1-\delta V_3, \delta V_3) \\ z &= (0, \delta V_2, 1-\delta V_3). \end{aligned}$$

This implies, for example,

$$V_1 = \frac{1}{3}(1-\delta V_3) < \frac{1}{3}(1-\delta V_3) + \frac{1}{3}\delta V_2,$$

which is a contradiction. Other cases with equalities involved are similarly shown to draw a contradiction.

Hence, we have the following proposition.

Proposition 4.1. *In the majority bargaining game, if the status profile is either one of* (D, D, D), (N_m, N_m, N_m), (D, D, N_b), (N_m, N_m, N_b), *and* (N_b, N_b, N_b), *then*

$$V_1 = V_2 = V_3 = \frac{1}{3}$$

holds for all $\delta \in (0, 1)$.

4.1.2. *The case of no communication tool between majority and minority.* This game corresponds to the classical two-person bargaining game with random proposers. Consider (D, D, N_m). Since only 1 and 2 can communicate with each other, and their agreement stands, N_m is a dummy agent, obtaining nothing.

The continuation values of Agents 1 and 2 are given by

$$V_1 = \frac{1}{3}x_1 + \frac{1}{3}y_1 + \frac{1}{3}\delta V_1 \tag{4.4}$$
$$V_2 = \frac{1}{3}x_2 + \frac{1}{3}y_2 + \frac{1}{3}\delta V_2. \tag{4.5}$$

Solving this system with some other constraints, we obtain the following result.

Proposition 4.2. *In the majority bargaining game, if the status profile is either one of* (D, D, N_m) *and* (N_m, N_m, D), *then we have*

$$\begin{aligned} x &= \left(\frac{3-2\delta}{3-\delta}, \frac{\delta}{3-\delta}, 0\right), \\ y &= \left(\frac{\delta}{3-\delta}, \frac{3-2\delta}{3-\delta}, 0\right), \end{aligned}$$

and

$$V = \left(\frac{1}{3-\delta}, \frac{1}{3-\delta}, 0\right).$$

Moreover, as δ *goes to one,* x*,* y*, and* V *all converge to* $(1/2, 1/2, 0)$*.*

4.1.3. *The case of two bilinguals.* This case occurs if the status profile is either one of (N_{b1}, N_{b2}, D) and (N_{b1}, N_{b2}, N_m). In this case, an important role is played by an additional substage prior to the bargaining game where two bilinguals simultaneously decide which language to use. Note that this substage is different from the first stage where the agents choose whether to become a bilingual or not.

Take (N_b, N_b, N_m) for the sake of argument. This game is more complicated than the previous case since the substage in which Agents 1 and 2 have moves to determine the language they use matters in a non-trivial way.

First of all, suppose that both Agents 1 and 2 choose m to accommodate 3. In this case, the outcome is the same as that of the standard case analyzed above, i.e.,

$$V_1 = V_2 = V_3 = \frac{1}{3}$$

holds for all $\delta \in (0, 1)$.

Next, suppose that one of the bilinguals, say, 1 chooses sign language, and if the other bilingual, 2, chooses oral language (or the both languages). In this situation, Agent 3 is still accommodated, and the above analysis applies. Agent 2 takes a balance between Agents 1 and 3 so that $V_1 = V_3$ holds. Moreover, since Agent 1 understands Agent 3's offer, Agent 3 takes a balance between 1 and 2 so that $V_1 = V_2$ holds. Thus, we have

$$V_1 = V_2 = V_3 = \frac{1}{3}$$

for all $\delta \in (0, 1)$.

If, on the other hand, two bilinguals both choose sign language, then Agent 3 (N_m) would never understand the two, and therefore, Agents 1 and 2 share the pie by themselves. In an equilibrium, it must be the case that A and R are indifferent for the responder as before. However, since neither Agent 1 nor 2 needs to worry about Agent 3's incentive, we have only two equations that correspond to this incentive constraint.

$$x_2 = z_2 = \delta V_2 \tag{4.6}$$

$$y_1 = z_1 = \delta V_1. \tag{4.7}$$

As for Agent 3, Agents 1 and 2 give nothing in their proposals. Therefore, we have

$$x_3 = y_3 = 0.$$

We have three resource constraints (4.1)-(4.3).

