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A macroeconomic model of liquidity crises*

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Abstract

We develop a model of liquidity crises based on debt overhang and credit networks. Firms need liquidity for their operations. The default of a group of firms may cause a chain reaction of defaults of banks and firms through a credit network. Our model is consistent with the observation that the decline in output during the Great Recession is mostly attributable to the deterioration in the labor wedge rather than to the deterioration in productivity.

Keywords: Systemic crises; liquidity demand; credit network; debt overhang.

JEL Classification numbers: E30, E44, G01.

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1 Introduction

Liquidity plays an important role in facilitating economic transactions. Thus, an abrupt reduction in liquidity, or a liquidity crisis, tends to cause a deep recession.[1] Indeed it has been argued that the recession in the late 2000s, the Great Recession, was induced and exacerbated by a liquidity crisis.[2]

In this study, we augment the real-business-cycle model with banks, and we consider how a liquidity crisis emerges and how its effect is propagated throughout the economy. In the quantitative analysis, we argue that the predictions of our model are roughly consistent with what happened in the Great Recession. In particular, our model accounts for the fact that the decline in output in the Great Recession was mostly due to the deterioration in the labor wedge rather than that in productivity (Arellano, Bai, and Kehoe, 2016; Brinca, Chari, Kehoe, and McGrattan, 2016).[3]

Our model has three key features. First, firms have to make wage payments in advance of production, so liquidity is essential for the operation of firms.[4] In addition, we assume that firms rely on banks for the provision of liquidity. These assumptions help the model generate a deep recession following a liquidity crisis.

Second, firms and banks both have long-term debt. As long as short-term debt is not senior to long-term debt, debt overhang may arise. The ability of firms and banks to obtain short-term funds is restricted by their amount of long-term debt. Firms and banks with severe debt overhang go into default, which may trigger a liquidity crisis.

Third, the lending markets are segmented and represented by a credit network, which is exogenously given and is held fixed over time. We suppose that the credit network reflects "relationship banking," although the lending markets are assumed to be perfectly competitive.[5] The form of credit network determines how a crisis is propagated and becomes systemic.

Combining these elements, a liquidity crisis emerges in our model in the following way. Suppose that some firms receive a negative productivity shock and default on their long-term debt. These

[1]See, for instance, Borio (2009).

[2]See, for instance, Lucas and Stokey (2011). For overviews of the crisis, see Adrian and Shin (2010), Brunnermeier (2009), Kacperczyk and Schnabl (2010), and Gorton (2010), among many others.

[3]The labor wedge is the wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. See, for instance, Chari, Kehoe, and McGrattan (2007) and Shimer (2009).

[4]For an analysis on corporate demand for liquidity, see, for instance, Holmström and Tirole (2011).

[5]See, for instance, Freixas and Rochet (2008) for the concept of relationship banking.

defaults hurt the net worth of the banks that hold the debt issued by these firms, and they may result in the defaults of such banks. These defaults, in turn, lead to the failure of some other firms that are supposed to obtain short-term funds from these banks. Thus, chain defaults may occur, and the crisis is propagated through the credit network.

In the quantitative analysis of our model, we illustrate that it replicates some important features of the Great Recession. In particular, it is consistent with the observation emphasized by Brinca, Chari, Kehoe, and McGrattan (2016), for instance, that the decline in output during the Great Recession is mostly due to the deterioration of the labor wedge rather than the deterioration of the level of TFP. The intuition behind this result is simple. In our model, most firms have a normal level of productivity even during a crisis. Massive defaults occur because of the unavailability of liquidity rather than a decline in productivity. Thus, the decline in the aggregate productivity level remains small. On the other hand, the decline in employment caused by the lack of liquidity increases the labor wedge.

Related literature

Our model is related to several strands of literature. The first one is the theory of bank runs developed by Bryant (1980) and Diamond and Dybvig (1983).[6] In this literature, a crisis occurs when there is a run on existing deposits, whereas in our model, the crisis occurs because of an evaporation of short-term loans. In fact, both of these aspects are present in actual liquidity crises, and the two approaches are considered to be complementary.

The second strand of literature is that on debt overhang, such as Myers (1977), Lamont (1995), Philippon (2010), and Occhino and Pescatori (2010). Liquidity crises occur in our model as a result of debt overhang. Firms and banks with a large amount of long-term debt are not able to obtain short-term funds. Among the above studies, Lamont (1995) is closest to this study in spirit. In a two-period model, Lamont (1995) shows that debt overhang can lead to multiple equilibria with different levels of investment. The key for multiple equilibria in the model of Lamont (1995) is the aggregate demand externality due to monopolistic competition, similar to Kiyotaki (1988). The mechanism operating in our model is rather different. First, our model is perfectly competitive. Second, we focus on a crisis induced by a fundamental shock rather than

[6]For more recent developments of this theory, see Uhlig (2010), Ennis and Keister (2009, 2010), Keister (2016), Kato and Tsuruga (2012), and Gertler and Kiyotaki (2015), among many others.

on self-fulfilling expectations. Third, financial intermediation and the credit network are crucial elements here but not in Lamont's (1995) model. In addition, we would like to note that the levels of long-term debt for firms and banks are endogenously determined in our model but are exogenously given in most of the existing models of debt overhang.

We embed banks in the real business cycle framework in a manner similar to those of Gertler and Karadi (2011), Gertler, Kiyotaki, and Queralto (2012), and Gertler and Kiyotaki (2015), among others. In particular, as in these studies, we assume that the amount of deposits that banks can collect is limited by their moral hazard constraint.

Our model is also related to the literature on contagions of crises through credit networks, such as Allen and Gale (2000) and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015). The focus of these studies is on comparisons of different forms of credit networks regarding financial stability, and, thus, their models are kept simple in other respects.[7] On the other hand, we consider only one type of credit network and do not assess its desirability or its empirical relevance. Instead, it is embedded in a standard real-business-cycle framework and is adequate for analyzing macroeconomic implications. In this sense, we view our study as complementary to the work in this literature.

The rest of the paper is organized as follows. In the next section, the basic structure of the model economy is described. The characterization of the equilibrium is given in section 3. Concluding remarks are given in section 4. All the proofs are given in the Appendix.

2 The model economy

Time is discrete and continues to infinity: $t = 0, 1, 2, \cdots$. There is a unit mass of identical and infinitely lived households who consume, save, and supply labor. In addition, in every period, N types of "firms" and N types of "banks" are born in each household. For simplicity, we assume that firms and banks live only for two periods.[8] We assume that all agents are price takers and all markets are perfectly competitive.

[7] For instance, their models use three-period endowment economies.

[8] This assumption is surely restrictive; for instance, as we shall see, crises in our model do not have persistent effects due to this and other assumptions. We nevertheless make this assumption since our focus is on how a crisis occurs and its relatively short-term effects on the economy. One way to introduce multiple periods of life for firms and banks in our model is to follow an approach similar to those of Gertler and Karadi (2011) and Gertler and Kiyotaki (2015). Such an extension is left for our future research.

Type-i firms and banks operate in sector i, for $i = 1, \cdots, N$. As the measure of households is unity, the respective measures of each type of firms and banks are also unity. Firms and banks are connected through a credit network, which is specified below. We abstract from capital accumulation and assume that the total supply of capital is fixed at a constant, N, the number of sectors in the economy.

Financial intermediation is introduced within the representative household framework in the standard way.[9] Funds flow from households to banks and from banks to firms. We assume that firms cannot obtain funds directly from households.[10] Banks obtain funds in the form of equity from the households to which they belong. They also collect funds from other households in the form of deposits.

Regarding loans and deposits, the following three assumptions are important for our argument. First, both loans and deposits take the form of risky debt, where borrowers make a fixed repayment as long as they are solvent.[11] Second, there are loans and deposits with different maturity periods. Specifically, we assume that firms need two types of loans: inter-period ("long-term") and intra-period ("short-term"). Corresponding to these financial needs of firms, banks also collect short-term and long-term deposits. Third, all loans (deposits) have the same seniority, regardless of their maturity. Thus, the recovery rates for short-term and long-term loans (deposits) are identical.[12]

Under these assumptions, bankruptcies of firms and banks can occur due to debt overhang. Providers of short-term funds are hesitant to lend to borrowers with large amounts of long-term debt. The bankruptcy of a relatively small number of firms and banks may bring about a systemic crisis because of propagation through a credit network.

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[9] This setting is similar to, for instance, Christiano, Motto, and Rostagno (2010), Gertler and Karadi (2011), and Gertler and Kiyotaki (2015).

[10] Theories that account for why some firms need to borrow from banks include, among others, delegated monitoring (Diamond 1984) and superior auditing technology of relationship banks (Diamond and Rajan 2000, 2001). Gu, Mattesini, Monnet, and Wright (2013) develop a theory of banking based on limited commitment.

[11] As is well known, with asymmetric information and costly state verification, the optimal contract does take the form of risky debt (e.g., Townsend 1979, Gale and Hellwig 1985).

[12] It is commonly assumed in the debt-overhang literature that long-term loans (deposits) are senior to short-term loans (deposits). Assuming such a seniority rule does not change our results qualitatively. Indeed, it will strengthen the mechanism causing a liquidity crisis in our model.

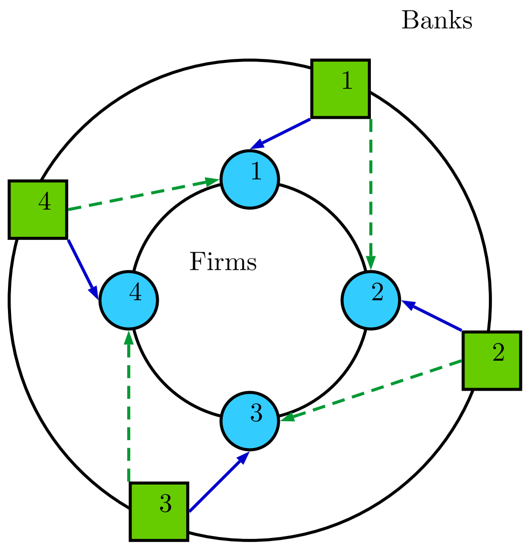

Figure 1: Credit network. Firms are represented by small circles on the inner ring, and banks are represented by squares on the outer ring. The numbers in the circles and squares correspond to sectors. Solid arrows express flows of long-term funds, and dashed arrows express those of short-term funds.

Credit network: We consider a very simple network of firms and banks that is an extension of the network called "ring" (e.g., Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015) to our model. Firms and banks are represented by nodes on two concentric rings, where firms are on the inner ring and banks on the outer ring. Here, nodes correspond to sectors, so we may use the terms "sector" and "node" interchangeably in what follows. Banks in node i make long-term loans to firms in node i and short-term loans to firms in node $i+1$, for $i = 1, 2, \cdots, N$ (modulo N). Figure 1 shows an example of our credit network when $N = 4$. Squares and circles represent banks and firms, respectively. Each number in a square or circle indicates the index of a node. Solid and dashed arrows represent long-term and short-term loans, respectively. For example, banks in node 1 provide long-term loans to firms in node 1 and short-term loans to firms in node 2.

2.1 Model overview

Before going into the formal analysis, it may be useful to provide an intuitive overview on how a liquidity crisis occurs in our model. We consider productivity shocks as exogenous disturbances, and we abstract from sunspot shocks, although sunspot equilibria may exist in our model, as in the bank-run model of Diamond and Dybvig (1983). Our focus on the productivity shock (or fundamental shocks, more broadly) is based on the argument, as in Allen and Gale (1998), that historical evidence does not support theories with sunspot shocks as the main cause of financial crises.

