Weighted Voting and Information Acquisition in Committees

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Abstract

In a weighted voting system, votes are assigned based on predetermined criteria, such as population for the decision-making at the EU Council, shares at shareholder meetings, and financial contribution at the IMF and the World Bank. The criterion is typically unrelated to the voters' ability to make a correct judgment. Do these unequal decision power distributions undermine the accuracy of group decisions? The goal of this paper is to analyze how the distribution of votes affects the accuracy of group decisions. I introduce an information aggregation model in which voters are identical except for voting shares. If the information is free, the accuracy of group decisions is always higher under unweighted majority rule than any weighted majority rules. When acquiring information is costly, by contrast, I show that the accuracy of group decisions may be higher under some weighted majority rules than under unweighted majority rule. This may justify giving someone greater decision power even if the person is no more capable than others. More generally, I characterize the equilibrium and find the optimal weight distribution to maximize the accuracy of group decisions. Asymmetric weight distributions may be optimal when the cost of improving signal is moderately high.

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1 Introduction

Collective decision-making frequently involves situations in which actors have different numbers of votes. Some institutions assign unequal voting weights explicitly. Examples include the Council of the European Union, the U.S. Electoral College, shareholder meetings, the International Monetary Fund and the World Bank, and the International Energy Agency. In addition to the cases in which weighted voting is used as a formal rule, there are also cases that can be interpreted as weighted voting. For example, parties in parliamentary systems and factions within the party are generally characterized as highly unified. Thus, each party and faction can be seen as a weighted voter. Also, seniority arrangements may be seen as a weighted voting rule. In a legislative party, senior group members often have greater influence over the group decisions, which can be interpreted as weighted voting rule where senior members have higher weight than junior members.[1] How does the distribution of votes affect the accuracy of group decisions? Several scholars have considered a setting in which voters have common interests and the only purpose of voting is information aggregation. Nitzan and Paroush (1982) and Shapley and Grofman (1984) show that the optimal collective decision rule assigns greater weights to the voters with higher ability to make a correct decision.[2] Their main result implies that equally weighted majority rule is optimal when voters are identical.

However, in the real world, the criteria for the weight distribution vary and are typically unrelated to the voters' ability to make a correct judgement, i.e., voters with greater weights are not necessarily a priori more likely than others to make correct judgements. For example, votes are assigned based on population of each member country at the

[1] In addition to those examples, decision making in regulatory organizations can be weighted voting. Bureaucratic and regulatory organizations make numerous errors. For example, the FDA may approve a faulty drug or reject a good drug. How do the errors of regulators depend upon the administrative structures? More specifically, those organizations may be hierarchical in the sense that some agents' opinions are more respected than others. For example, in Japanese bureaucracy, officials have a significantly greater say than the younger officials. Also, it has been reported that opinion of the chairman of the Federal Reserve Board is more influential than other members of the board.

[2] If p_j is the probability that voter j is correct in any given judgment, and if the judgments are independent, then the maximum likelihood rule for two alternatives is to use weighted majority rule, where the weight on individual j's vote is $log\frac{p_j}{1-p_j}$ (Nitzan and Paroush, 1982; Shapley and Grofman, 1984).

E.U. Council, the number of shares that each shareholder owns at shareholder meetings, financial contribution of each member county at the IMF and the World Bank, and each member country's 1973 oil consumption at IEA.[3] These systems may be fair, but do they sacrifice the accuracy of group decision? The aim of this paper is to analyze the influence of heterogeneous voting shares on the accuracy of group decisions.

To this effect, I set up a model with the following features: Voters are identical except for voting shares[4]; Voters have common interests and the only purpose of the voting is information aggregation; Information is a public good in the sense that the social benefits of one voter acquiring information exceed the private benefits. The designer chooses the distribution of weights so that the mechanism not only aggregates information efficiently, but also induces the voters to acquire information.

My main result is that the group decisions may be more likely to be correct under heterogeneous voting shares compared to the case in which every voter has one vote. This is because of the improvement of information possessed by the group: When information acquisition is costly, the voters with greater weights have higher incentives to invest in information than they would have under equal voting shares, and more investment means that more accurate information is aggregated, which makes the group decision more accurate.

To put it another way, an unequal distribution of voting power can sometimes be a solution to the problem of under-provision of a public good, i.e., the information. This may give a justification to delegating authority to someone even if she is no more capable than others when information acquisition is costly. More generally, I characterize the equilibrium and find the optimal weight distribution to maximize the accuracy of group decisions. Asymmetric weight distributions may be optimal when the cost of improving signal is moderately high.

[3] One exception may be the plural voting system that John Stuart Mill proposed. Mill argued that more educated citizens should be given more votes than uneducated citizens (Mill, (1859).

[4] Since the weight distribution is a parameter chosen by the designer, by voters being identical I mean that voters are identical except for weight shares in the this paper.

2 Related Literature

Weighted Voting

The literature of the weighted voting can be classified into two categories: the studies that assume common interest among agents and those that assume conflicting interests. First, consider a setting in which voters have common interests and the only purpose of the voting is information aggregation, which is perhaps best known as the setting of the Condorcet Jury Theorem model. In research on weighted voting in this common-interest framework, Nitzan and Paroush (1982) and Shapley and Grofman (1984), assuming heterogeneous abilities to make a correct judgement, show that the optimal collective decision rule assigns greater weights to the voters with higher ability to make a correct decision.[5] Their main result implies that equally weighted majority rule is optimal when voters are identical.

Those results are based on the assumption that voters are non-strategic and little is known about the strategic behavior in this setting. The current paper, to my knowledge, is the first study to analyze the strategic aspect of weighted voting in the common-interests setting. In contrast to the previous work, this paper shows that heterogeneous voting shares may be optimal even when voters are identical.

The reason why my result is different from those of the previous literature is as follows. The previous literature is only concerned with the negative effect of heterogeneous voting shares: Heterogeneity causes inefficiency in aggregating the information. By introducing the strategic behavior and costly information, the current paper is concerned with both negative and positive effects: Heterogeneity causes inefficiency in information aggregation but it also gives highly weighted voters incentives to acquire information. I show that sometimes the loss in efficiency is more than compensated by the greater amount of information acquired in equilibrium, relative to the case of equal distribution of voting shares.

[5] If p_j is the probability that voter j is correct in any given judgment, and if the judgments are independent, then the maximum likelihood rule for two alternatives is to use weighted majority rule, where the weight on individual j's vote is $\ln \frac{p_j}{1-p_j}$ (Nitzan and Paroush, 1982; Shapley and Grofman, 1984).