Solving this system of equations, we obtain

$$x = (x_1, x_2, x_3) = \left(\frac{3-2\delta}{3-\delta}, \frac{\delta}{3-\delta}, 0\right), \tag{4.8}$$

$$y = (y_1, y_2, y_3) = \left(\frac{\delta}{3-\delta}, \frac{3-2\delta}{3-\delta}, 0\right), \tag{4.9}$$

$$z = (z_1, z_2, z_3) = \left(\frac{\delta}{3-\delta}, \frac{\delta}{3-\delta}, 3\frac{1-\delta}{3-\delta}\right). \tag{4.10}$$

Thus, we have

$$\begin{aligned} V_1 &= \frac{1}{3-\delta}, \\ V_2 &= \frac{1}{3-\delta}, \\ V_3 &= \frac{1-\delta}{3-\delta}. \end{aligned}$$

Observe that $V_1 = V_2 > 1/3$, and that as δ goes to one, x, y, z, and V all converge to

$$(\frac{1}{2}, \frac{1}{2}, 0).$$

Note that in the language choice substage prior to the bargaining game, choosing sign language is a weakly dominant strategy if we view this substage as a 2×2 game. We simply say that the players take a weakly dominant strategy in the substage of language selection, and we assume so in the sequel.

Proposition 4.3. *In the majority bargaining game, suppose that the status profile is either one of* (N_b, N_b, D) *and* (N_b, N_b, N_m)*. Then, in a stationary subgame perfect equilibrium where the players take a weakly dominant strategy in the substage of language selection, the expected payoff profile is given by*

$$(\frac{1}{3-\delta}, \frac{1}{3-\delta}, \frac{1-\delta}{3-\delta})$$

Moreover, as δ *goes to one, the profile converges to* $(1/2, 1/2, 0)$*.*

4.1.4. *A deaf, a bilingual, and a monolingual.* This case occurs when we have (D, N_b, N_m). First, $V_1 = V_3 = V$ holds since if $V_1 > V_3$ (resp. $V_1 < V_3$) holds, then N_b favors 3 (resp. 1). Therefore, N_b uses the following strategy: N_b uses both sign and oral languages and offers $(\delta V, 1-\delta V, 0)$ and $(0, 1-\delta V, \delta V)$ with probability 1/2 each.

Next, $x = (1-\delta V_2, \delta V_2, 0)$ and $z = (0, \delta V_2, 1-\delta V_2)$ hold since Agents 1 and 3 cannot communicate with each other.

Thus, we have

$$V = \frac{1}{3}(1-\delta V_2) + \frac{1}{6}\delta V \tag{4.11}$$

$$V_2 = \frac{2}{3}\delta V_2 + \frac{1}{3}(1-\delta V). \tag{4.12}$$

Solving this equation, we obtain the following result.

Proposition 4.4. *In the majority bargaining game, if the status profile is* (D, N_b, N_m)*, then we have*

$$\begin{aligned} V_1 = V_3 = V &= \frac{2}{3}\frac{(18-\delta)(1-\delta)}{(6-\delta)(6-5\delta)} \\ V_2 &= \frac{1}{3}\frac{4-\delta}{6-5\delta}. \end{aligned}$$

Moreover, as δ *goes to one,* V *and* V_2 *converge to 0 and 1, respectively.*

4.1.5. *Summary of the majority bargaining game: the payoff table.* The payoff table in the limit of δ going to one is given by Table 1.

	(D,D)	(D,N_b)	(N_b,N_b)	(D,N_m)	(N_b,N_m)	(N_m,N_m)
D	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{2}$	0	0
N_b	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	1	$\frac{1}{2}$	$\frac{1}{3}$
N_m	0	0	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$

Table 1. Payoff Table: Majority Bargaining Games

In this table, the entry that corresponds to row $i \in \{D, N_b, N_m\}$ and column $(j,k) \in \{D, N_b, N_m\}^2$ is the payoff of agent i who encounters (j,k).