Productivity shocks are denoted by a vector $s_t = (s_{1,t}, s_{2,t}, \cdots, s_{N,t})$, where $s_{i,t}$ is the productivity of firms in node i in period t (firms in the same node have a common productivity level). In our model, a negative productivity shock to just one sector of the economy can lead to a systemic financial crisis, even if firms in the rest of the economy are all sound and productive. To illustrate this property, suppose specifically that firms in node i experience a negative productivity shock in period t. Firms have long-term debt, which constitutes a fixed cost for them. Thus, with a sufficiently large (negative) productivity shock, firms in node i are not able to repay their long-term debt in full (even if they chose the profit-maximizing level of output), and they declare bankruptcy.

The failure of firms in node i is propagated to other sectors through the credit network. Given that firms in node i default on the long-term debt that they owe to banks in node i, those banks become insolvent, that is, their net worth becomes zero. Under our assumption on financial frictions, similar to Gertler and Karadi (2011) and Gertler and Kiyotaki (2015), banks need to have strictly positive levels of net worth in order to engage in financial intermediation. It follows that banks in node i are now unable to provide short-term loans to firms in node $i + 1$.

Then, in spite of the fact that the productivity of firms in node $i + 1$ has not fallen at all, they cannot obtain short-term working capital, which is necessary to produce a positive amount of output, from their banks. Thus these firms have to default, which, in turn, causes defaults of banks in node $i+1$. In this way, the bankruptcy of a relatively small number of firms (i.e., firms in one node/sector) causes a chain reaction of defaults of firms and banks, that is, a systemic crisis. Here, in particular, it is a “liquidity crisis” because the failure of all firms other than those in node i is caused by their inability to obtain short-term loans (liquidity).

Without a prompt intervention of the government, the chain reaction of defaults would continue, and all firms and banks would go bankrupt. As a simple form of policy intervention, we suppose that the government gives a subsidy to banks (only) in node $i + n$ (modulo N). Here, n is a positive integer that measures the “delay” of the policy response. With such a policy, banks in nodes $i + n, \cdots, i - 1$ (modulo N) and firms in nodes $i + n + 1, \cdots, i - 1$ (modulo N) would be able to avoid bankruptcy.

2.2 Households

Now, we turn to the formal presentation of the model. A representative household has the flow budget constraint:

$$
\begin{aligned}
c_t + \sum_{i=1}^{N} d_{it}^L + \sum_{i=1}^{N} d_{it}^S + \sum_{i=1}^{N} e_{it} \\
= \sum_{i=1}^{N} \tilde{\xi}_{i,t}^B (R_{i,t-1}^D d_{i,t-1}^L + R_{i,t}^B d_{i,t}^S) + w_t l_t + \sum_{i=1}^{N} \tilde{R}_{i,t}^E e_{i,t-1} + \sum_{i=1}^{N} \pi_{i,t}^F - T_t, \qquad (1)
\end{aligned}
$$

where c_t denotes the amount of consumption in period t, l_t the amount of labor supplied to firms (in other households), $\pi_{i,t}^F$ the profits earned by its member firms, $i = 1, \cdots, N$, and T_t the lump-sum taxes paid to the government.

The household provides funds to banks in two ways. First, it provides equity, $e_{i,t}$, to its member banks $i = 1, \cdots, N$. As shown below, a moral hazard problem of banks requires banks to hold some equity for financial intermediation. The return on equity is stochastic, and its realized rate is denoted by $\tilde{R}_{i,t}^E$. Second, each household puts deposits in the banks of other households. Deposits are of two types: long-term (inter-period), $d_{i,t}^L$, and short-term (intra-period), $d_{i,t}^S$. Their rates of interest are $R_{i,t-1}^D$ and $R_{i,t}^B$, respectively. Note that the long-term deposit rate between periods $t-1$ and t, $R_{i,t-1}^D$, is determined in period $t-1$.

If the banks in node i (call them "banks i") are insolvent in period t, the depositors recover only a fraction $\xi_{i,t}^B \in [0, 1)$ of their claims to banks i, where $\xi_{i,t}^B$ will be given later in equation (16) in Section 2.4. Let $\tilde{\xi}_{i,t}^B$ denote the stochastic recovery rate of depositors of banks i in period t:

$$
\tilde{\xi}_{i,t}^B = \begin{cases} 1, & \text{if bank } i \text{ is solvent,} \\ \xi_{i,t}^B, & \text{if bank } i \text{ is insolvent.} \end{cases}
$$

The consumption goods produced in the N sectors are perfect substitutes, so $c_t = \sum_{i=1}^{N} c_{i,t}$, where $c_{i,t}$ is the consumption good produced in sector i. As we shall see below, under the laissez-faire policy, all firms go bankrupt during a liquidity crisis, and, as a result, $c_t = 0$. To keep the level of consumption bounded away from zero in equilibrium, we assume that there are other sources of output that are determined outside of our model. Suppose for simplicity that the level of such output is constant and denoted by $\underline{c} > 0$. Thus, the total amount of output (consumption) is $c_t + \underline{c}$ in each period t, and the utility is defined for $\{c_t + \underline{c}\}_{t=0}^{\infty}$.

Given stochastic processes ($\tilde{\xi}_{i,t}^B$, $R_{i,t-1}^D$, $R_{i,t}^B$, $\tilde{R}_{i,t}^E$, w_t, $\pi_{i,t}^F$), the household chooses non-negative processes (c_t, $d^L_{i,t}$, $d^S_{i,t}$, $e_{i,t}$, l_t) to solve:

$$\max_{(c_t, d^L_{i,t}, d^S_{i,t}, e_{i,t}, l_t)\geq 0} E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t + \underline{c}) + \gamma \ln(1 - l_t)], \tag{2}$$

subject to the sequence of the flow budget constraint (1).

The first-order conditions for $d^L_{i,t}$ and $e_{i,t}$ are given by

$$1 = E_t \left[\lambda_{t,t+1} \tilde{\xi}^B_{i,t+1} R^D_{i,t}\right] = E_t \left[\lambda_{t,t+1} \tilde{R}^E_{i,t+1}\right], \tag{3}$$

where $\lambda_{t,t+1}$ is the stochastic discount factor

$$\lambda_{t,t+1} = \beta \frac{c_t + \underline{c}}{c_{t+1} + \underline{c}}.$$

For a bounded solution for $d^S_{i,t}$ to exist, $\tilde{\xi}^B_{i,t}$ and $R^B_{i,t}$ must satisfy

$$\tilde{\xi}^B_{i,t} R^B_{i,t} \leq 1. \tag{4}$$

2.3 Firms

There are N sectors of production. In each sector, a mass of firms with measure 1 is born in every period. Firms in all sectors produce a single homogeneous consumption good. All firms in the same sector are identical. We sometimes refer to firms in sector i as firms i.

Consider a representative firm in sector i that is born in period $t-1$. It purchases physical capital, $k_{i,t-1}$, in the first year of its life, and it produces output, $y_{i,t}$, in the second. Its production technology is described as:

$$y_{i,t} = s_{i,t} m^{\nu}_{i,t} k^{\alpha-\nu}_{i,t-1} l^{1-\alpha}_{i,t}, \tag{5}$$

where $s_{i,t}$ denotes the (sector-specific) productivity of firms in sector i, $k_{i,t-1}$ the capital input, $l_{i,t}$ the labor input, and $m_{i,t}$ the managerial input. The productivity level of each sector, $s_{i,t}$, is a random variable that is realized at the beginning of period t. Whereas the capital input $k_{i,t-1}$ is determined in period $t-1$, the other inputs are determined in period t.

Each firm supplies one unit of managerial input inelastically, so $m_{i,t} = 1$ in equilibrium. The firm cannot obtain the other inputs, $k_{i,t-1}$ and $l_{i,t}$, directly from the household to which it belongs. Instead, it has to purchase these inputs at the market. Similarly, each household cannot directly consume what its member firms produce. Thus, firms have to sell their products to other households in the market. The earnings of a firm are transferred back to the household to which it belongs.

To produce output in the second period (period t), the firm needs to receive both "long-term" and "short-term" loans. The long-term (inter-period) loans are needed to purchase capital, $k_{i,t-1}$, in the first period. Let q_{t-1} denote the price of capital in period $t-1$. Thus, the amount the firm needs to borrow is $L_{i,t-1} = q_{t-1}k_{i,t-1}$. Under our assumption on the credit network, this amount should be obtained from banks in node i. Let $R^L_{i,t-1}$ denote the (gross) interest rate on inter-period loans for firms in node i.

Our assumption on the need for short-term loans is motivated by historical evidence that financial crises triggered severe declines in output (e.g., Reinhart and Rogoff, 2009, 2014). This evidence suggests that liquidity plays an essential role in the production process. To incorporate this idea, we assume that firms have a limited ability to commit to pay for the factors of production. Specifically, we assume that firms have to pay wages in advance of production.[13] This assumption creates demand for liquidity (short-term loans) by firms, and, without it, firms are not able to produce output.

To hire labor services of amount $l_{i,t}$, the firm needs to pay $w_t l_{i,t}$ before production. Hence, the firm needs to borrow $W_{i,t} \geq w_t l_{i,t}$ from banks in node $i-1$ (under our assumption on the credit network). The short-term (gross) interest rate for firms i is $R^F_{i,t}$. After production, the firm sells its capital to newborn firms at price q_t.

Then, the profit maximization problem of the firm is stated as follows. Given the stochastic processes ($\lambda_{t-1,t}$, q_{t-1}, q_t, w_t, $R^L_{i,t-1}$, $R^F_{i,t}$), the firm chooses ($k_{i,t-1}$, $l_{i,t}$, $W_{i,t}$) so as to solve the profit maximization problem:[14]

$$\begin{aligned} \max_{k_{i,t-1}\geq 0} \quad & E_{t-1}\left[\lambda_{t-1,t}\left\{\max_{(l_{i,t},W_{i,t})\geq 0} \pi^F_{it}(k_{i,t-1}, l_{i,t}, W_{i,t})\right\}\right], \\ \text{s.t.} \quad & w_t l_{i,t} \leq W_{i,t}, \end{aligned} \tag{6}$$

where

$$\pi^F_{i,t}(k,l,W) = \max\{\hat{\pi}^F_{i,t}(k,l,W),\ 0\}, \tag{7}$$

$$\hat{\pi}^F_{i,t}(k,l,W) = s_{i,t}k^{\alpha-\nu}l^{1-\alpha} + q_t k - R^L_{i,t-1}q_{t-1}k - R^F_{it}W. \tag{8}$$

[13]This assumption is also common in the New Keynesian model (e.g., Christiano, Eichenbaum, and Evans, 2005).

[14]In this setup of the firm's problem, we rule out the possibility that it borrows the wage payment $W_{i,t}$ in advance in period $t-1$. Depending on the parameter values, if the firm is allowed to borrow in advance, it may want to do so in order to avoid the possibility of a liquidity shortage. To simplify the analysis, however, we abstract from such a possibility here as a shortcut to creating corporate demand for liquidity.

In equation (8), we use the fact that the firm chooses $m_{i,t} = 1$.

Here, $\hat{\pi}^F_{i,t}$ denotes the "profit of the firm," whereas $\pi^F_{i,t}$ is the "payment to the owner (household) of the firm." To simplify exposition, we slightly abuse notation by using $\hat{\pi}^F_{i,t}$ to denote $\max_{(l_{i,t},W_{i,t})\geq 0} \hat{\pi}^F_{i,t}(k_{i,t-1}, l_{i,t}, W_{i,t})$ and π^F_{it} to denote $\max_{(l_{i,t},W_{i,t})\geq 0} \pi^F_{i,t}(k_{i,t-1}, l_{i,t}, W_{i,t})$.