The other category in weighted voting is the studies that assume conflicting interests among voters. As in the first category, some studies assume non-strategic voters and others assume strategic voters. The former's central question has been how to measure the power distribution among voters. A voter's ability to affect the group decision is not always proportional to her weight. To measure the power, scholars employ power indices such as the Shapley-Shubik index and the Banzhaf index (Shapley and Shubik, 1954; Banzhaf, 1965, 1966).[6][7] As applications, there is a large amount of literature employing power indices to study the voting weights in the Council of the European Union, the IMF and the World Bank.[8]

The latter, the studies that assume strategic voters, typically analyze legislative bargaining as a game of weighted voting. Most of them employs proposal-based bargaining models, developed by Baron and Ferejohn (1989).[9] Banks and Duggan (2000) provide an existence result for a generalized Baron and Ferejohn model that encompasses weighted voting. Winter (1996) and McCarty (2000) study variants of the Baron-Ferejohn model with veto players. Ansolabehere et al. (2005) characterize the generalized Baron and Ferejohn model under weighted voting. Others employ demand-based bargaining models (Morelli, 1999) and two-stage proposal-based bargaining models (Montero, 2003). By focusing on the aspect of conflicting interests among voters, those studies of conflicting interests analyze distributional politics, i.e., how the distribution of votes affects who gets what. In contrast, the current paper, by focusing on the aspect of the common-interest among voters, studies the likelihood that the group make a correct decision, i.e., how the

[6]For a review, see Roth (1988) and Felsenthal and Machover (1998).

[7]Other cooperative solution concepts applied to weighted voting games include bargaining sets, bargaining aspirations, the kernel, and the competitive solution. See Schofield Schofield (1976, 1978, 1982), McKelvey et al. (1978), Bennett (1983), Holler (1987), and Morelli and Montero (2003).

[8]For the Council of the European Union, see Nurmi and Meskanen (1999), Sutter (2000), Le Breton et al. (2012), BramsandAffuso(1985),Bergetal.(1993),Kaisa Herne and HannuNurmi (1993), Hosli (1993, 1995). Mika Widgren (1994, 2000), R. J. Johnston(1995), Jan-ErikLaneandReinertMaeland(1995, 1996),Laneet al. (1995, 1996),MatthiasBrucknerandTorstenPeters(1996), AnthonyL. Teasdale(1996), GeorgeTsebelis and Geoffrey Garrett(1996), Ulrich Bindseil and CordulaHantke(1997), Dan S. FelsenthalandMosheMachover(1997, 2000, 2001), AnnickLaruelleandWidgren(1998),KonigandBrauninger (1998), GarrettandTsebelis(1999a, 1999b,2001), Hollerand Widgren(1999). For the IMF, see Dreyer and Schotter (1980), (Leech, 2002), Alonso-Meijide and Bowles (2005), and Strand and Rapkin (2006), Aleskerov et al. (2008, 2010).

[9]Power indices are based on the idea that all coalitions are equally likely to form, regardless of how expensive they are. Under the competitive bargaining, cheap coalitions will form more often than expensive ones.

distribution of votes affects the accuracy of group decisions.

Condorcet Jury Theorem

In addition to the literature of weighted voting, this paper contributes to the literature of Condorcet Jury Theorem. The framework within which this paper addresses the collective choice problems is a variant of the Condorcet Jury Theorem model, in which voters have common interests and the only purpose of the voting is information aggregation (Condorcet, 1976).[10]

The Condorcet Jury Theorem model with strategic voting is pioneered by Austen-Smith and Banks (1996). They question Condorcet's assumption of sincere voting by showing that sincere voting do not constitute a Nash equilibrium in general. In response, McLennan (1998) and Wit (1998) demonstrate that allowing mixed strategies sustains Condorcet's argument that groups are more likely to make correct decisions than individuals. Feddersen and Pesendorfer (1998) support Condorce's argument on the group size. They show that the accuracy of a group decision becomes higher as the group grows larger under nonunanimity rules while not under unanimity rule. Under unanimity rule, the probabilities that a group makes a wrong decision stay bounded away from zero regardless of the size of jury. Moreover, increasing the size of the group does not help and actually may increase the probability of convicting an innocent defendant. On the other hand, under nonunanimous rule, both types of mistakes converge to zero as the jury grows large.

The current paper falls into the Condorcet Jury models with costly information acquisition, in which agents privately gather costly information, and then aggregate it to produce a collective decision. Because information is a public good and it causes the collective

[10] Condorcet (1976) claims that majority rule is superior to dictatorship for a society even when the dictator has common interests with others. He considers the situation in which there are two alternatives on an epistemic issue and a society is deciding which one to select. Key feature are (1) that all individuals have common interests to the extent that all agree on the alternative when the true state of the world is revealed and (2) that each individual receives a signal about true state of the world. The resulting claim that a group of voters using majority is more likely to choose the right action than an arbitrary single voter is known as the Condorcet Jury Theorem. Condorcet also argues that the decisions by majorities get better as the size of the groups becomes larger, which some scholars consider part of the Condorcet Jury Theorem.

action problem among agents, the information will be under-provided relative to the social optimum.[11] Mukhopadhaya (2003) shows that a larger committee may actually make poorer decisions because of the collective action problem. He assumes majority rule and focuses on symmetric mixed strategy equilibria. Cai (2009) studies a case in which agents acquire their policy preferences and information structures, which are captured by normal random variables. He shows that when information cost is high, preference heterogeneity can provide agents additional incentives to gather information.

In research on the jury model with costly information, a few papers analyze a problem similar to the current paper. Persico (2004) focuses on the rules that are symmetric, i.e., voters are treated equally in aggregating information, while the current paper allows for asymmetric rules.[12] In his set up, the ex ante optimal threshold rule is ex post efficient, i.e., it is efficient at aggregating information reported from a statistical point of view. In stark contrast, in my setup the ex ante optimal rule may be ex post inefficient. Gerardi and Yariv (2008) allow for a broader class of voting rules than Persico (2004) but their analysis is also restricted to symmetric rules.[13] Gerardi and Yariv yield the same insight as the current paper that the optimal rule may be ex post inefficient. They characterize the equilibrium only for extreme values of a parameter while the current paper does so for more general range of parameters.

Hegemonic Stability Theory

This paper is related to the literature on the hegemonic stability theory. The central idea behind hegemonic stability theory is that the world needs a single dominant state, a hegemonic state, to create and enforce the rules of fee trade among the most important

[11] Li (2001) considers a similar setting to Condorcet Jury model. He analyses a setting in which a fixed number of jurors each purchase the precision of a noisy signal, which is public information. It is shown that there is an incentive to distort the rule away from the rule that would be optimal if information was exogenously given.

[12] Also, Persico focuses on general threshold rules by which one of the alternatives is selected if and only if a certain number of voters support that alternative while the current paper focuses on simple majority rule by which one of the alternatives is selected if and only if more than half of the total votes are cast for that alternative.

[13] More specifically, Gerardi and Yariv (2008) allow for other rules than threshold rules but focus on symmetric rules. In contrast, the current paper focuses on simple majority rule by which one of the alternatives is selected if and only if more than half of the total votes are cast for that alternative but allows for asymmetric rules.

members of the system (See Gilpin 1981, 1994; Grunberg 1990; Kennedy 1987, Keohane1984; kindleberger 1973; Krasner 1976; Strange 1987). That is, a hegemon provides to the international economy in the form of public goods and other states free ride on the benefits. In international economic affairs, for example, an open trading system, well-defined property rights, common standards of measures including international money, consistent macroeconomic policies, proper action in case of economic crisis, and stable exchange rates, are said to be public goods. The problem is an under-provision of those public goods in the absence of external enforcement. The hegemonic stability theory is based on the basic idea that this collective action problem is solved by the unequal distribution of benefits. Whereas the hegemonic stability theory explains that a hegemon has an incentive to provide public goods because it is the largest beneficiary, this paper explains that highly weighted voters have an incentive to do so because they have greater influence in collective decisions.