4.2. **Unanimity bargaining games.** Once the majority bargaining games are solved, the analysis of the unanimity bargaining games as set up in Subsection 3.2 either is reduced to some cases of the majority bargaining games or becomes trivial. The payoff table in the limit of δ going to one is given by Table 2. The way to read the table is the same as that for the majority bargaining game.

	(D,D)	(D,N_b)	(N_b,N_b)	(D,N_m)	(N_b,N_m)	(N_m,N_m)
D	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0
N_b	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$
N_m	0	0	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$

Table 2. Payoff Table: Unanimity Bargaining Games

As we have pointed out earlier, the reason that we consider these two games is *not* because they are the only possible games played in the community. Rather, people in the community must have played a variety of games. In language selection games, however, it is often taken for granted that the more people learn a certain language, the more incentive one has to learn this language. We show that this claim needs to be examined. As we have seen in the case of the unanimity bargaining game, this is not the case. The game exhibits strategic substitutability, i.e., the more people learn a certain language, the less incentive one has to learn it. At the same time, however, it is shown that even if the unanimity bargaining game is played, people have more incentives to learn sign language in Martha's Vineyard Island than in the mainland U.S. In other words, an incentive to become a bilingual is larger in the former than in the latter, but the bilinguals crowd out each other; given the population of the deaf, an incentive to become a bilingual is smaller when there are many bilinguals than when there are few.

One may casually argue that the more people use a certain language, the more incentive one has to learn it. This claim is not accurate at least in the present framework. We may state that the more people can use *only* a certain language, the more incentive one has to learn it. Again, we do not wish to make a general claim like this. The point is that the incentive to learn language is relative to the game they play, and that it does not contradict with the fact that even on the Island, there were people who did not use sign language.

5. The Analysis of the First Stage: Evolutionary Approach

Using Tables 1 and 2, we examine the incentive of the non-deaf people to learn sign language. Suppose that the fraction of the deaf is exogenously determined, and denoted by α. Suppose next that the fraction of bilinguals in the non-deaf population is given by β. Then the probability that one meets, say, (D, N_b) is calculated as

$$2(1-\alpha)\alpha\beta.$$

Table 3 shows the probabilities/frequencies of matching with respective pairs for all the cases.

	(D, D)	(D, N_b)	(N_b, N_b)	(D, N_m)	(N_b, N_m)	(N_m, N_m)
Probability	α^2	$2(1-\alpha)\alpha\beta$	$(1-\alpha)^2\beta^2$	$2(1-\alpha)\alpha(1-\beta)$	$2(1-\alpha)^2$	$(1-\alpha)^2(1-\beta)^2$

Table 3. Frequency of matching

5.1. **The majority bargaining game.** Using Tables 1 and 3, we calculate the payoffs π_B^J and π_M^J of taking N_b and N_m, respectively, for the majority bargaining game.

$$\begin{aligned}
\pi_B^J &= \frac{1}{3}\alpha^2 + (1-\alpha)\alpha\beta + \frac{1}{3}(1-\alpha)^2\beta^2 + 2(1-\alpha)\alpha(1-\beta) + (1-\alpha)^2 + \frac{1}{3}(1-\alpha)^2(1-\beta)^2\alpha^2 \\
\pi_M^J &= (1-\alpha)\alpha(1-\beta) + \frac{1}{3}(1-\alpha)^2 + \frac{1}{3}(1-\alpha)^2(1-\beta)^2\alpha^2.
\end{aligned}$$

Therefore, we have

$$\pi_B^J - \pi_M^J = \alpha\left(1 - \frac{2}{3}\alpha\right) + \frac{1}{3}(1-\alpha)^2\beta. \tag{5.13}$$

The coefficient of β is positive. Thus, the more bilinguals there are, the more incentive each agent has to learn sign language. The majority bargaining game exhibits strategic complement.

In order to analyze this situation, it is more appropriate to use evolutionary game theory than deductive game theory since the propagation of language, especially learning language in their childhood, is from parents to children and based on adaptation rather than deliberation. Let us use the best response dynamics due to [**?**] as an example.