Due to the limited liability, if $\hat{\pi}^F_{i,t} < 0$, then the firm chooses to default, and, as a result, the payment to the owner is zero, $\pi^F_{i,t} = 0$. In such an event, the firm is indifferent about the choice of $(l_{i,t}, W_{i,t})$ because any choice would result in $\pi^F_{i,t} = 0$. Here, we resolve this indeterminacy issue by simply assuming that the firm chooses not to operate in the event of default:

$$l_{i,t} = W_{i,t} = 0, \qquad \text{if} \max_{(l_{i,t},W_{i,t})\geq 0} \hat{\pi}^F_{i,t}(k_{i,t-1}, l_{i,t}, W_{i,t}) < 0. \tag{9}$$

This assumption could be justified by, for instance, assuming an (infinitesimally small and non-defaultable) fixed cost for production.

Since long-term and short-term loans have the same seniority, their recovery rates are identical and are denoted by $\tilde{\xi}^F_{i,t}$. This rate is equal to one as long as $\hat{\pi}^F_{i,t} \geq 0$, but, otherwise, the firm can only repay a fraction $\xi^F_{i,t} \in [0, 1)$ of its total debt. Since firms do not produce output in the event of default, the recovery rate of the loans to the firm, $\tilde{\xi}^F_{i,t}$, is given by

$$\tilde{\xi}^F_{i,t} = \begin{cases} 1, & \text{if } \pi^F_{i,t} > 0, \\ \xi^F_{i,t}, & \text{if } \pi^F_{i,t} = 0, \end{cases}$$

where

$$\xi^F_{i,t} = \frac{q_t}{R^L_{i,t-1} q_{t-1}}.$$

2.4 Banks

Consider a representative bank in node i that is born in period $t-1$. As we see below, banks cannot operate without initial capital, so we assume that the household provides its newborn banks with funds $e_{i,t-1}$ as equity. In period $t-1$, the bank collects inter-period deposits $d^L_{i,t-1}$ (from other households) and makes inter-period loans $L_{i,t-1}$ to firms i, where

$$L_{i,t-1} = d^L_{i,t-1} + e_{i,t-1}.$$

In period t, the bank collects intra-period deposits $d^S_{i,t}$ and makes intra-period loans $W_{i+1,t}$ to firms $i+1$ so that $d^S_{i,t} = W_{i+1,t}$.

The government gives the bank lump-sum subsidies, $T_{i,t} \geq 0$, which are financed by lump-sum taxes on households. The determination of those subsides shall be discussed below in Section 2.6. If the bank chooses $(L_{i,t-1}, W_{i+1,t})$, its "profit," $\hat{\pi}^B_{i,t}$, is then given by

$$\hat{\pi}^B_{i,t}(L_{i,t-1}, W_{i+1,t}) = T_{it} + \tilde{\xi}^F_{i,t} R^L_{i,t-1} L_{i,t-1} + \tilde{\xi}^F_{i+1,t} R^F_{i+1,t} W_{i+1,t} - \left[R^B_{i,t} W_{i+1,t} + R^D_{i,t-1}(L_{i,t-1} - e_{it-1})\right]. \tag{10}$$

To take into account frictions associated with financial intermediation, we assume that banks are subject to a moral hazard problem similar to the one considered by Gertler and Karadi (2011).[15] This assumption implies that only a fraction of the bank's revenue can be pledged to its depositors. Here, we assume that the moral hazard problem is associated only with the short-term loans $W_{i+1,t}$.

To be specific, suppose that the bank can divert a fraction ψ of its revenue from short-term loans $\tilde{\xi}^F_{i+1,t} R^F_{i+1,t} W_{i+1,t}$. Then, the pledgeable amount of the bank's revenue is given by

$$T_{it} + \tilde{\xi}^F_{i,t} R^L_{i,t-1} L_{i,t-1} + (1-\psi)\tilde{\xi}^F_{i+1,t} R^F_{i+1,t} W_{i+1,t}.$$

This result sets the upper bound of the amount of deposits that the bank can collect. That is,

$$T_{it} + \tilde{\xi}^F_{i,t} R^L_{i,t-1} L_{i,t-1} + (1-\psi)\tilde{\xi}^F_{i+1,t} R^F_{i+1,t} W_{i+1,t} \geq \left[R^B_{i,t} W_{i+1,t} + R^D_{i,t-1}(L_{i,t-1} - e_{i,t-1})\right], \tag{11}$$

which is referred to as the moral hazard constraint for banks i.

The bank declares default if (11) is violated. That is, the payment to its owner, $\pi^B_{i,t}$, is written as

$$\pi^B_{i,t}(L_{i,t-1}, W_{i+1,t}) = \max\left\{\hat{\pi}^B_{i,t}(L_{i,t-1}, W_{i+1,t}),\ \psi\tilde{\xi}^F_{i+1,t} R^F_{i+1,t} W_{i+1,t}\right\}. \tag{12}$$

Note that when the bank defaults, the owner of the bank receives $\pi^B_{i,t} = \psi\tilde{\xi}^F_{i+1,t} R^F_{i+1,t} W_{i+1,t}$, which is positive as long as $W_{i+1,t} > 0$. This property is a consequence of our assumption that banks can divert a part of their revenue from short-term lending. As shown by Lemma 2 below, however, $W_{i+1,t} = 0$, and, hence, $\pi^B_{i,t} = 0$ whenever bank i defaults.

For a given value of $L_{i,t-1}$, let $\Gamma_{i,t}(L_{i+1,t-1})$ denote the set of short-term loan values, $W_{i+1,t}$, that is feasible for the bank:

$$\Gamma_{i,t}(L_{i,t-1}) \equiv \{0\} \cup \{W \geq 0 :\ \text{Condition (11) is satisfied.}\} \tag{13}$$

[15] We make this assumption for a technical reason as well. Without such an assumption, the bank's size would become infinite in this model.

Then, the profit maximization problem of the bank is defined as follows. Given stochastic processes $(\lambda_{t-1,t}, e_{i,t-1}, \tilde{\xi}^F_{i,t}, \tilde{\xi}^F_{i+1,t}, R^D_{i,t-1}, R^L_{i,t-1}, R^B_{i,t}, R^F_{i+1,t})$, the bank chooses $(L_{i,t-1}, W_{i+1,t})$ so as to solve:

$$\max_{L_{i,t-1} \geq e_{i,t-1}} E_{t-1}\left[\lambda_{t-1,t}\left\{\max_{W_{i+1,t}\in\Gamma_{i,t}(L_{i,t-1})} \pi^B_{i,t}(L_{i,t-1}, W_{i+1,t})\right\}\right], \tag{14}$$

where the function π^B_t is defined in (12) and the correspondence Γ_t is defined in (13). As in the case of firms, we use $\pi^B_{i,t}$ to denote $\max_{W_{i+1,t}\in\Gamma_{i,t}(L_{i,t-1})} \pi^B_{i,t}(L_{i,t-1}, W_{i+1,t})$.

Let $\tilde{\xi}^B_{i,t}$ be the recovery rate for depositors of banks in node i in period t. From the discussion above, it follows that

$$\tilde{\xi}^B_{i,t} = \min\{\xi^B_{i,t},\ 1\}, \tag{15}$$

where

$$\xi^B_{i,t} = \frac{T_{it} + \tilde{\xi}^F_{i,t} R^L_{i,t-1} L_{i,t-1} + (1-\psi)\tilde{\xi}^F_{i+1,t} R^F_{i+1,t} W_{i+1,t}}{R^B_{i,t} W_{i+1,t} + R^D_{i,t-1}(L_{i,t-1} - e_{i,t-1})}. \tag{16}$$

For depositors to make a positive amount of short-term deposits, $d^S_{i,t} > 0$, the intra-temporal rate of return, $\tilde{\xi}^B_{i,t} R^B_{i,t}$, must be no less than one, $\tilde{\xi}^B_{i,t} R^B_{i,t} \geq 1$, which, together with (4), implies that

$$\tilde{\xi}^B_{i,t} R^B_{i,t} = 1 \tag{17}$$

must hold whenever $d^S_{i,t} > 0$ in equilibrium.

The realized rate of return on bank equity, $\tilde{R}^E_{i,t}$, is given by

$$\tilde{R}^E_{i,t} e_{i,t-1} = \pi^B_{i,t}.$$

2.5 Productivity Shocks

Firms are subject to sector-specific productivity shocks, $s_t = \{s_{1,t}, s_{2,t}, \cdots, s_{N,t}\} \in [0, s_{\max}]^N$, where $s_{i,t}$ is the productivity level of firms in sector i in period t and $s_{\max}$ is the exogenous upper limit. To simplify the analysis, we assume that in each period only one (randomly selected) sector experiences a shock.

Specifically, let $I(t) \in \{1, \ldots, N\}$ be an i.i.d. random variable that realizes in period t with probabilities $\Pr(I(t) = i) = 1/N$ for all i. This variable determines the sector that is hit by the shock in period t. Given $I(t)$, the productivity level $s_{I(t),t} \in [0, s_{\max}]$ is determined according to a probability distribution $G(\cdot)$. We assume that the distribution $G(\cdot)$ is identical over time and across sectors. The productivity levels of the other sectors are unity: $s_{i,t} = 1$ for $i \neq I(t)$. As an example, consider a firm that is born in sector i in period $t-1$. The probability distribution of its productivity in period t, $s_{i,t}$, is given by

$$\Pr(s_{i,t} \leq z) = \begin{cases} \frac{1}{N} G(z), & \text{for } z < 1, \\ \frac{N-1}{N} + \frac{1}{N} G(z), & \text{for } z \geq 1. \end{cases} \tag{18}$$

To define the support of s_t, let $s^{(i)}(z)$ be an N-vector of the following form:

$$s^{(i)}(z) \equiv \{\underbrace{1}_{s_{1,t}}, \cdots, \underbrace{1}_{s_{i-1,t}}, \underbrace{z}_{s_{i,t}}, \underbrace{1}_{s_{i+1,t}}, \cdots, \underbrace{1}_{s_{N,t}}\}. \tag{19}$$

That is, it denotes the productivity levels of the N sectors when sector i experiences shock $s_{i,t} = z$. Then, the support of s_t, Ω, is expressed as

$$\Omega = \left\{ s^{(i)}(z) \,|\, i = 1, 2, \cdots, N, \text{ and } z \in [0, s_{\max}] \right\}. \tag{20}$$

Let $F(s)$ be the probability distribution implied by our assumption on the probability shocks.

2.6 Government

We assume that there is a government whose role is to limit the chain reaction of defaults. For this purpose, it collects taxes from the household and gives subsidies to banks.