3 A Motivating Example: Weighted Voting at the IMF

This section describes the governance structure of the IMF and then provides a few insights into the literature.

Decision Making

At the IMF, there are two decision-making bodies: the Board of Governors and the Executive Board. The Board of Governors is the highest decision body of the IMF. It consists of one governor and one alternate governor for each member county. While the Board of Governors has delegated most of its powers to the IMF's Executive Board, it makes important decisions such as quota increases, special drawing right allocations, the admittance of new members, compulsory withdrawal of members, and amendments to the Articles of Agreement and By-Laws.

The Executive Board, on the other hand, is the IMF main decision-making body and takes care of the general operations of the IMF.[14] The Executive Board is composed of one Director from each of the eight countries with the largest quotas - the U.S., Japan, Germany, France, UK, China, Russian Federation, and Saudi Arabia- and 16 other Directors each of whom represents a certain constituencies consisting of 4 to 22 countries. Thus, it consists of twenty-four member countries and they represent all 188 member countries.

Essentially, decisions can be taken by the required majority of votes at either the Board of Governors or the Executive Board. At the Board of Governors, all countries have direct representation. The voting weight of each country is made up of two components: a fixed component of 250 'basic' votes which is the same for each county, and a variable component that depends on the country's quota, i.e., its financial contribution. As of 2012, the United States has 16% of total votes, Japan has 6.23%, Germany has 5.81%, France has 4.29%, UK has 4.29%. On the other hand, at the Executive Board, each of the 24 Executive Directors has the number of votes equal to the sum of the votes in the constituency she represents.

Literature

The IMF's decisions have two aspects: the conflicting interests among the member countries and the common interest shared by the member countries. Firstly, the literature that looks at the aspect of conflicting interests tend to be interested in the link between the distribution of voting share and the resulting distribution of power over actual decisions. In this context, this inequitable representation is often criticized as an unfair reflection of the differences in the member countries' financial contributions, especially for the enormous voting power of the United States and small representation of developing countries (Buria 2003, 2005; Kapur and Nam 2005; Kelkar, Yadav, and Chaundhry 2004; Rato 2006; Bird and Rowlands 2006).

For example, Leech (2002) examines whether the inequality of voting power is a fair

[14]The Board discusses everything from the IMF staff's annual health checks of member countries' economies to economic policy issues relevant to the global economy. The board normally makes decisions based on consensus but sometimes formal votes are taken. At the end of most formal discussions, the Board issues what is known as a summing up, which summarizes its views. Informal discussions may be held to discuss complex policy issues still at a preliminary stage.

reflection of the differences in their contributions and how the votes should be weighted. Leech demonstrates that the distribution of voting powers are considerably more unequal than their financial contributions. In particular, the power of the USA is much greater than its nominal 17% of the votes under the existing weighted voting system.[15] He further presents a new algorithm to find what the weights should be in order to achieve a given desired power distribution.

On the other hand, the literature that looks at the aspect of the common interest is primarily concerned with the quality of the IMF decision making, especially the effectiveness of its crisis management policies.[16] The effectiveness of these IMF-sponsored programs to assist countries in financial crisis has been questioned especially since the East Asian financial crisis of 1997. Many criticized how the IMF handled the crisis (Sachs, 1997; Feldstein, 1998; Stiglitz, 2002; *IFIAC Report*, 2000). For example, Sachs (1997) is critical of the IMF's advice to Asian countries in crisis: He argues that the IMF's insistence upon market reform conditions only worsen the crisis and the IMF should have only provided sufficient funds. Feldstein (1998) criticizes the IMF's crisis management for two reasons: Its recommendation of austerity measures were inconsistent with the Asian countries' economic needs; Its conditions for disbursement of credit were too targeted. Stiglitz (2002) also criticizes that the countries in crisis have been left worse off through the IMF's bad

[15] This results are for ordinary decisions requiring a simple majority.

[16] For the effectiveness of other aspects of the IMF policies, such as the accuracy of IMF projection, see Conway (2006).

advice. [17][18][19]

The current paper suggests that the unequal distribution of voting share, which is often criticized as unfair from the perspective of conflicting interests, may have a positive effect on the quality of the IMF's decision making from the perspective of common interest.[20] Unequal distribution increases highly weighted countries more influential, which encourages them to put effort to improve their information needed for group decision. As a consequence, the organization may be more likely to make a correct decision. More generally, I analyze how the weighted voting system influences the quality of group decisions in terms of the efficiency of information aggregation, rather than fairness.

[17] The IMF practices conditionality when it lends in the hope that its loans do not simply subsidize the bad policies that led a country to borrow from the Fund in the first place. In return for continued disbursements of IMF loans, countries must follow austere policy conditions designed to ensure debt repayment, stabilize the economy, and promote national prosperity. There is little evidence, however, of the success of IMF conditionality. While there is some evidence that balance of payments problems are curtailed, there is less evidence of success with respect to inflation. Regarding economic growth, recent evidence shows that IMF programs hurt. The single most consistent effect that IMF programs appear to have is to exacerbate income inequality. The reasons for failure are debated. On the one hand, people who believe that states have an important role to play in promoting economic growth argue that IMF austerity policies imposed through IMF conditionality are the problem. (critics on the left that basic IMF approach to economic problems is incorrect) Instead of reducing the size of the state in the face of an economic crisis, the IMF should support government stimulus packages, like those employed in the developed world during times of economic trouble. On the other hand, people who are skeptical of non-market solutions for economic problems believe that the IMF hurts countries in the opposite manner. (critics on the right that compliance with conditionality was a problem. Better implementation would lead to better results.) They agree with IMF austerity but think that the IMF fails to strictly enforce conditionality. As a result, IMF loans provide a subsidy for continued economic mismanagement by developing country governments. For the more detailed review, see Vreeland (2007) ch. 4.

[18] Hutchison (2001) examines the effects of whether participation in IMF programs has an positive effect on economic outcomes for countries with financial crises. He concludes that the IMF is not a significantly worse crisis manager than having no manager at all, but that IMF programs are in general growth-retarding. Garuda (2000) estimate the impact of IMF programs on income distribution. He first classifies thirty-nine countries into three groups: Group 1 is a set of the countries that are least likely to participate in IMF programs based on preexisting economic conditions while Group 3 is the one most likely. He finds that the poor are likely to gain the most compared to the non-program countries in Group 1 countries, and the effect is opposite in Group 3 countries.

[19] See Mohsin (1990), Conway (1994), and Haque and Khan (1998) for earlier discussion on the effectiveness of the IMF conditionality, and see Hajro and Joyce (2004) for the impact of IMF programs on infant mortality as an indicator of poverty, and see Barro and Lee (2004) and Dreher (2006) for the economic effects of the loan programs such as the effect on the economic growth, and see Dreher and Gassebner (2012) for the effect of the IMF program on the government crises, and see Dreher and Walter (2010) for the effect of IMF programs on the likelihood and outcome of currency crises.

[20] This may be a counter argument to Stiglitz (2002), which believes that the distribution of voting share at the IMF should change to improve the policies the IMF imposes as part of conditionality policies.

4 Basic Model: Costless Information

Following the convention, I use a jury analogy to explain the model. In the political context, deciding whether to convict the defendant would be choosing between two candidate for office, whether to build a nuclear power plant, whether to launch a space shuttle, whether to approve the drug, deciding whether to send troops to Iraq, and so on.