Suppose that the initial condition is $\beta = 0$. A non-deaf agent prefers bilingual to monolingual at $\beta = 0$ if

$$d < d^* \equiv \alpha \left(1 - \frac{2}{3}\alpha\right). \tag{5.14}$$

Due to strategic complementarity, once the first non-deaf agent begins to learn sign language, all other agents start learning it, i.e., this process continues until every agent learns sign language.

If we use the historical data from the case of Martha's Vineyard, the threshold d^* on the island was about twenty times as high as that in mainland U.S., and d^* in Chilmark was about two hundred and forty times as high as that in mainland U.S. This wide margin might make people without disability adjust to the hereditary deafness on the island.

5.2. **The unanimity bargaining game.** It is commonly assumed that language acquisition exhibits strategic complementarity. But, it depends on the context. In the case of the unanimity bargaining game, we have strategic substitute, and its analysis becomes different from the case of strategic complement.

Using Tables 2 and 3, we calculate the payoffs of taking N_b and N_m, respectively, for the unanimity bargaining game.

$$\begin{aligned} \pi_B^U &= \frac{1}{3} + \frac{4}{3}\alpha(1-\alpha)(1-\beta) \\ \pi_M^U &= \frac{1}{3}(1-\alpha)^2. \end{aligned}$$

Therefore, we have

$$\pi_B^U - \pi_M^U = \frac{1}{3}\left[\alpha(6-5\alpha) - 4(1-\alpha)\alpha\beta\right]. \tag{5.15}$$

The coefficient of β is negative. Thus, the more bilinguals there are, the less incentive each agent has to learn sign language. The unanimity bargaining game exhibits strategic substitute.

In this case, the threshold d^{**} to make the first agent have an incentive to learn sign language is given by

$$d^{**} = \frac{1}{3}\alpha(6-5\alpha).$$

Like in the case of majority bargaining games, d^{**} in the island was about forty times as high as that in mainland U.S., and d^{**} in Chilmark was about two hundred forty times as high. Unlike the previous case, however, since the unanimity bargaining game exhibits strategic substitute, d^{**} alone cannot explain the dynamics of people's choices.

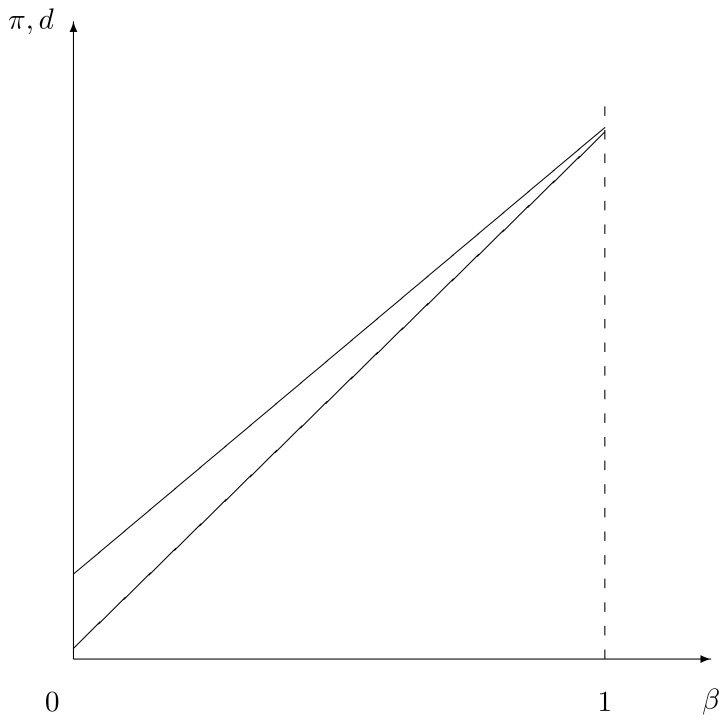

FIGURE 5. Majority case

Suppose that $d < d^{**}$ holds. Then people without disability start learning sign language, but it stops at

$$\overline{\beta} \equiv \frac{\alpha(6-5\alpha)-3d}{4\alpha(1-\alpha)}$$

if $d > \frac{1}{3}\alpha(2-\alpha)$; otherwise, all of the people without disability become bilinguals.