Specifically, we consider the following kind of government intervention. In each period t, given that a productivity shock hits node $I(t)$, the government gives transfers to all banks at node $I(t)+n$ (modulo N), where the amount of transfers is determined so as to exactly cover their losses if the firms at $I(t)+n$ default on their inter-temporal loans. That is, the transfers from the government to sector $i \in \{1, \cdots, N\}$ in period t are written as

$$T_{i,t} = \begin{cases} (1 - \tilde{\xi}^F_{i,t}) R^L_{i,t-1} L_{i,t-1}, & \text{for } i = I(t) + n, \\ 0, & \text{for } i \neq I + n. \end{cases} \tag{21}$$

Note that if the firms at node i are solvent, then $\tilde{\xi}^F_{i,t} = 1$ so that there are no subsidies to banks in node i, $T_{i,t} = 0$. The lump-sum taxes on the household are given by

$$T_t = \sum_{i=1}^{N} T_{i,t}.$$

Here, the government policy is parameterized by an exogenously given number, $n \in \{0, \cdots, N-1\}$, which measures how quickly and accurately the government can respond to a crisis. As we shall see below, the bankruptcy of firms in node $I(t)$ leads to the bankruptcy of banks in nodes $i = I(t), \cdots, I(t)+n-1$ and to that of firms in nodes $i = I(t)+1, \cdots, I(t)+n$. Thus, when $n = 0$, no banks will fail (even when firms $I(t)$ do). As n gets larger, the number of sectors where firms and banks go into default increases in the event of a crisis. Our interpretation of a large value of n is that the government's intervention is delayed and/or inadequate. The limiting case where the government chooses not to intervene at all (the laissez-faire case) is represented by $n = \emptyset$.

The type of policy considered here saves both depositors and shareholders of banks. This property might be inconsistent with banking regulations in practice, which typically intend to save only depositors and not shareholders when banks default (e.g., the deposit insurance system). One reason that we consider a policy that saves both is, of course, to keep our analysis simple. However, another reason is that during a time of crisis it is often difficult to design a policy that saves depositors selectively, particularly when policy needs to respond quickly. For example, both depositors and bank shareholders were saved by the Troubled-Asset Relief Program (TARP) from 2008–2014.

3 Equilibrium

We consider an equilibrium in which a crisis occurs due to a productivity shock. For a given government policy $n \in \{0, 1, \cdots, N-1, \emptyset\}$, the competitive equilibrium is defined in a standard manner. Households, firms, and banks solve their optimization problems (2), (6), and (14), respectively, and all markets clear.

Furthermore, we restrict our attention to the Markov equilibrium where all endogenous variables are written as functions of the current state of nature $s_t \in \Omega$.[16] In what follows, we use $s = (s_1, s_2, \cdots, s_N) \in \Omega$ to denote the state in the "current period," $s_- = (s_{1,-}, s_{2,-}, \cdots, s_{N,-}) \in \Omega$ to denote that in the "previous period," and $s' = (s'_1, s'_2, \cdots, s'_N) \in \Omega$ to denote that in the "next period." We denote the equilibrium value of a variable x at state s by $x(s)$. Furthermore, let $I(s)$ denote the node hit by the productivity shock for $s \in \Omega$.

The main focus of this study is to analyze "liquidity crises," by which we mean chain defaults of firms and banks mediated through credit networks. Bank failures reduce the supply of short-term loans (liquidity), which causes the defaults of firms. The defaults of these firms, in turn, lower the

[16] That is, we adopt the minimal state variable criterion (McCallum, 1983) for equilibrium selection.

revenues of banks, which leads them to default.

Reflecting this understanding, we say that there is a crisis in state $s \in \Omega$ if banks in some sectors default (i.e., if $\exists i \in \{1, \cdots, N\}$ such that $\tilde{\xi}_i^B(s) < 1$). For each sector i, we define two sets Ω_i^b and Ω_i^g as

$$\Omega_i^b = \{s \in \Omega \,|\, \tilde{\xi}_i^B(s) < 1\}, \tag{22}$$

$$\Omega_i^g = \{s \in \Omega \,|\, \tilde{\xi}_i^B(s) = 1\}, \tag{23}$$

that is, banks in sector i go bankrupt if and only if $s \in \Omega_i^b$. We also define Ω^b as

$$\Omega^b = \bigcup_{i=1}^{N} \Omega_i^b. \tag{24}$$

This variable is the set of states s where there are defaults of banks in some sectors. That is, according to our definition, a liquidity crisis occurs if and only if $s \in \Omega^b$. A state $s \in \Omega^b$ is called a crisis state.

3.1 Basic characterization of equilibrium

In this subsection, we characterize some basic features of the (Markov) equilibrium in our model, which are summarized in Lemmas 5 and 6.

First, observe that the inter-period rates of loans and deposits must be equal across all sectors:

$$R_i^L(s) = R^L(s), \qquad \forall i, \tag{25}$$

$$R_i^D(s) = R^D(s), \qquad \forall i. \tag{26}$$

This result follows from the fact that firms and banks in different sectors are identical in their first periods of life, when they obtain inter-period loans and deposits.

Then, the inter-period rates of loans and deposits are proportional:

$$R^L(s) = \Theta R^D(s). \tag{27}$$

Here $\Theta \geq 1$ is a constant defined by

$$\Theta \equiv \frac{\int_{\Omega_i^g} [c(s') + \underline{c}]^{-1} \, dF(s')}{\int_{\Omega_i^g} \tilde{\xi}_i^R(s') [c(s') + \underline{c}]^{-1} \, dF(s')}, \tag{28}$$

where $\tilde{\xi}_i^R(s)$ is the recovery rate of inter-period loans to firms adjusted for the policy intervention $\{T_i(s)\}$ (see equation 21), which is given by

$$\tilde{\xi}_i^R(s) \equiv \begin{cases} \tilde{\xi}_i^F(s), & \text{if } i \neq I(s)+n, \\ 1, & \text{if } i = I(s)+n. \end{cases}$$

A formal derivation of equation (27) is given in Appendix A.7. However, the intuition is simply explained. Equation (27) is derived from the profit maximization of banks. The cost of obtaining inter-period funds (deposits) for banks in state s is given by $R^D(s)$. Banks in each node i lend these funds to firms in the same node with the policy-adjusted recovery rate $\tilde{\xi}_i^R(s')$, which depends on the state in the next period, $s' \in \Omega$. With compensation for the risk of default of firms, the inter-period loan rate, $R^L(s)$, must be greater than or equal to the inter-period deposit rate, $R^D(s)$, that is, $\Theta \geq 1$. The fact that Θ is independent of the current state follows from our focus on the Markov equilibrium. The integrands of the right-hand side of equation (28) depend only on the next-period state s', and, hence, their integrals are independent of the current-period state s.

We start with the following result, which is useful for computing the equilibrium. We provide all proofs in Appendix.

Lemma 1. *The equilibrium values of the following products of variables, $R^D(s)q(s)$, $R^D(s)e(s)$, and $[c(s)+\underline{c}]R^D(s)$, are constant and independent of the past, current, and future states.*

The following lemma shows that banks that default do not provide short-term loans. Remember that Ω_i^b is the set of states where banks at node i default, as defined in (22).

Lemma 2. *Defaulting banks do not provide short-term loans: $W_{i+1}(s) = 0$ for all $s \in \Omega_i^b$ and $i \in \{1, \cdots, N\}$.*

Our next result plays a key role for chain defaults of firms and banks. It states that if banks at node i default, then firms at node $i+1$ default as well. These firms have to default, because, as shown in Lemma 2, they cannot obtain short-term loans from node-i banks and, thus, are unable to produce output.

Lemma 3. *The default of banks at node i leads to the default of firms at node $i+1$: $\tilde{\xi}_{i+1}^F(s) < 1$ for all $s \in \Omega_i^b$ and $i \in \{1, \cdots, N\}$.*

The next lemma shows that, as long as banks recover their long-term loans fully, they are solvent, even if they do not supply any short-term loans.

Lemma 4. *If* $\tilde{\xi}_i^F(s) = 1$, *then* $\tilde{\xi}_i^B(s) = 1$.

Lemmas 3-4 implies that defaults are propagated "forwards" but not "backwards" along the credit network, that is, the defaults of banks in node i lead to the defaults of firms in node $i+1$, but the defaults of firms in node i do not induce the defaults of banks in node $i-1$.

To generate liquidity crises, we need to make some assumptions on parameters. In particular, we need to rule out the case in which banks are solvent even when their long-term loans are defaulted on and the case in which firms with normal levels of productivity become insolvent even when the banks supplying short-term loans to them are solvent.

Assumption 1. We restrict the parameter values such that the following conditions are satisfied. (i) If $\tilde{\xi}_i^F(s) < 1$, then $\tilde{\xi}_i^B(s) < 1$, and (ii) if $i \neq I(s)$ and $\tilde{\xi}_{i-1}^B(s) = 1$, then $\tilde{\xi}_i^F(s) = 1$, where $I(s)$ denotes the node hit by the productivity shock.

Under Assumption 1 and using previous lemmas, we can characterize the set of states Ω as follows. As long as firms in the node hit by the productivity shock are solvent, there are no defaults. If these firms default, on the other hand, a chain default will occur, the extent of which is determined by the policy parameter n.

Lemma 5. *Let Assumption 1 hold. Then, for a fixed policy parameter* n, *there are two possible equilibrium outcomes for each state* $s \in \Omega$.

(i) No firms or banks default: $\tilde{\xi}_i^F(s) = \tilde{\xi}_i^B(s) = 1 \ \forall i$.

(ii) Firms in the node hit by the productivity shock default: $\tilde{\xi}_{I(s)}^F(s) < 1$. *If* $n \geq 1$, *chain defaults occur:* $\tilde{\xi}_i^F(s) < 1$ *for* $i = I, I+1, \cdots, I+n$ *(modulo* N*) and* $\tilde{\xi}_i^B(s) < 1$ *for* $i = I, I+1, \cdots, I+n-1$ *(modulo* N*). Firms and banks in the other nodes are solvent. If* $n = 0$, *the defaults of firms in node* $I(s)$ *do not cause any other defaults. In the laissez-faire case,* $n = \emptyset$, *all firms and banks default.*

The proof is straightforward and, thus, is omitted. Roughly, case (i) is given by Lemma 4 and Assumption 1, and case (ii) follows from Assumption 1 and Lemma 3.

Since our focus here is on a liquidity crisis, unless otherwise stated, we shall restrict the policy parameter to be $n \neq 0$ so that defaults of firms in node $I(s)$ cause defaults of some banks and firms in other nodes.[17] Then, Lemma 5 implies that the set of states Ω is divided into two disjoint subsets, Ω^g and Ω^b, where Ω^g is the set of states where (i) of Lemma 5 holds and Ω^b is those states associated with (ii). (This definition is consistent with our former definition of Ω^b in equation 24.)

It follows from Lemma 5 that

$$\Theta = 1.$$

To see this result, suppose that the productivity shock hits node I. In the states where banks I go bankrupt, both banks and firms in node i go bankrupt for $i = I, \cdots, I+n-1$ (modulo N), whereas both banks and firms survive in nodes $i = I+n+1, \cdots, I-1$ (modulo N). In node $I+n$, firms go bankrupt, but banks are bailed out by the policy in such a way that their long-term loans are fully recovered (i.e., $\tilde{\xi}^R_{I+n} = 1$). Thus, equation (28) implies that $\Theta = 1$.

Another implication of Lemma 5 is given in the next lemma, which says that the level of aggregate output is identical across crisis states $s \in \Omega^b$.

Lemma 6. *Let Assumption 1 hold. Then, for each policy parameter* $n \in \{1, \cdots, N-1, \emptyset\}$, *there exists a constant* $c^b > 0$ *such that*

$$c(s) = c^b, \qquad \forall s \in \Omega^b.$$

3.2 Detailed description of equilibrium

In this subsection, we formally derive the equilibrium conditions.