A finite set of jurors $N = \{1, 2, \ldots, n\}$ (n odd) is to make a collective decision $d \in \{A, C\}$ where A and C correspond to acquittal or conviction, respectively. The unknown state is $\omega \in \{G, I\}$: the defendant is either guilty (G) or innocent (I), with prior distribution $\Pr(G) = \Pr(I) = \frac{1}{2}$.

All the jurors and the designer have identical preferences over the choice $d \in \{A, C\}$ and state ω.[21] The common utility is given by

$$u(d, \omega) = \begin{cases} -\frac{1}{2} & \text{if } (d, \omega) = (C, I) \\ -\frac{1}{2} & \text{if } (d, \omega) = (A, G) \\ 0 & \text{otherwise.} \end{cases}$$

Both jurors and the designer maximize expected utility.[22]

Given the state of the world $\omega \in \{G, I\}$, each juror j simultaneously receives a private signal $s_j \in \{g, i\}$. Conditional on the state, signals are independent across jurors. Let $p \in (1/2, 1)$ represent the probability that each juror observes the correct signal.[23]

Once jurors receive the signals, the group decision is made as follows. Let $\mathbf{w} = (w_1, \ldots, w_n)$ be a vector of non-negative weights with $w_j \geq 1$. Throughout the paper, I focus on the simple majority rule with weight $\mathbf{w}$ such that $w_1 = \ldots = w_m = w$ and $w_{m+1} = \ldots = w_n = 1$. That is, jurors $1, \ldots, m$ are weighted by $w \geq 1$ and jurors $m{+}1 \ldots n$ are not weighted.[24] Formally, the group decision rule is defined as $f^{\mathbf{w}} : \{g, i\}^n \to \{A, C\}$

[21] I assume jurors are all identical except for their weights in order to see if weighted rule can be better than unweighted rule for some parameters even if jurors are all identical.

[22] Juror j prefers conviction to acquittal if and only if she places at least probability $\frac{1}{2}$ that the defendant is guilty. We say that the outcome of the trial is correct if either the defendant is guilty and convicted or he is innocent and acquitted.

[23] For the sake of simplicity, I assume that the probability that each juror observes the correct signal when the true state is G is equal to the one when the true state is I.

[24] Note that (m, w) is well-defined so that $n + m(w - 1)$ is odd.

such that

$$f^{\mathbf{w}}(s_1,\ldots,s_n)=\begin{cases} C & \text{if } \sum_{j=1}^{n} w_j I_{s_j=g} \geq \frac{\sum w_j+1}{2}=\frac{n+m(w-1)+1}{2} \\ A & \text{otherwise.}^{25} \end{cases}$$

I define a mechanism as the following game:

Stage 1 The mechanism designer chooses the weight distribution $\mathbf{w}$, i.e., the number of weighted jurors m and their weight w, which becomes common knowledge among the jurors.

Stage 2 Each juror independently receives signal.

Stage 3 If $\frac{n+m(w-1)+1}{2}$ or more weighted average of the jurors receive guilty signal, the defendant is convicted. Otherwise, acquitted.

As a corollary of Nitzan and Paroush (1982)'s result, the aggregation rule that maximizes the probabilities of convicting a guilty defendant and acquitting an innocent defendant is to distribute the weights equally.[26]

In Section 4, it is assumed that jurors receive information for free. In Section 5, I incorporate the stage in which jurors decides whether to purchase highly accurate information or receive less accurate information for free. The goal of the both sections is to find the optimal weight distribution $\mathbf{w}$, i.e., (m, w), to maximize the designer's expected utility of the collective decision.

5 Costly Information

The model with costless information illustrates that the weighted rule is inefficient at aggregating reported information compared to unweighted rule. In this section, I incorporate

[25]For the sake of simplicity, I exclude the possibility $\sum_{j=1}^{n} w_j v_j = \frac{n+m(w-1)}{2}$.

[26]This corollary is based on the assumption that signal accuracies are identical for all jurors. Otherwise, the optimal allocation of weights is proportional to each juror's log-likelihood ratio.

the stage where the jurors decide whether or not to invest in information before voting. Because information is a public good, information is under provided relative to the social optimum. When the information is costless, the mechanism designer needs to care only about the efficiency of aggregating information. By contrast, when the information is costly, he needs to care about whether a rule gives jurors incentives to acquire information, as well as whether it aggregates information efficiently.

At Stage 2, each juror j simultaneously and independently makes a decision $t_j \in \{0, 1\}$ about signal acquisition where 1 and 0 correspond to invest and not invest, respectively: She chooses whether to purchase a highly accurate signal at a cost c (> 0) or receive a low-quality signal for free. [27] The conditional probabilities of high-quality signals and low-quality signals are $P(s_j = i|I, t_j = 1) = P(s_j = g|G, t_j = 1) = p_H$ and $P(s_j = i|I, t_j = 0) = P(s_j = g|G, t_j = 0) = p_L$, respectively, where $p_H > p_L \geq \frac{1}{2}$.[28] Denote the weighted juror's signal accuracy by $p \in \{p_H, p_L\}$ and unweighted jurors' signal accuracy by $p' \in \{p_H, p_L\}$. The mechanism designer does not take into account the cost c incurred by a juror who purchases a high-quality signal.

Denote the probability that the jury makes a correct decision by

$$V(w, \mathbf{t}) := \Pr(G)\Pr(C|G) + \Pr(I)\Pr(A|I).$$

Since $\Pr(G) = \Pr(I) = \frac{1}{2}$ and $\Pr(C|G) = \Pr(A|I)$, $V(w, \mathbf{t}) = \Pr(C|G)$.[29] Also, since the cost of convicting an innocent defendant and the one of acquitting a guilty defendant are both $\frac{1}{2}$, the designer's expected utility is $-\frac{1}{2} + \frac{1}{2}\Pr(C|G)$. Therefore, in order to maximize the expected utility, the designer maximizes the probability of making a correct decision $V(w, \mathbf{t})$.

In the following, I restrict attention to the symmetric equilibria: Jurors of the same weight play the same strategy. Because all jurors with the same weight face a similar

[27] Information acquisition could be reinterpreted as information processing. In that case, the cost c captures the effort that each decision maker puts into updating his beliefs given the available information.

[28] For the sake of simplicity, I assume that the probability that each juror observes the correct signal when the true state is G is equal to the one when the true state is I.

[29] Since the signal accuracy is symmetric, i.e., $\Pr(s_j = g|\omega = G) = \Pr(s_j = i|\omega = I)$, and I focus on the simple majority rule, $\Pr(C|G) = \Pr(A|I)$.

decision problem, it is natural to assume that jurors with the same weight use the same decision rule in equilibrium. We restrict the analysis to this kind of equilibrium, which I refer to as a symmetric equilibrium.

The purpose of this section is to (1) characterize the symmetric equilibria given the weight and (2) find the optimal weight for the designer. For now, I consider the case in which there is at most one weighted juror, $m = 1$. For the future research, I intend to extend the analysis to the case in which more than one jurors may be weighted, $1 < m < \frac{n+1}{w+1}$ (See Appendix D).

Note that w is well-defined. Since n is odd and $m = 1$, w needs to be odd in this case. We start with a simple example which suggests that weighting may improve the quality of the group decision and jurors' payoffs.