6. PREJUDICE: INDUCTIVE APPROACH

In 1970s and 80s, there were a series of attempts to refine Nash equilibrium, the central solution concept in noncooperative game theory, from the viewpoint of the rationality hypothesis. This so called refinement program reached its highest when [**?**] proposed the concept of stable sets. It is often assumed that players have sufficient knowledge of the structure of the game they play (at least in a probabilistic manner), and that they deduce

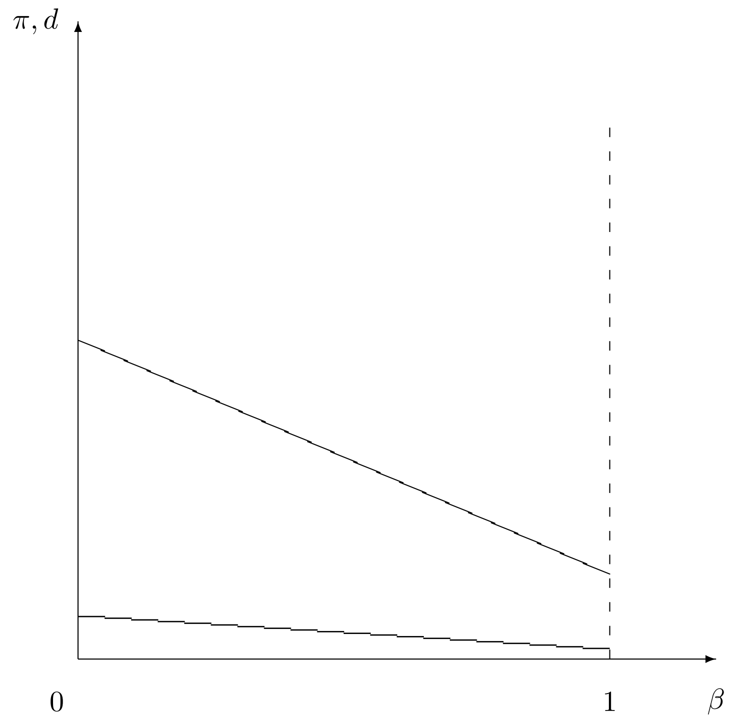

Figure 6. Unanimity case

what they should do (strategies) by reasoning processes. Here, let us call this type of game theory *deductive game theory*.

Its powerful hypothesis has not been well challenged. Recent studies on bounded rationality, or behavioral economics, only attest to it. In order to explain the phenomena that seem to deviate from the rational behavior, one needs to modify the rule of the game and/or preferences of the players therein. But, this modification itself reinforces the rationality hypothesis of game theory.

This rationality hypothesis, including the hypothesis behind recently developed behavioral economics, limits the scope of application of game theory to some issues associated with disability studies, which have been concerned with concepts like prejudices and social inclusion. Take the concept of prejudice as an example. According to the Cambridge Dictionary online, prejudice is “an unfair and unreasonable opinion or feeling, especially when formed without enough thought or knowledge.” This concept, by nature, involves some sort of wrong beliefs and limited reasoning. If it is assumed that players know the structure of the world, then it leads to the world with prejudices. On the other hand, if one constructs a model in which players have wrong beliefs, such a model presumes prejudices as an outset, and we cannot study how and in what circumstances prejudices emerge.

To understand societal phenomena like the emergence of prejudices without assuming it as an outset, we consider a situation where players do not know the entire structure of the game, obtain information through the course of the play of the game, and try to construct a model of the game. A *model* of the game itself is described as a game, which may or may not be the same as the original game. A model is *coherent* with a set of information (*a* priori knowledge and experiences) if it does not contradict with prior knowledge, if any, and the information acquired through the play of the game (experiences). We apply this concept to the game we have analyzed so far and study what type of model people might construct through the play of the game.