Remember that the distribution of s'_i conditional on the current state s is identical across sectors and independent of s, as shown in (18). It follows that inter-temporal choices of capital are identical across sectors (i.e., $k_i(s) = k(s)$ for all i and s). Similarly, since all banks are identical in their first period of life, $d^L_i(s) = d^L(s)$ and $e_i(s) = e(s)$ for all i and s.

Since the total supply of capital is N, and the measures of households, firms i, and banks i are all equal to unity, the market clearing conditions for the capital stock, managerial inputs, loans,

[17] It is straightforward to include the case $n = 0$ in our argument below, with slight complication of notation. For instance, when $n = 0$, there are no states where banks default, so $\Omega^b = \emptyset$ according to our definition (24). Thus, Ω is no longer written as $\Omega^g \cup \Omega^b$ (as long as Ω^g denotes the set of states without any defaults of firms and banks). We would need to introduce another set of states where only firms in node $I(s)$ default. Thus, our restriction to the case where $n \neq 0$ is for the sake of notational simplicity as well.

deposits, labor, and consumption are given as

$$k_i(s) = m_i(s) = 1, \quad d_i^L(s) = d^L(s), \quad e_i(s) = e(s),$$
$$L_i(s) = q(s) = d^L(s) + e(s),$$
$$W_{i+1}(s) = d_i^S(s), \quad l(s) = \sum_{i=1}^{N} l_i(s),$$

and

$$c(s) = \sum_{i=1}^{N} s_i l_i(s)^{1-\alpha} \tag{29}$$

for all $s \in \Omega$.

We derive the equilibrium conditions in two steps. First, given the equilibrium values of $R^D(s_-)q(s_-)$ and $R^D(s_-)e(s_-)$, the conditions for the "intra-temporal variables," $\{w(s), l_i(s), \tilde{\xi}_i^F(s), \tilde{\xi}_i^B(s), R_i^F(s), R_i^B(s)\}$, for $i = 1, 2, \cdots, N$, are derived in Section 3.2.1. Then, the equilibrium conditions for the "inter-temporal variables," $R^D(s)$, $R^E(s)$, $q(s)$, and $e(s)$, are obtained in Section 3.2.2.

3.2.1 Intra-temporal conditions for Markov equilibrium

Here, the intra-temporal variables $\{w(s), l_i(s), \tilde{\xi}_i^F(s), \tilde{\xi}_i^B(s), R_i^F(s), R_i^B(s)\}$, for $i = 1, 2, \cdots, N$, are solved for each s. In this subsection, we fix a state $s \in \Omega$ and take the values $R^D(s_-)q(s_-)$ and $R^D(s_-)e(s_-)$ as already determined.

The utility maximization of the representative household implies that for all $s \in \Omega$,

$$w(s) = \frac{\gamma[c(s) + \underline{c}]}{1 - l(s)}, \tag{30}$$

where $c(s)$ is given by (29).

Consider banks in node i and firms in node $i+1$. We consider separately the states where banks i are solvent and short-term loans are provided to sector $i+1$ (Case 1) and the states where banks i are insolvent and short-term loans are not provided to sector $i+1$ (Case 2).

Case 1: Short-term loans are provided to firms $i+1$. Suppose that $W_{i+1}(s) > 0$. It follows from Lemma 2 and condition (17) that

$$\tilde{\xi}_i^B(s) = 1, \qquad R_i^B(s) = 1. \tag{31}$$

The fact that firms $i+1$ hire labor in state s ($W_{i+1}(s) > 0$) implies that they are solvent (see equation 9):

$$\tilde{\xi}^F_{i+1}(s) = 1. \tag{32}$$

Then, the profit maximization problem of the firms (6) yields the first-order conditions with respect to l_{i+1}, which can be solved as

$$l_{i+1}(s) = \left(\frac{(1-\alpha)s_{i+1}}{R^F_{i+1}(s)w(s)} \right)^{\frac{1}{\alpha}}. \tag{33}$$

The moral hazard constraint of banks i, (11), implies that

$$W_{i+1}(s) = w(s)l_{i+1}(s) \leq \frac{[\tilde{\xi}^F_i(s) - 1]R^D(s_-)q(s_-) + R^D(s_-)e(s_-) + T_i(s)}{1 - (1-\psi)R^F_{i+1}(s)}. \tag{34}$$

The denominator of the right-hand side of (34) is positive, as shown in the following lemma.

Lemma 7. *If $W_{i+1}(s) > 0$, then $R^F_{i+1}(s)$ satisfies $1 \leq R^F_{i+1}(s) < \frac{1}{1-\psi}$.*

When the moral hazard constraint is non-binding, $R^F_{i+1}(s) = 1$. Thus, the following condition must be satisfied in equilibrium.

$$R^F_{i+1}(s) = \max\left\{ 1, \frac{1}{1-\psi}\left[1 - \frac{[\tilde{\xi}^F_i(s) - 1]R^D(s_-)q(s_-) + R^D(s_-)e(s_-) + T_i(s)}{w(s)l_{i+1}(s)} \right] \right\} \tag{35}$$

Note that $R^F_{i+1}(s)$ in (35) satisfies $R^F_{i+1}(s) \geq R^B_i(s) = 1$.

To be consistent with our presumption that $W_{i+1}(s) > 0$, the variables derived above, $\{l_{i+1}(s)$, $\tilde{\xi}^F_{i+1}(s)$, $R^F_{i+1}(s)$, $\tilde{\xi}^B_i(s)$, $R^B_i(s)\}$, must satisfy

$$\xi^F_{i+1}(s) \geq 1, \quad \text{and} \quad \xi^B_i(s) \geq 1, \tag{36}$$

where

$$\xi^F_{i+1}(s) = \frac{s_{i+1}l_{i+1}(s)^{1-\alpha} - R^F_{i+1}(s)w(s)l_{i+1}(s) + q(s)}{R^D(s_-)q(s_-)}, \tag{37}$$

$$\xi^B_i(s) = \frac{T_i(s) + \tilde{\xi}^F_i(s)R^D(s_-)q(s_-) + (1-\psi)\tilde{\xi}^F_{i+1}(s)R^F_{i+1}(s)w(s)l_{i+1}(s)}{W_{i+1}(s) + R^D(s_-)[q(s_-) - e(s_-)]}. \tag{38}$$

If the conditions in (36) are satisfied, the values $\{l_{i+1}(s), \tilde{\xi}^F_{i+1}(s), R^F_{i+1}(s), \tilde{\xi}^B_i(s), R^B_i(s)\}$ obtained by solving equations (31), (32), (33), and (35) indeed constitute a Markov equilibrium at state s. Otherwise, we should consider Case 2 in the next paragraph.

Case 2: Short-term loans are not provided to firms $i+1$. In the above calculations, if some of the conditions in (36) are violated, then short-term loans are not supplied to firms in node $i+1$, and their output is zero. Consequently, $\tilde{\xi}_i^B(s)$, $\tilde{\xi}_{i+1}^F(s)$, $l_{i+1}(s)$, and $W_{i+1}(s)$ are given by

$$\tilde{\xi}_i^B(s) = \min\left\{ \frac{T_i(s) + \tilde{\xi}_i^F(s) R^D(s_-) q(s_-)}{R^D(s_-)\{q(s_-) - e(s_-)\}},\ 1 \right\}, \tag{39}$$

$$\tilde{\xi}_{i+1}^F(s) = \frac{q(s)}{R^D(s_-) q(s_-)}, \tag{40}$$

$$l_{i+1}(s) = 0, \qquad W_{i+1}(s) = 0. \tag{41}$$

The rates for intra-temporal loans $\{R_i^B(s), R_{i+1}^F(s)\}$ are not uniquely determined, but they can take any values as long as the following conditions are satisfied:

$$\tilde{\xi}_i^B(s) R_i^B(s) \leq 1, \quad \text{and} \quad \tilde{\xi}_{i+1}^F(s) R_{i+1}^F(s) \leq R_i^B(s).$$

3.2.2 Inter-temporal conditions for Markov equilibrium

Here, we derive the equilibrium conditions for the variables $\{R^D(s), \tilde{R}_i^E(s), q(s), e(s)\}$ for each state $s \in \Omega$, which involves inter-temporal optimization.

Let us start with the condition for $q(s)$. Consider the profit maximization problem of firms, (7). Then, for each $s \in \Omega$ and $i \in \{1, \cdots, N\}$, the first-order condition with respect to $k_i(s)$ is given by

$$\int_{s' \in \Omega_i^f} \frac{1}{[c(s') + \underline{c}]} \, dF(s') R^D(s) q(s) = \int_{s' \in \Omega_i^f} \frac{1}{[c(s') + \underline{c}]} \{r_i(s') + q(s')\} \, dF(s'), \tag{42}$$

where Ω_i^f is the set of states where firms i are solvent and $r_i(s)$ is the return to capital:

$$\Omega_i^f \equiv \{s \in \Omega \,|\, \tilde{\xi}_i^F(s) = 1\},$$

$$r_i(s) \equiv \begin{cases} \left(\frac{\alpha - \nu}{1-\alpha}\right) \frac{[(1-\alpha)s_i]^{\frac{1}{\alpha}}}{[R_i^F(s) w(s)]^{\frac{1-\alpha}{\alpha}}}, & \text{if } W_i(s) > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{43}$$

A formal derivation of equations (42) and (43) is given in Appendix A.8. Note that equation (42) requires that the product $R^D(s) q(s)$ be identical for all $s \in \Omega$ in Markov equilibrium.

Next, let us turn to the equilibrium conditions for the return on bank equity, $\tilde{R}_i^E(s)$, and the inter-temporal deposit rate, $R^D(s)$, which are derived from the household's first-order conditions (3). Let us write the stochastic discount factor in (3) as $\lambda(s, s')$, so that

$$\lambda(s, s') = \beta \frac{c(s) + \underline{c}}{c(s') + \underline{c}}.$$

Then, (3) implies that

$$E\Big[\lambda(s,s')\tilde{\xi}_i^B(s')R^D(s)\,\big|\,s\Big] = E\Big[\lambda(s,s')\tilde{R}_i^E(s')\,\big|\,s\Big] = 1. \tag{44}$$

From Lemma 2 and equation (12), the realized profit of a bank in node i in each state $s' \in \Omega$ can be expressed as

$$\pi_i^B(s') = \begin{cases} \hat{\pi}_i^B(s'), & \text{for } s' \in \Omega_i^g, \\ 0, & \text{for } s' \in \Omega_i^b, \end{cases}$$

where

$$\hat{\pi}_i^B(s') = \Psi_i(s')\Lambda_i(s'),$$

and

$$\Lambda_i(s') \equiv \left\{\Big[\tilde{\xi}_i^F(s') - 1\Big]\,R^D(s)q(s) + T_i(s') + R^D(s)e(s)\right\},$$
$$\Psi_i(s') \equiv \begin{cases} \frac{\psi R_{i+1}^F(s')}{1-(1-\psi)R_{i+1}^F(s')}, & \text{if } W_{i+1}(s') > 0, \\ 1, & \text{if } W_{i+1}(s') = 0. \end{cases}$$

Here, $\Lambda_i(s')$ is written as a function of s' only, because, as stated in Lemma 1, the products $R^D(s)q(s)$ and $R^D(s)e(s)$ take constant values (independent of s).