Example 1. *Let* $N = \{1, 2, 3, 4, 5\}$, $\Pr(G) = 0.5$, $q = 0.5$, $c = 0.05$, $p_H = 0.8$, *and* $p_L = 0.6$.

First, consider the unweighted simple majority rule. Suppose that no jurors acquire high-quality signal. Since

$$EU_j[t_j = 1] - EU_j[t_j = 0] = -0.01544,$$

no jurors have an incentive to deviate. In this situation, the designer's expected utility is -0.31744.

Second, consider the weighted simple majority rule where juror 1 has three votes while each of the others has one vote. Suppose that only juror 1 acquires high-quality signal. Since

$$\begin{aligned} &EU_1[t_1 = 1] - EU_1[t_1 = 0] = 0.03448 > 0 \\ &EU_j[t_j = 1] - EU_j[t_j = 0] = -0.04056 < 0 \qquad \text{for } j \neq 1, \end{aligned}$$

no jurors have an incentive to deviate. In this situation, the designer's expected utility is

-0.19456.

In this example, no jurors have incentives to invest without weighting while juror 1 does so with weight $w = 3$. As a consequence, the designer's expected utility is higher under the weighted rule.

This example demonstrates that the probability of making a correct decision (therefore the designer's expected utility) is higher under weighted aggregation rule than unweighted aggregation rule.

To see how individual weights affect the jury's decision, I first examine how individual weights affect the probability of each juror's being pivotal. Let $\Pr(\text{piv}_j|\omega, w, \mathbf{t}_{-j})$ be the probability that juror j becomes pivotal given ω. Then, juror j invests in information if and only if $\Pr(G)\Pr(\text{piv}_j|G, w, \mathbf{t}_{-j})(p_H - p_L) \geq c$. [30]

Lemma 1. *Consider $w \geq n$. the weighted juror is decisive and unweighted jurors never becomes pivotal for any w. Consider $w \leq n-2$. As w becomes larger, the weighted juror is more likely to become pivotal. As w increases, the unweighted jurors are*

1. *less likely to be pivotal if (1) $p < \frac{\left(\frac{p'}{1-p'}\right)^{w+1}}{1+\left(\frac{p'}{1-p'}\right)^{w+1}}$ and $n < N(p, p', w)$ or (2) $p \geq \frac{\left(\frac{p'}{1-p'}\right)^{w+1}}{1+\left(\frac{p'}{1-p'}\right)^{w+1}}$.*
2. *independent of w if $p < \frac{\left(\frac{p'}{1-p'}\right)^{w+1}}{1+\left(\frac{p'}{1-p'}\right)^{w+1}}$ and $n = N(p, p', w)$*
3. *more likely to be pivotal if $p < \frac{\left(\frac{p'}{1-p'}\right)^{w+1}}{1+\left(\frac{p'}{1-p'}\right)^{w+1}}$ and $n > N(p, p', w)$*

[30] A juror j has an incentive to pay for the highly accurate information if and only if $\Pr(G, \text{piv}_j|w, \mathbf{t}_{-j})(p_H - p_L) \geq c$ because

$$\begin{aligned}
EU_j[t_j = 1] - EU_j[t_j = 0] &= \Pr(G, \text{piv}_j|w, \mathbf{t})\{0 \cdot p_H - (1-q)(1-p_H)\} + \Pr(I, \text{piv}_j|w, \mathbf{t})\{0 \cdot p_H - q(1-p_H)\} - c \\
&\quad - \Pr(G, \text{piv}_j|w, \mathbf{t})\{0 \cdot p_L - (1-q)(1-p_L)\} - \Pr(I, \text{piv}_j|w, \mathbf{t})\{0 \cdot p_L - q(1-p_L)\} \\
&= (1-q)(p_H - p_L) \cdot \Pr(G, \text{piv}_j|w, \mathbf{t}) + q(p_H - p_L) \cdot \Pr(I, \text{piv}_j|w, \mathbf{t}) - c \\
&= (p_H - p_L) \cdot \Pr(G, \text{piv}_j|w, \mathbf{t}) - c.
\end{aligned}$$

The last equality holds because $\Pr(G, \text{piv}_j|w, \mathbf{t}) = \Pr(I, \text{piv}_j|w, \mathbf{t})$. Note that the probability of juror j's being pivotal is independent of of his investment.

for $p, p' \in \{p_L, p_H\}$ *where*

$$N(p,p',w) := w \cdot \frac{\left(1+\frac{1-p'}{p'}\right)\left(\left(\frac{p'}{1-p'}\right)^{w+1}+\frac{p}{1-p}\right)}{\left(1-\frac{1-p'}{p'}\right)\left(\left(\frac{p'}{1-p'}\right)^{w+1}-\frac{p}{1-p}\right)} + 2 \cdot \frac{\frac{p}{1-p}\cdot\frac{1-p'}{p'}+\left(\frac{p'}{1-p'}\right)^{w+1}}{\left(1-\frac{1-p'}{p'}\right)\left(\left(\frac{p'}{1-p'}\right)^{w+1}-\frac{p}{1-p}\right)}.$$

Proof. See Appendix A $\square$

It is surprising that, as well as the weighted juror, even unweighted jurors may be more likely to be pivotal as the weight increases. To understand this phenomenon, it is crucial to remark that unweighted jurors are more likely to be pivotal as the expected weight of correct signals becomes closer to the simple majority of the total weight.

Consider the event that the weighted juror receives a wrong signal. As w becomes larger, the expected weight of correct signals conditional on this event decreases, i.e., becomes closer to the simple majority of the total weight. This increases the chances that unweighted jurors are pivotal. On the other hand, consider the event that the weighted juror receives a correct signal. As w becomes larger, the expected weight of correct signals conditional on this event increases, i.e., becomes farther away from the simple majority of the total weight. This decreases the chances that unweighted jurors are pivotal. When the weighted and unweighted jurors receive the same quality signals and n is large, for example, the former effect is greater than the latter, which means that unweighted jurors are more likely to be pivotal as w increases.

As a corollary of Lemma 1, I have sufficient conditions of (p_H, p_L, w, n) for unweighted jurors being less likely to be pivotal as w increases for all $p, p' \in \{p_H, p_L\}$ as follows.

Corollary 1. *If* $p_H < \frac{\left(\frac{p_L}{1-p_L}\right)^{w+1}}{1+\left(\frac{p_L}{1-p_L}\right)^{w+1}}$ *and* $n < N(p = p_L, p' = p_H, w)$*, unweighted jurors are less likely to be pivotal as* w *increases for all* $(p, p') \in \{p_H, p_L\} \times \{p_H, p_L\}$.