Also, when we discuss inclusive education, one of its purposes is to enlighten students without disability that our society consists of a variety of people through the interaction between people with and without disability. In order to tell a story like this, we are faced with the limit of the rationality hypothesis since rational players are aware of this possibility, and their knowledge acquisition is by eliminating some possible states of the world out of scope rather than by adding new insight from their experiences and information they obtain.

In order to construct a theory of disability and economy, incorporating some ideas that have been discussed in disability community, we introduce a new theory, inductive game theory, which was first developed by [**?**]. In this theory, players do not know the structure of the game they play. Instead, they accumulate experiences and try to understand the society, constructing a model of the society. The constructed model may or may not correspond to the "true" model, if the true model ever exists.

This way, inductive game theory is able to describe the situation where people, based on their limited experiences, form a false model of the world. This false image may include prejudice as a typical example.

For this purpose, let us consider a specific situation. Suppose first that nobody without disability learns sign language, playing unanimity bargaining games. It may not occur to them that there is a choice of learning it.

Suppose next that each player in N_m sees the structure of the subgame of the bargaining stage, including the payoff therein. However, they do not see its structure unless they enter the subgame they actually play. For example, suppose the player in question is in N_m. Then it is only after this player meets a player in D and a player in N_m to enter the subgame where (N_m, N_m, D) is a type profile that this player sees the payoff structure of this subgame.

Suppose now that the player plays the unanimity bargaining game several times. The experience he/she obtains is the following:

- met with (D, D), and no agreement was reached;
- met with (D, N), and no agreement was reached;
- met with (N, N), and an agreement was reached, and each member obtaines 1/3.

Note that as experiences, the player thinks the non-deaf people who do no use sign language are N instead of N_m.

One of the simplest payoff function that is coherent with the above knowledge and experiences is given by

$$U_N(i,j) = \begin{cases} \frac{1}{3} & \text{if } i,j = N, \\ 0 & \text{otherwise,} \end{cases}$$

where $U_N(i,j)$ is the subgame perfect equilibrium payoff when this player meets a player of type i and a player of type j. Then the situation in their eyes can be summarized as in Table 4. Notice that there is no distinction between N_b and N_m, while in reality, everyone is in N_m.

Of course, this is a simplistic model compared to the true model laid out in the present paper. However, its explanatory power is as good as the true model given the experiences they have. Moreover, this model may be regarded as a better model than the true model precisely because it describes the world in a simpler manner than the true one. Prejudices emerge.

	$\{D, D\}$	$\{D, N\}$	$\{N, N\}$
D	0	0	0
N	0	0	$\frac{1}{3}$

Table 4. Payoff Table: Unanimity Bargaining Games from non-deaf's viewpoint

This suggests, for example, the importance of inclusive education, i.e., both children with and without disability learn at the same school, side by side.

In Japan, children with disability are often go to special support education schools. There are pros and cons. In case of children with hearing impairment, they had better be educated by sign language. [?] indicates that deaf children quickly acquire a vocabulary of about 1,000 words by the age of five, almost as much as hearing children, if the deaf children use sign language, and that if they are given oral instructions only, then they learn only several dozens.

On the other hand, in order to reduce prejudices of children without hearing impairment, it is important to include them in the same (nursery) school where teachers (childcare givers) can use both oral and sign languages. If one sees there is a bilingual of this kind, they have an experience that among those who can communicate, there are both bilingual and monolingual people, i.e., N_b and N_m, and they realize that the payoff table like 4 is a false image of the world.

7. Concluding Remarks

The present paper takes Martha's Vineyard Island as an example. It must be clear to the reader by now that disability is a concept that is relative to the society. Similar, yet different in details, analyses are applicable to a variety of issues our society is faced with. One thing in common is that disability is a social construct.

The present paper is suggestive rather than definitive, conveying the idea that disability-related issues are societal phenomena, emerging from interactions of people therein, and that game theory provides us with a useful approach to these issues.

Disability-related issues call for a new approach in game theory, especially when we discuss the issues of prejudices and inclusion. Inductive game theory is one such an example.

My hope is that the field of disability and economy develops a new frontier of research that will foster the creation of a society for all.

References