Then, the rate of return on bank equity, realized at state $s' \in \Omega$, is given by

$$\tilde{R}_i^E(s') = \frac{\pi_i^B(s')}{e(s)} = \begin{cases} \frac{\Psi_i(s')\Lambda_i(s')}{R^D(s)e(s)}R^D(s), & \text{for } s' \in \Omega_i^g, \\ 0, & \text{for } s' \in \Omega_i^b, \end{cases}$$

Thus, the expected value of the return on bank equity becomes

$$E\Big[\lambda(s,s')\tilde{R}_i^E(s')\,\big|\,s\Big] = \beta\left\{\int_{s'\in\Omega_i^g} \frac{[c(s)+\underline{c}]}{[c(s')+\underline{c}]}\frac{\Psi_i(s')\Lambda_i(s')}{R^D(s)e(s)}\,dF(s')\right\}R^D(s). \tag{45}$$

Similarly, the expected value of the inter-termporal deposit rate is given by

$$\begin{aligned} &E\Big[\lambda(s,s')\tilde{\xi}_i^B(s')R^D(s)\,\big|\,s\Big] \\ &\quad= \beta\left\{\int_{s'\in\Omega_i^g} \frac{[c(s)+\underline{c}]}{[c(s')+\underline{c}]}\,dF(s') + \int_{s'\in\Omega_i^b} \frac{[c(s)+\underline{c}]}{[c(s')+\underline{c}]}\tilde{\xi}_i^B(s')\,dF(s')\right\}R^D(s). \end{aligned} \tag{46}$$

Using (45) and (46), the first equation in (44) can be rewritten as

$$\begin{aligned} &\int_{s'\in\Omega_i^g} \frac{1}{[c(s')+\underline{c}]}\Psi_i(s')\frac{\big[\tilde{\xi}_i^F(s')-1\big]R^D(s)q(s)+T_i(s')+R^D(s)e(s)}{R^D(s)e(s)}\,dF(s') \\ &\quad= \int_{s'\in\Omega_i^g} \frac{1}{[c(s')+\underline{c}]}\,dF(s') + \int_{s'\in\Omega_i^b} \frac{1}{[c(s')+\underline{c}]}\xi_i^B(s')\,dF(s'), \end{aligned} \tag{47}$$

and the second equation as

$$\beta\left\{\int_{s'\in\Omega_i^g}\frac{1}{[c(s')+\underline{c}]}\,dF(s')+\int_{s'\in\Omega_i^b}\frac{1}{[c(s')+\underline{c}]}\xi_i^B(s')\,dF(s')\right\}[c(s)+\underline{c}]R^D(s)=1. \tag{48}$$

Note here that, given that $R^D(s)q(s)$ is constant, equation (47) implies that $R^D(s)e(s)$ is constant, and equation (48) implies that $[c(s)+\underline{c}]R^D(s)$ is constant in our Markov equilibrium.

The description of the equilibrium conditions is now complete. To summarize, the Markov equilibrium in our economy is given by a collection of (positive) functions, namely, $\{c(s)$, $l_i(s)$, $w(s)$, $e(s)$, $r(s)$, $q(s)$, $R^D(s)$, $R_i^B(s)$, $R_i^F(s)$, $\tilde{\xi}_i^F(s)$, and $\tilde{\xi}_i^B(s)\}$ that satisfy (29), (30), (31) or (39), (32) or (40), (33) or (41), (35), (42), (43), (47), and (48).

3.3 Numerical results

In this subsection, we report the results of our numerical experiments, and we argue that the predictions of our model are consistent with the recent financial crisis (the Great Recession) in some important respects.

Remember that state $s\in\Omega$ in our model takes the form of $s^{(i)}(z)$ in (19), which denotes that firms in node i are hit by a productivity shock $z\in[0,s_{\max}]$. We conjecture that the crisis states, Ω^b, are characterized by a threshold value of productivity $\underline{z}\in[0,s_{\max}]$, defined by

$$\xi_i^F\left(s^{(i)}(\underline{z})\right)=1,$$

where $\xi_i^F(s)$ is defined in (37). Using $\underline{z}$, the sets of states Ω^g and Ω^b are described as

$$\Omega^g=\{s\in\Omega\,|\,z(s)\geq\underline{z}\},\quad\text{and}\quad\Omega^b=\{s\in\Omega\,|\,z(s)<\underline{z}\},$$

where $z(s)\equiv s_{I(s)}$, i.e., the productivity in node $I(s)$, and $I(s)$ denotes the node hit by the productivity shock at state s. This conjecture is verified in all of the numerical exercises conducted here.

When the productivity shock is greater than $\underline{z}$, the equilibrium outcome is as described by case (i) of Lemma 5, and $c(s)$ varies with the value of $z(s)$. When it is less than $\underline{z}$, case (ii) of Lemma 5 occurs, and $c(s)=c^b$, which is independent of $z(s)$.

Let $c^g_{\min}$ be the minimum value of output in those states without defaults:

$$c^g_{\min}\equiv\min_{s\in\Omega^g}c(s).$$

This is the amount of output when the productivity shock is equal to the threshold value $\underline{z}$. The difference between c^b and $c^g_{\min}$ can be sizable (depending on parameter values). In that case, a small change in the productivity shock from slightly above $\underline{z}$ to below $\underline{z}$ generates a sharp and large drop of aggregate output from $c^g_{\min}$ to c^b.

The parameter values are set as $\beta = 0.96$, $\nu = 0.05$, $\alpha = 0.3$, $\gamma = 0.6533$, $\underline{c} = 1.3609$, $\psi = 0.1$, and $N = 12$.[18] The productivity shock $\ln(z)$ is assumed to follow a normal distribution with mean 0 and standard deviation 0.02.

For the policy parameter, we consider two cases: $n = 1$ and $n = \emptyset$. The policy with $n = \emptyset$ is the laissez-faire policy, which does not allow for government intervention at all. With this policy, once a crisis occurs, all firms and banks go bankrupt, and total output goes down to $\underline{c}$. With $n = 1$, on the other hand, the government responds to the crisis rather quickly. The failure of banks occurs only in one node (and that of firms occurs in two nodes). We roughly interpret the case of $n = 1$ as corresponding to the U.S. experience of the Great Recession. We consider the laissez-faire case $n = \emptyset$ as a counterfactual case that might have happened in the absence of a policy intervention. We set the value of $\underline{c}$ so that the decline in GDP is comparable to that during the Great Depression in such a case (about 40 percent according to, for instance, Cole and Ohanian, 1999).

For the numerical simulation, we assume the following realization of the productivity shock:

$$z_t = \begin{cases} 1 & \text{for } t \neq 2009, \\ z \ (< \underline{z}) & \text{for } t = 2009. \end{cases} \tag{49}$$

Thus, a liquidity crisis occurs in 2009. Since our model does not have endogenous state variables and the exogenous shock is i.i.d. across periods, the crisis does not have any persistent effects. In equation (49), the threshold value $\underline{z}$ depends on the policy parameter n. Note that we do not need to specify the exact value of z for 2009, as long as $z < \underline{z}$, because of Lemma 6.

Let us start with the laissez-faire case: $n = \emptyset$. In this case, the threshold is given by $\underline{z} = 0.9575$ and $\Pr(z < \underline{z}) = G(\underline{z}) = 0.015$. That is, a crisis would occur, on average, once in 67 years. With our parameterization, it results in about a 40 percent drop in total output, $y_t = c_t + \underline{c}$, when z_t changes from one to below $\underline{z}$. As illustrated in Figure 2, the predicted fall in output is far greater than the actual decline in detrended output during the Great Recession.

Next, consider the case where $n = 1$. In this case, the threshold value of productivity is given

[18] One period in this model corresponds to a year. γ is chosen such that the labor supply in the deterministic steady state equals 0.3. $\underline{c}$ is chosen such that $\underline{c}$ is 60 percent of the total consumption in the steady state.

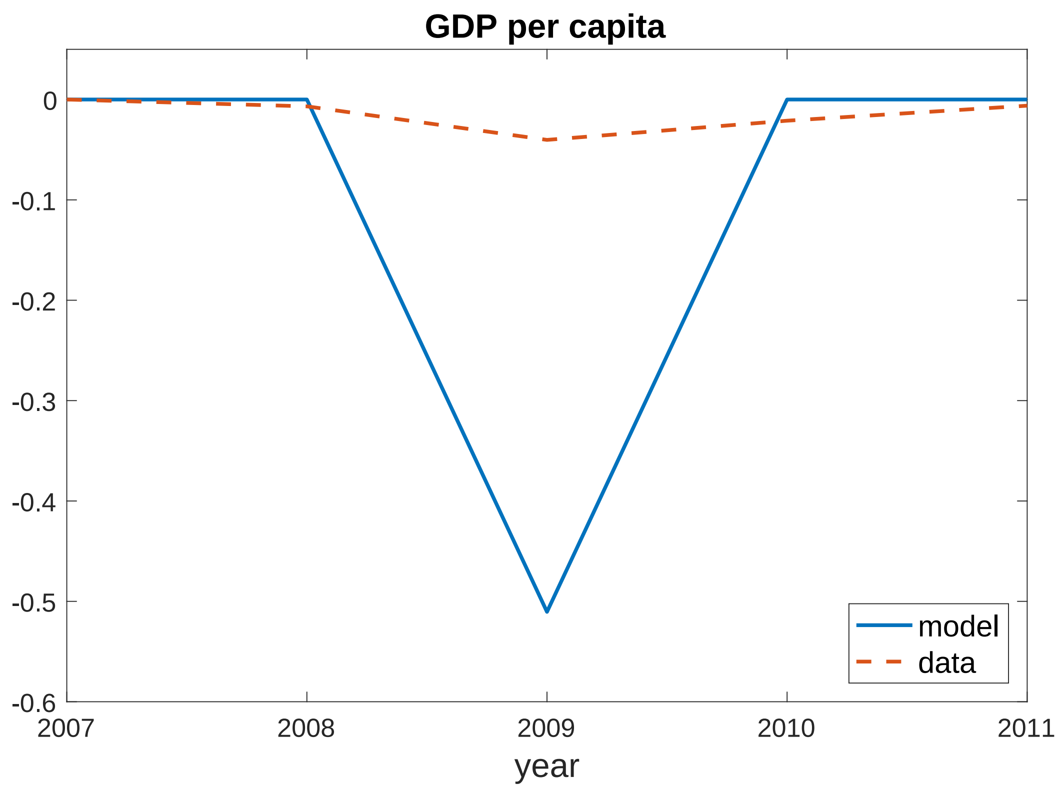

Figure 2: Output under the laissez-faire policy: $n = \emptyset$. We use the data sets provided by Karabarkounis (2014) and Cocius, Prescott, and Uederfeldt (2009). The data on real GDP is Hodrick-Prescott filtered with the smoothing parameter 6.25. Its value in 2007 is normalized to zero. Both series are logged.