Proof. Suppose $p_H < \frac{\left(\frac{p_L}{1-p_L}\right)^{w+1}}{1+\left(\frac{p_L}{1-p_L}\right)^{w+1}}$. Then, it follows that $N(p = p_H, p' = p_H, w) > N(p_L, p_L, w) > N(p = p_L, p' = p_H > N(p = p_L, p' = p_H, w) > 0$. Thus, if $p_H < \frac{\left(\frac{p_L}{1-p_L}\right)^{w+1}}{1+\left(\frac{p_L}{1-p_L}\right)^{w+1}}$ and $n < N(p = p_L, p' = p_H, w)$, then $n < N(p, p', w)$ for all $(p, p') \in$

$\{p_H, p_L\} \times \{p_H, p_L\}$. By Lemma 1, it follows that $\Pr(\text{piv}_j | G, w, \mathbf{t}_{-j})$ is decreasing in w for $j \neq 1$ for all $(p, p') \in \{p_H, p_L\} \times \{p_H, p_L\}$. $\square$

By Corollary 1, Lemma 2 shows the existence and sufficient conditions of (p_H, p_L) for unweighted jurors being less likely to be pivotal as w increases for all $w = 1, 3, \ldots, n-2$, regardless of jurors' investment behavior, i.e., for all $p, p' \in \{p_H, p_L\}$.

Lemma 2. *Suppose* $p_H < \frac{\left(\frac{p_L}{1-p_L}\right)^{w+1}}{1+\left(\frac{p_L}{1-p_L}\right)^{w+1}}$. *For every* n*, there exists* (p_H^*, p_L^*) *such that, if* $p_H \leq p_H^*$ *and* $p_L \leq p_L^*$*, unweighted jurors are less likely to be pivotal as* w *increases for all* $w = 1, 3, \ldots, w-2$ *and for all* $(p, p') \in \{p_H, p_L\} \times \{p_H, p_L\}$.

Proof. Suppose $p_H < \frac{\left(\frac{p_L}{1-p_L}\right)^{w+1}}{1+\left(\frac{p_L}{1-p_L}\right)^{w+1}}$. For all $w = 1, \ldots, w-2$,

$$\lim_{p_H \to \frac{1}{2}} \lim_{p_L \to \frac{1}{2}} N(p = p_L, p' = p_H, w) = \infty. \tag{1}$$

By (1), it follows that for given n there exists p_H^* such that

$$\lim_{p_L \to \frac{1}{2}} N(p = p_L, p' = p_H, w) > n + 1 \tag{2}$$

for every $p_H \leq p_H^*$. (2) implies that there exists p_L^* such that

$$N(p = p_L, p' = p_H, w) > n \tag{3}$$

for every $p_L \leq p_L^*$. By Corollary 1, (3) implies that unweighted jurors are less likely to be pivotal as w increases, regardless of jurors' investment behavior, i.e., for all $(p, p') \in \{p_H, p_L\} \times \{p_H, p_L\}$. $\square$

Lemma 2 shows that unweighted jurors are less likely to be pivotal as $w(\leq n-2)$ increases if p_H and p_L are sufficiently small. In the following, I focus on the cases of sufficiently small p_H and p_L, where unweighted jurors are less likely to be pivotal as $w(\leq n-2)$ increases.

Assumption 1. *The accuracy of high-quality signal p_H and the one of low-quality signal p_L are sufficiently low so that unweighted jurors are less likely to be pivotal as $w(\leq n-2)$ increases.*

Proposition 1 describes the equilibria given c and w.

Define $(c_1, c_2, c_3, c_4, c_5, c_6)$ as follows:

$$
\begin{aligned}
c_1 &:= f_{-1}(n-2, (1,1,\ldots,1)) \\
c_2 &:= f_j(1, (1,1,\ldots,1)) \qquad \text{for any } j \in N \\
c_3 &:= f_{-1}(n-2, (1,0,\ldots,0)) \\
c_4 &:= f_{-1}(1, (1,0,\ldots,0)) \\
c_5 &:= f_1(1, (1,0,\ldots,0)) = f_1(1, (0,0,\ldots,0)) \\
c_6 &:= f_1(n-2, (1,0,\ldots,0)) = f_1(n-2, (0,0,\ldots,0))
\end{aligned}
$$

where $f_j(w, \mathbf{t}_{-j}) := (p_H - p_L) \cdot \Pr(G, \mathrm{piv}_j | w, \mathbf{t}_{-j})$ for $j \in N$.[31]

Proposition 1. *Consider sufficiently small p_H and p_L such that unweighted jurors are less likely to be pivotal as $w(\leq n-2)$ increases. Then, Figure 1 illustrates the equilibria*

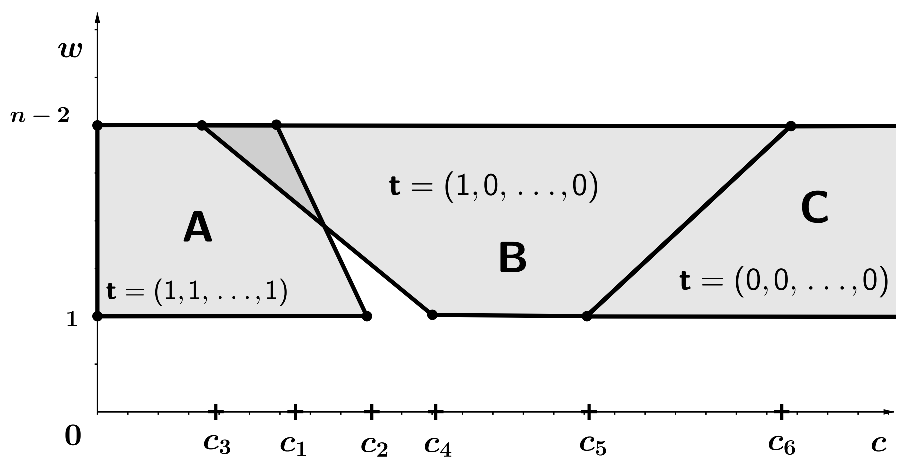

Figure 1: $\{(c,w) | w_1(c,\mathbf{t}) \leq w \leq w_{-1}(c,\mathbf{t})\}$ for $\mathbf{t} = (1,1,\ldots,1)$

[31] $f_{-1} := f_j$ for $j \neq 1$.

where $\mathbf{t} = (1,1,\ldots,1),(1,0,\ldots,0),(0,0,\ldots,0)$.[32] *The area A represents* (c,w) *for which* $\mathbf{t} = (1,1,\ldots,1)$ *is an equilibrium; the area B represents* (c,w) *for which* $\mathbf{t} = (1,0,\ldots,0)$ *is an equilibrium; the area C represents* (c,w) *for which* $\mathbf{t} = (0,0,\ldots,0)$ *is an equilibrium.*

Proof. See Appendix B. □

Proposition 1 illustrates the equilibrium behavior, i.e, who invests in information in equilibrium, given the weight (w) and the cost of improving signal (c). Based on Proposition 1, we find the optimal weight for the designer.

Proposition 2. *The optimal weight* $\hat{w}$ *for the designer is*

$$\hat{w} = \begin{cases} 1 & \text{if } c \in [0, c_2] \cup [c_4, c_5] \cup [c_7, \infty) \\ w^{**}(c,\mathbf{t}) & \text{if } c \in (c_2, c_4) \\ w^{*}(c,\mathbf{t}) & \text{if } c \in (c_5, c_7] \end{cases}$$

if $p_H\left(1-(1-p_L)^{n-1}\right) + (1-p_H)p_L^{n-1} - \sum_{x \geq \frac{n+1}{2}} \binom{n}{x} p_L^x (1-p_L)^{n-x} < 0$ *(Figure 2), and*

$$\hat{w} = \begin{cases} 1 & \text{if } c \in [0, c_2] \cup [c_4, c_5] \cup [c_6, \infty) \\ w^{**}(c,\mathbf{t}) & \text{if } c \in (c_2, c_4) \\ w^{*}(c,\mathbf{t}) & \text{if } c \in (c_5, c_6) \end{cases}$$

if $p_H\left(1-(1-p_L)^{n-1}\right) + (1-p_H)p_L^{n-1} - \sum_{x \geq \frac{n+1}{2}} \binom{n}{x} p_L^x (1-p_L)^{n-x} \geq 0$ *(Figure 3).*

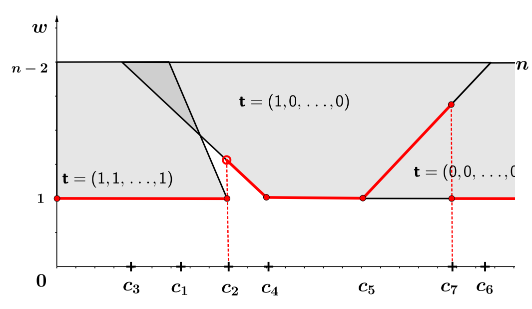

Figure 2: Optimal Weights

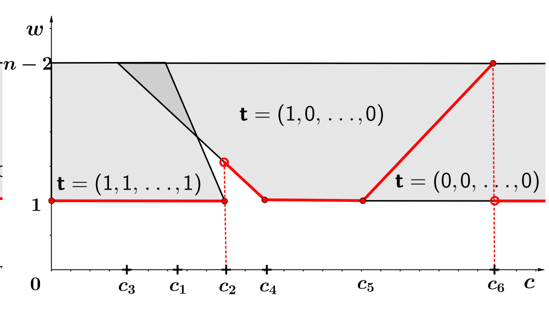

Figure 3: Optimal Weights

[32] $w_{-1} := w_j$ for $j \neq 1$

Proof. See Appendix C. □

Proposition 2 shows that if the cost of improving the signal is moderately high, the designer may maximize the probability of the jury's making a correct decision by distributing weights unequally.

6 Discussions

In this section, I describe the possible implications to political theory and possible future research.

6.1 Implications

Equal Suffrage and Weighted Voting

This paper may shed light on political philosophies. light Equal suffrage has been one of the most important concepts for political theorists. It has been commonly accepted that political equality is a central feature of a democratic system (See Barry Holden, *The Nature off Democracy*, 1974, p.19; Giovanni Sartori, *Democratic Theory*, 1965, ch 14; Ivor BVrown, *The Meaning of Democracy*, 1926, p.44, George Edwards III, *Why the Electoral College is Bad for America*, 2004, ch 2, James S. Fishkin, *Democracy and Deliberation*, p.29). Robert Dahl, for example, argues that equality in voting is a crucial part of a democratic system: "every member must have an equal and effective opportunity to vote, and all voters must be counted as equal." A constitution for democratic government, he adds, "must be in conformity with one elementary principle: that all members are to be treated (under the constitution) as if they were equally qualified to participate in the process of making decisions about the policies the association will pursue. Whatever may be the case on other matters, then, in governing this association all members are to be considered as politically *equal*." (*On Democracy*, Robert A. Dahl (2000) p.37).

On the other hand, John Stuart Mill advocated the weighted voting system whereby educated and more responsible persons would be given more votes than the uneducated. As much as this weighted system may be unfair to uneducated citizens, there is no guarantee that the educated have better sense about what is good for the society than the uneducated. This paper shows that Mill's argument may hold even if the educated are not more likely to make a correct judgement than the uneducated. Moreover, it suggests that the quality of society's decisions may be higher under the concentration of power compared to the one in democratic societies.

Correspondence with the argument of John Stuart Mill

Both the current paper and Mill ((1859) consider the same environment. In particular, both assume the common-interest. For Mill it is vital that voters should vote in accordance with their ideas of the general interest; that is they should vote for whichever candidates they feel most likely to improve the citizens and efficiently manage the affairs of the country in the interests of all. In fact, Mill uses an analogy with jury service:

> "[The citizen's] vote is not a thing in which he has an option; it has no more to do with his personal wishes than the verdict of a juryman. It is strictly a matter of duty; he is bound to give it according to his best and most conscientious opinion of the public good."
>
> —John Stuart Mill, *Representative Government*, 299

Both the current paper and Mill address weighted voting systems but the arguments differ in the weight assignment. Mill argues that the particularly intelligent or well educated should be given two or more votes. Mill's concern is that the uneducated poor–the numerical majority–will make a terrible mistake. Mill wants to ensure that representative democracy contains certain safeguards to prevent it from being dictated to by stupidity and class interest. (*Representative Government*, 284). The current paper also supports weighted voting by demonstrating that the group may be more likely to make a correct judgement under a weighted voting system than under an equal representation. The difference in those studies is that, while Mill argued that weights should be distributed based on the education level, the current paper argues that weighted voting systems may work even if voters are all equally capable of making a judgement.

Thus, by considering the same situation as Mill's, the current paper supports the unequal representation advocated by Mill. Moreover, this paper uses the same setting whereby Condorcet supported democracy.

6.2 Future Research

There are many directions this framework suggests pursuing. One example is to analyze the optimal size of the committee. In the current model, the size of the committee is exogenously given and the designer cannot choose. Another possibility is adding heterogeneity amongst agents, in the form of differential preferences, may affect the optimal design. Indeed, in our model, both the designer and all of the players share the same utility parameter q.

The Size of Nations

The current model may contribute to the literature on the size of nations and the regime type. A large body of literature deals with the size of nations, the regime type, and the relationship linking these two variables (See Alesina and Spolaore (1997), Alesina et al. (1997), Alesina and Wacziarg 1998). Political theorists have argued that democracy cannot survive in a large state. In particular, Plato, Aristotle and Montesquieu worried about the political costs of large states: Plato wrote that "*the number of citizens should be sufficient to defend themselves agains the injustice of their neighbors,*" (*Laws*, Book V); Aristotle argued that a polity should be no larger than a size in which everybody knows personally everybody else because "*experience has shown that it is difficult, if not impossible, for a* *populous state to be run by good laws*" (*The Politics*); Montesquieu wrote that "*In a large republic, the common good is sacrificed to a thousand considerations. It is subordinated to various exceptions. It depends on accidents. In a small republic, the public good is more strongly felt, better known, and closer to each citizen.*" (*The Sprit of the Laws*).

On the other hand, Madison objected that a large size, far from being a problem, was actually an advantage for a democracy. His point was that ht enlarger territory becomes in size, the greater will be its variety of parties and interests, and hence the smaller will be the chance that "*a majority of the whole will have a common motive to invade the rights of other citizens; or if such a common motive exists, it will be more difficult for all who feel it to discover their own strength, and to act in unison with each other.*" In other words, according to Madison, in larger states rent-seeking groups who want to "*invade the rights of other citizens*" will have a harder time to overcome problems of collective action. Moreover, according to Madison, " *the influence of factious leaders may kindle a flame within their particular States, but will be unable to spread a general conflagration through the other States.*"

The current model may support the former thinkers, the advocates for small nations. The current model may demonstrate that weighted voting becomes superior to equal suffrage as the group size becomes larger: Because incentives to free-ride becomes stronger as the group size become larger, the designer may want to giver higher weight to some voters to improve their incentives to acquire accurate information at the cost of efficiency in information aggregation. More generally, the current model gives the optimal distribution of weights for a given size of the group. That is, on the assumption that the distribution of weights can be interpreted as the distribution of the power, the model suggests the optimal regime design for a given size of the country.