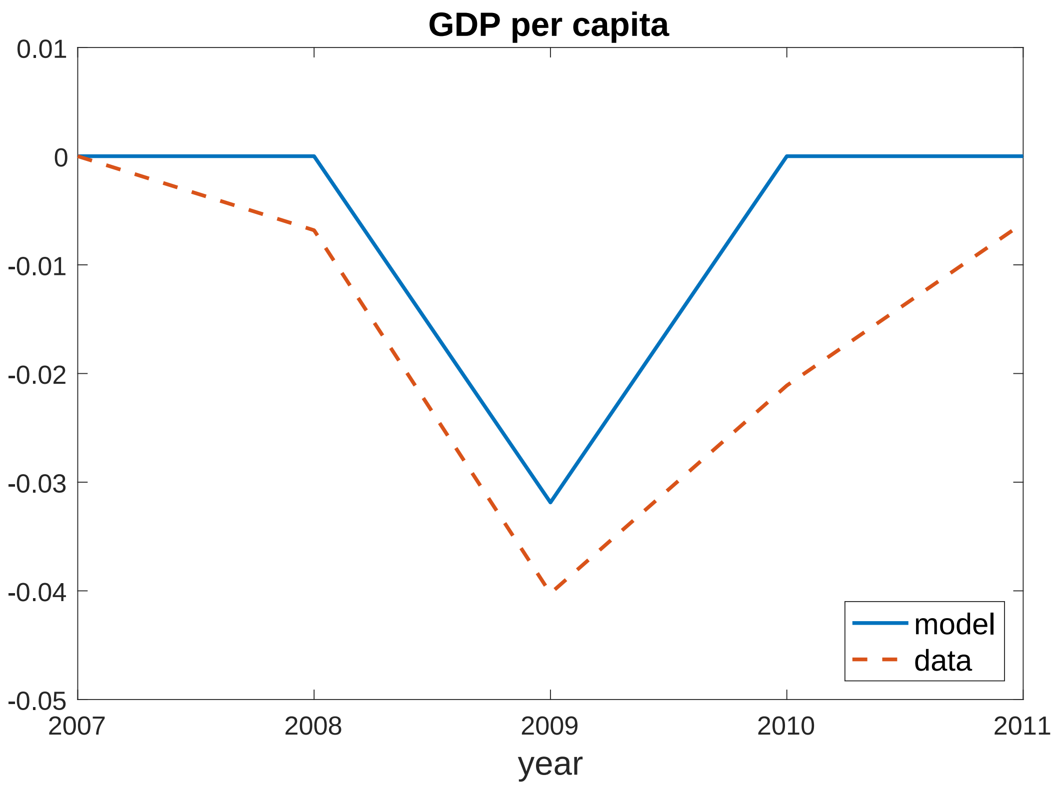

Figure 3: Output under the benchmark policy: $n = 1$. We use the data sets provided by Karabarkounis (2014) and Cocius, Prescott, and Uederfeldt (2009). The data on real GDP is Hodrick-Prescott filtered with the smoothing parameter 6.25. Its value in 2007 is normalized to zero. Both series are logged.

by $\underline{z} = 0.9564$ with $G(\underline{z}) = 0.013$. Thus, a crisis occurs, on average, once in 77 years. With $n = 1$, policy "quickly" responds in order to limit the crisis so that banks (firms) default only in one (two) node(s). As a result, the decline in output during the crisis is much smaller, about 3.2 percent. Figure 3 compares the model's predicted path of output and the actual path of GDP per person. The data on GDP are detrended using the Hodrick-Prescott (HP) filter, and its value in 2007 is normalized to zero. The smoothing parameter for the HP filter is set to 6.25, following Ravn and Uhlig (2002). The figure shows that the detrended level of GDP in 2009 is about four percent lower than the 2007 level in the data, which is similar to the prediction of the model. However, this result is just a direct consequence of our choice of parameter values, in particular, n and N.

To examine its applicability, let us see some other predictions of the model that are not targeted in the calibration. Specifically, motivated by the business cycle accounting approach developed by Chari, Kehoe and McGrattan (2007), we look at the efficiency and labor wedges. The efficiency wedge is the measured TFP, which is given in our model by $\ln y_t - (1-\alpha)\ln l_t$, where $y_t \equiv c_t + \underline{c}$. The labor wedge is defined as the gap between the marginal rate of substitution of consumption for leisure and the marginal product of labor, which is given in our model by $\ln\left[\frac{\gamma y_t}{1-l_t}\right] - \ln\left[(1-\alpha)\frac{y_t}{l_t}\right]$.

Figures 4 and 5 plot the paths of the TFP and labor wedge predicted by the model and estimated from the data. The data on both the TFP and the labor wedge are detrended by the HP filter, and their values in 2007 are normalized to zero. As argued in, for instance, Brinca, Chari, Kehoe, and McGrattan (2016), during this period, the fluctuations in TFP are much smaller than those in the labor wedge. These two figures show that the predictions of the model are consistent with the data. The TFP in 2009 is 0.57 percent lower than the 2007 level in the data, whereas it is 0.49 percent lower in the model's prediction. The labor wedge in 2009 is 5.54 percent worse than the 2007 level in the data, whereas it is 5.46 percent worse in the model's prediction.

The reason that our model generates a relatively large deterioration in the labor wedge and a small decline in TFP is simple. First, a crisis occurs in our model because firms are unable to obtain liquidity. Concerning productivity, firms in all nodes but one have the normal level of productivity even during a crisis. Thus, we do not need a large decline in the aggregate TFP to trigger a crisis. Second, during a crisis, firms without liquidity reduce employment and stop operating. Thus, a liquidity crisis is directly translated to deterioration in the labor wedge in the macro data.

4 Conclusion

In this study, we have developed a model of liquidity crises with the following features. First, liquidity is essential for the operation of firms. Second, both banks and firms have long-term debt, which may cause debt overhang and obstruct the flow of short-term funds (liquidity). Firms without enough liquidity have to default. Third, the lending market is segmented and represented by a credit network. Then, the defaults of firms and/or banks in one sector may cause chain defaults propagated through the credit network, resulting in a systemic financial (liquidity) crisis.

As argued, for instance, by Brinca, Chari, Kehoe, and McGrattan (2016), the labor wedge has played a much more important role than the efficiency wedge (TFP) in the Great Recession. This observation is consistent with our model. During a crisis, most firms have a normal level of productivity, so that the decline in the aggregate TFP remains small. In addition, defaulting firms

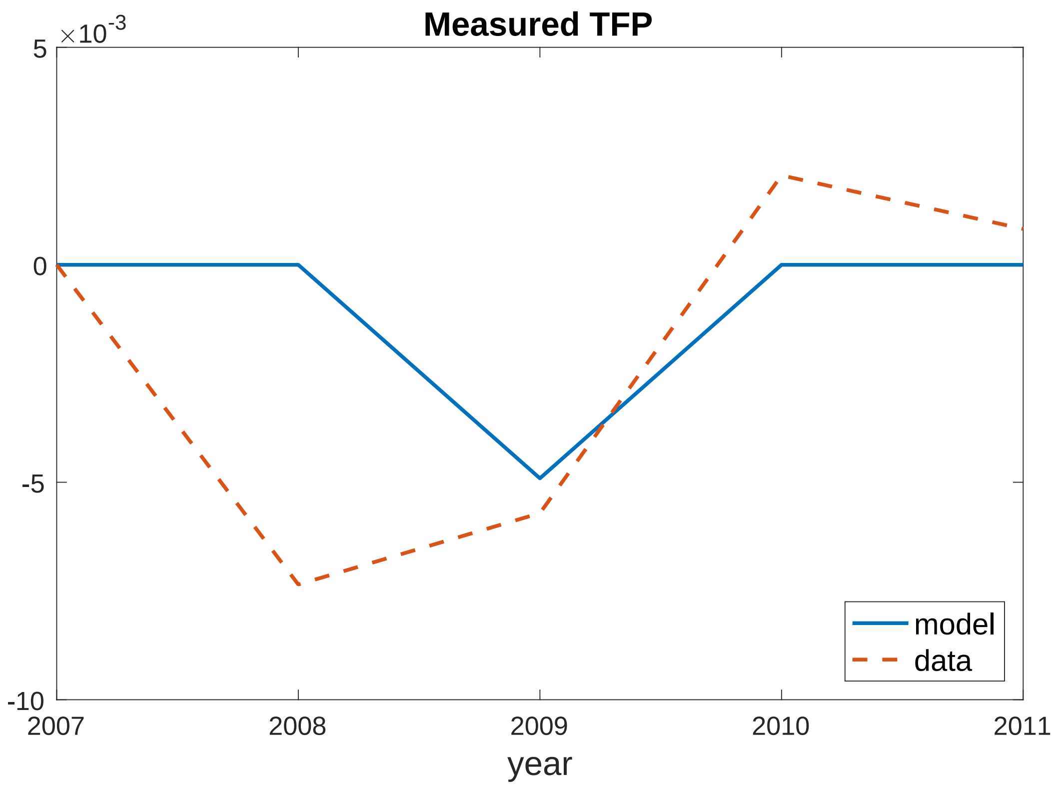

Figure 4: TFP under the benchmark policy: $n = 1$. We use the data sets provided by Karabarkounis (2014) and Cocius, Prescott, and Uederfeldt (2009). The data on TFP is Hodrick-Prescott filtered with the smoothing parameter 6.25. Its value in 2007 is normalized to zero. Both series are logged.

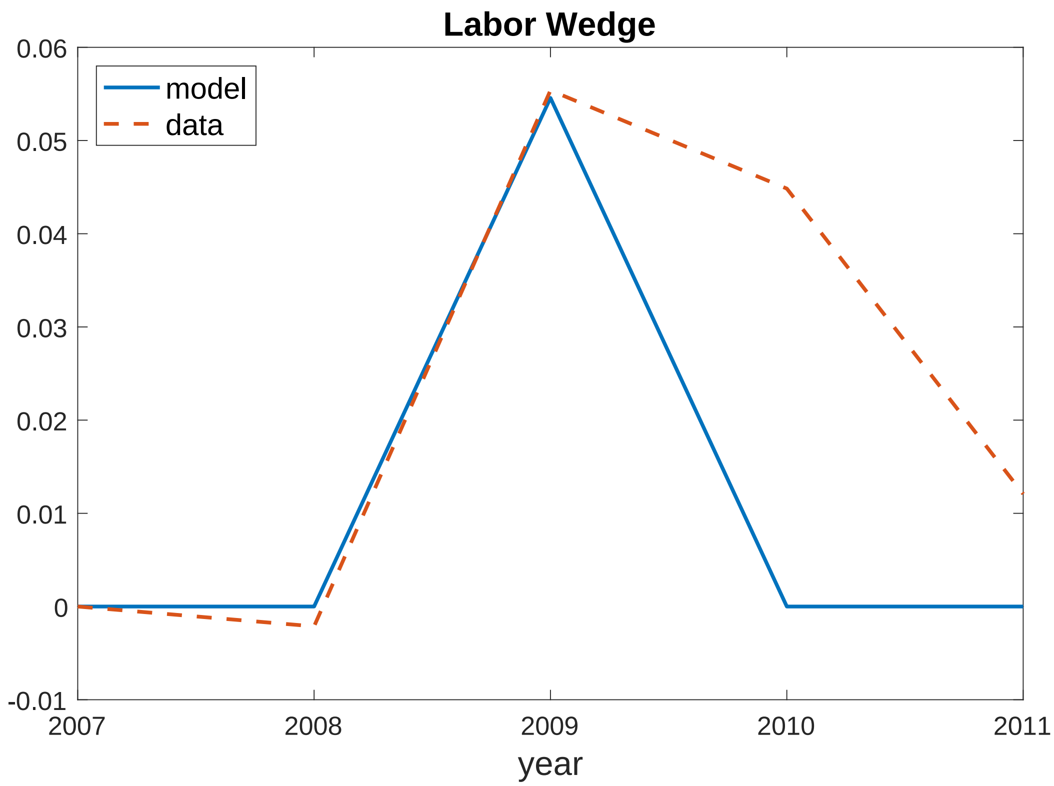

Figure 5: Labor wedge under the benchmark policy: $n = 1$. We use the data sets provided by Karabarkounis (2014) and Cocius, Prescott, and Uederfeldt (2009). The data on the labor wedge is Hodrick-Prescott filtered with the smoothing parameter 6.25. Its value in 2007 is normalized to zero. Both series are logged.

stop producing, which causes a deterioration of the labor wedge. We also illustrate that our model reproduces quantitatively the fluctuations in output, TFP, and the labor wedge during the Great Recession.

For the sake of transparency and tractability, we have deliberately kept our model as simple as possible. It should be extended in several dimensions for a further understanding of financial crises. Here, we discuss some of the potential extensions, which are left for future research.

First, our specification of government policy may be too simplistic. In particular, government intervention during a crisis has only benefits without any costs. To conduct a meaningful normative analysis, we would need both benefits and costs.

Second, we have assumed the simplest form of credit network here. However, as discussed, for instance, in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), the form of network affects the stability of the financial system.

Third, the effect of a financial crisis is purely temporary in our model. However, historical evidence, such as Reinhart and Rogoff (2009, 2014), shows the opposite: a financial crisis has a very persistent effect. Persistence can be introduced in our model in different ways. For instance, we should allow firms and banks to live more than two periods. To do so, we could follow the approach of Gertler and Kiyotaki (2015), or alternatively, it may be useful to adopt the framework of Lagos and Wright (2005).

Lastly but not least, we have chosen a reduced-form approach to financial contracts. For instance, we have assumed that all financial contracts are in the form of risky debt, financial intermediation by banks is necessary, firms need to borrow short-term funds to pay for their workers, and short-term debt is not senior to long-term debt. These assumptions are not derived from first principles. It would be desirable to extend the model so that these features arise endogenously as an equilibrium outcome.

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A Appendix

A.1 Proof of Lemma 1

In a Markov equilibrium, equations (42), (47), and (48) should be satisfied for all s, s', s_-. These equations imply that $R^D(s)q(s)$, $R^D(s)e(s)$, $[c(s) + \underline{c}]R^D(s)$ are constant.

A.2 Proof of Lemma 2

Suppose that $W_{i+1}(s) > 0$. Then, the assumption that $\tilde{\xi}_i^B(s) < 1$ in equilibrium implies that condition (11) is violated. Thus, $W_{i+1}(s)$ cannot be positive. Thus, $W_{i+1}(s) = 0$. The definition (10), together with the fact that $\tilde{\xi}_i^B(s) < 1$ and $W_{i+1}(s) = 0$, implies that $\hat{\pi}_i^B(s) < 0$.

A.3 Proof of Lemma 3

Since $\tilde{\xi}_i^B(s) < 1$, $\hat{\pi}_i^B(s) < 0$, and $W_{i+1}(s) = 0$. Thus,

$$\hat{\pi}_i^B(s) = T_i(s) + q(s) - R^D(s_-)[q(s_-) - e(s_-)] < 0,$$

and, hence,

$$q(s) < R^D(s_-)[q(s_-) - e(s_-)] - T_i(s).$$

Now,

$$\begin{aligned}
\hat{\pi}_{i+1}^F(s) &= q(s) - \Theta R^D(s_-)q(s_-) \\
&< R^D(s_-)[q(s_-) - e(s_-)] - \Theta R^D(s_-)q(s_-) - T_i(s) \\
&= -(\Theta - 1)R^D(s_-)q(s_-) - R^D(s_-)e(s_-) - T_i(s) \\
&< 0.
\end{aligned}$$

Thus, $\hat{\pi}_{i+1}^F(s) < 0$, and, hence, $\tilde{\xi}_{i+1}^F(s) < 1$.

A.4 Proof of Lemma 4

$\tilde{\xi}_{i+1}^F(s) < 1$ implies that $W_{i+1}(s) = 0$ due to (9). As (28) shows, $\Theta \geq 1$. $\tilde{\xi}_i^F(s) = 1$ and $\Theta \geq 1$ imply that

$$\begin{aligned}
\hat{\pi}_i^B(s) &= T_i(s) + \Theta R^D(s_-)q(s_-) - [R^D(s_-)q(s_-) - R^D(s_-)e(s_-)] \\
&= T_i(s) + (\Theta - 1)R^D(s_-)q(s_-) + R^D(s_-)e(s_-) > 0,
\end{aligned}$$

which means that $\tilde{\xi}_i^B(s) = 1$.

A.5 Proof of Lemma 6

Suppose that the productivity shock, z, hits node I. As long as $s \in \Omega^b$, it is the case that $l_I(s) = 0$, which is independent of the value of z. Hence, the aggregate consumption $c(s)$ and labor $l(s)$ are both independent of the value of z.

A.6 Proof of Lemma 7

When $W_{i+1}(s) > 0$, Lemma 2 implies that $\tilde{\xi}_i^B(s) = 1$, and (9) implies that $\tilde{\xi}_{i+1}^F(s) = 1$. In this case, the households' optimization implies that $R_i^B(s) = 1$. Thus, the bank profit is written as

$$\hat{\pi}_i^B(s) = T_i(s) + \tilde{\xi}_i^F(s)\Theta R^D(s_-)q(s_-) - R^D(s_-)[q(s_-) - e(s_-)] + [R_{i+1}^F(s) - R_{i+1}^B(s)]W_{i+1}(s).$$

In order for the bank to choose a nonnegative $W_{i+1}(s)$, it must be the case that $R_{i+1}^F(s) \geq R_i^B(s) = 1$. The moral hazard constraint is written as

$$[R_i^B(s) - (1-\psi)R_{i+1}^F(s)]W_{i+1}(s) \leq T_i(s) + \tilde{\xi}_i^F(s)\Theta R^D(s_-)q(s_-) - R^D(s_-)[q(s_-) - e(s_-)].$$

If $R_{i+1}^F(s) \geq \frac{1}{1-\psi}R_i^B(s)$, then the bank would choose to make $W_{i+1}(s)$ infinitely large, which is not possible in equilibrium. Thus, it must be the case that $R_{i+1}^F(s) < \frac{1}{1-\psi}R_i^B(s) = \frac{1}{1-\psi}$.

A.7 Proof of equation (27)

Consider a bank born in period $t-1$ and an arbitrary state in period $t-1$, $s_{t-1} = s_- \in \Omega$. We write s_t as $s \in \Omega$. Given $(e_{t-1}, L_{t-1}) = (e, L)$, define the following:

$$\begin{aligned}
\Gamma_i(e, L, s, s_-) &\equiv \{0\} \cup \{W > 0 : \text{ Condition (50) is satisfied.}\}, \\
\pi_i^B(e, L, W, s, s_-) &\equiv \max\left\{\hat{\pi}_i^B(e, L, W, s, s_-),\ 0\right\}, \\
\hat{\pi}_i^B(e, L, W, s, s_-) &\equiv T_i(s) + \tilde{\xi}_i^F(s)R^L(s_-)L + \tilde{\xi}_{i+1}^F(s)R_{i+1}^F(s)W \\
&\qquad - \left[R^D(s_-)L + R_i^B(s)W - R^D(s_-)e\right], \\
\Omega_i^{B+}(e, L, s_-) &\equiv \left\{s \in \Omega : \max_{W \in \Gamma_i(e,L,s,s_-)} \pi_i^B(e, L, W, s, s_-) > 0\right\},
\end{aligned}$$

where the condition is given by

$$T_i(s) + \tilde{\xi}_i^F(s) R^L(s_-) L + (1-\psi)\tilde{\xi}_{i+1}^F(s) R_{i+1}^F(s) W \\ \geq \left[R_i^B(s) W + R^D(s_-) L - R^D(s_-) e \right]. \tag{50}$$

Then, the maximand in (14) can be expressed as

$$\begin{aligned} \tilde{\pi}_i^B(e, L|s_-) &\equiv E\left[\lambda(s_-, s) \max_{W \in \Gamma(e,L,s,s_-)} \pi_i^B(e, L, W, s, s_-) \middle| s_- \right] \\ &= \int_{\Omega_i^{B+}(e,L,s_-)} \lambda(s_-, s) \max_{W \in \Gamma_i(e,L,s,s_-)} \pi_i^B(e, L, W, s, s_-)\, dF(s). \end{aligned}$$

Now, consider profit maximization with respect to L. Note first that in equilibrium, $\Omega_i^{B+}(e, L, s_-) = \Omega_i^g$. Second, on the boundary of $\Omega_i^{B+}(e, L, s_-)$, $\max_{W \in \Gamma_i(e,L,s,s_-)} \pi_i^B(e, L, W, s, s_-) = 0$. It then follows that

$$\frac{\partial \tilde{\pi}_i^B}{\partial L}(e, L|s_-) = R^L(s_-) \int_{\Omega_i^g} \tilde{\xi}_i^R(s) \lambda(s_-, s)\, dF(s) - R^D(s_-) \int_{\Omega_i^g} \lambda(s_-, s)\, dF(s) = 0.$$

$\tilde{\xi}_i^R(s)$ appears here because the government transfer $T_i(s)$ is dependent on L_i for $i = I + n$. Thus, $R^L(s_-) = \Theta R^D(s_-)$ for all s_-, where Θ is defined by (28). As $\tilde{\xi}_i^F(s) \leq 1$, it follows that $\Theta \geq 1$. In the above derivation of Θ, we used that $\lambda(s_-, s) = \beta \frac{[c(s_-)+\underline{c}]}{[c(s)+\underline{c}]}$.

A.8 Derivation of equation (42)

Define the following profit for a firm i:

$$\begin{aligned} \pi_i^F(k, s', s) &\equiv \max\ \{\hat{\pi}_i^F(k, s', s),\ 0\}, \\ \hat{\pi}_i^F(k, s', s) &\equiv \max_{(l,W) \geq 0} s_i' k^{\alpha-\nu} l^{1-\alpha} + q(s')k - R^L(s) q(s) k - R_i^F(s') W, \\ &\qquad \text{s.t.} \quad w(s') l \leq W, \end{aligned}$$

which can be rewritten as

$$\pi_i^F(k, s', s) = \begin{cases} \frac{[(1-\alpha)s_i']^{\frac{1}{\alpha}} k^{\frac{\alpha-\nu}{\alpha}}}{[R_i^F(s') w(s')]^{\frac{1-\alpha}{\alpha}}} + q(s')k - R^L(s) q(s) k, & \text{if } W > 0, \\ 0, & \text{otherwise,} \end{cases}$$

because $W > 0$ if $\hat{\pi}_i^F(k, s', s) \geq 0$ due to the Inada condition for the labor input.

Then the maximand in (6) is written as

$$\tilde{\pi}_i^F(k|s) \equiv E\left[\lambda(s, s') \pi_i^F(k, s', s) \middle| s \right] = \int_{\Omega_i^f} \lambda(s, s') \pi_i^F(k, s', s)\, dF(s').$$

Now, consider profit maximization with respect to k. An argument similar to the one in Appendix A.7 implies that the first-order condition with respect to k is written as

$$\frac{\partial \tilde{\pi}_i^F}{\partial k}(k|s) = \int_{\Omega_i^f} \lambda(s,s') \frac{\partial \pi_i^F(k,s',s)}{\partial k}\, dF(s') = 0.$$

It follows that

$$\frac{\partial \pi_i^F(k,s',s)}{\partial k} = \big[q(s') + r_i(s')\big] - R^L(s)q(s),$$

where $r_i(s)$ is defined in (43). Using $\lambda(s,s') = \beta[c(s)+\underline{c}]/[c(s')+\underline{c}]$, the FOC with respect to k is rewritten as

$$0 = \int_{s' \in \Omega_i^f} \frac{\beta[c(s)+\underline{c}]}{[c(s')+\underline{c}]} \Big\{ \big[q(s') + r_i(s')\big] - R^L(s)q(s) \Big\}\, dF(s'),$$

which leads to (42).