Heterogeneous Signal Accuracy

The next step is to incorporate the heterogeneity of signal accuracy to the current model. The model with heterogeneous signal accuracy may have several important implications. Firstly, if one accepts the interpretation that the distribution of weights represents the distribution of the power, a variant of my model may shed new lights on one of the most important topics in political science: By incorporating the heterogeneity of signal accuracy to the current model, one may be able to find the optimal distribution of the power, i.e., the optimal regime design.

Examples of classic arguments over regime design are those by Plato, Mill, and Rousseau. Mill's aristocratic liberalism falls between Plato's guardianship and Rousseau's democratic principle: Plato asserted that only a few selected experts should rule; Mill asserted that even uneducated citizens should be enfranchised but educated citizens should be given more votes than the uneducated; Rousseau supported democratic principle that all citizens have an equal say (except for female citizens). My model may explain when Plato's guardianship works, when Mill's aristocratic liberalism works, and when Rousseau's democratic principle works.

One way to generalize those three types of systems is to consider them to be a variant of weighted rule: In Plato's benevolent dictatorship, a few selected experts, *guardians*, have votes while others' weight is zero; In Rousseau's democratic system, everyone has an equal weight; In Mill's system, everyone has positive weight but their weights are allocated based on their education level. The question is what the optimal weighting is.[33]

[33] The following problem about Mill's system may help us understand the question. Mill's critics to his proposal about plural voting argue that, if uneducated revere the educated then I need not give the latter extra votes, for the uneducated can simply seek out their opinions. But if they do not respect such opinions then they would not accept plural voting. Plural voting is either unnecessary or unjustified. In fact, Mill himself recognized this point and made the following remark.

> "I may remark, that if the voter acquiesces in this estimate of his capabilities, and really wishes to have the choice made for him by a person in whom he places reliance, there is no need of any constitutional provision for the purpose; he has only to ask the confidential person privately what candidate he had better vote for."
>
> —John Stuart Mill, *Representative Government*, 294

The question is whether and when the power should be concentrated on a few selected people as Plato suggested, and whether and when everyone should be enfranchised whereas the distribution of the votes

Suppose that some voters inherently receive low-quality signal whose accuracy is $\frac{1}{2}$ whereas the others inherently receive high-quality signal whose accuracy is higher than $\frac{1}{2}$. Consider (1) a group of voters all of whom receive high-quality signal and (2) a group of voters some of whom receive low-quality signal and the others receive high-quality signal. Suppose that the voters with high-quality signal have more than one vote while the voters with low-quality signal have only one vote each. Under a certain range of parameters, the former group may be more likely to make a correct decision than the latter group. If that is the case, it implies that Mill's partial democracy works better than Plato's guardianship or Rousseau's democracy under such parameters. More generally, this variant of my model may provide the conditions for each of the thee regime types to be optimal.

Secondly, the model with heterogeneous signal accuracy may also examine Mill's proposal about plural voting from another perspective. According to Mill, plural voting has two benefits: By giving higher weight to highly educated people, it is efficient at aggregating information; By enfranchising uneducated citizens, it also helps poorly educated citizens educate themselves through participation. My model may demonstrate that those two benefits may contradict each other and, if so, provide conditions for plural voting to have those benefits all together.

Recall that my model has shown that there may be a trade-off between the efficiency of information aggregation and the incentives to acquire information under a certain range of parameters. Because "costly information acquisition" can be interpreted as the process for citizens to learn about the policy, it implies that there may a trade-off between the efficiency of information aggregation and the citizens' incentives to educate themselves.

Before getting to the model, note that weighted voting may reduce the performance of the group decision for two possible reasons: The inefficiency of information aggregation caused by weighting disproportionate to the voters' signal accuracy and the decrease in investment in information (education). We pay close attention to the second aspect. If

are not equal as Mill suggested, and whether and when all citizens should be equally enfranchised as Rousseau suggested.

some voters have higher weight than others, it may discourage other voters to acquire more accurate signal even though they would be willing to invest if all voters had an equal say.

Suppose that there are voters who inherently receive more accurate signal than the other voters. Consider (1) a group of voters all of whom are unweighted regardless of their signal accuracy and (2) a group of voters whose weight are determined based on their signal accuracy: voters who receive highly accurate signal have more than one vote while the others have only one vote each. There may be a range of parameters under which all voters in the former group invest in information while none of the unweighted voters do so in the latter group. In this case, the quality of information possessed by the latter group is lower than the the one possessed by the former. Thus, the latter group is more likely to make a wrong decision than the former even if the inefficiency caused by the weighting is adjusted.

Therefore, under some weighted rule, the accuracy of group decision becomes lower because the weighted rule discourages unweighted voters to invest in signal (educate themselves), which decreases the quality of information possessed by the group. This implies that making good use of highly educated people's opinions (efficient information aggregation) and the positive effect of the franchise on uneducated people's learning, both of which are supported by Mill, may contradict each other.

Appendix

A Proof of Lemma 1

Consider $w \leq n-2$.

Part 1

We first examine the weighted juror's incentive. Let p' denote the accuracy of unweighted jurors' signals where $p' \in \{p_H, p_L\}$. Then, the probability that the weighted juror becomes pivotal when the defendant is guilty is

$$\Pr(\text{piv}_1|G, w, \mathbf{t}_{-1}) = \sum_{\nu=\frac{n-w}{2}}^{\frac{n+w}{2}-1} \binom{n-1}{\nu} p'^{\nu}(1-p')^{n-1-\nu},$$

which is increasing in $w \in \{1, 3, 5, 7, \ldots, n-2\}$.[34][35] Note that this probability is independent of the weighted juror's signal accuracy. Thus, regardless of her and unweighted jurors' investment behavior, the weighted juror is more likely to be pivotal as w increases.

Part 2

Second, I examine an unweighted juror's incentive. Take an arbitrary unweighted juror j. The probability that an unweighted juror becomes pivotal decreases as w increases if and only if $\Pr(\text{piv}_j|G, w+2) < \Pr(\text{piv}_j|G, w)$. Suppose that the weighted juror's signal

[34] $\sum_{\nu=\frac{n-w}{2}}^{\frac{n+w}{2}-1} \binom{n-1}{\nu} p'^{\nu}(1-p')^{n-1-\nu}$ is increasing in w because $\frac{n-w}{2}$ is decreasing in w and $\frac{n+w}{2}-1$ is increasing in w.

[35] $\mathbf{t}_{-1} := \mathbf{t}_j$ for $j \neq 1$

accuracy is $p \in \{p_H, p_L\}$ and unweighted jurors' signal accuracy is $p' \in \{p_H, p_L\}$.

$$
\begin{aligned}
&\Pr(\mathrm{piv}_j|G, w+2) < \Pr(\mathrm{piv}_j|G, w) \\
&\Leftrightarrow p\binom{n-2}{\frac{n-w}{2}-2}p'^{\frac{n-w}{2}-2}(1-p')^{\frac{n+w}{2}} + (1-p)\binom{n-2}{\frac{n+w}{2}}p'^{\frac{n+w}{2}}(1-p')^{\frac{n-w}{2}-2} \\
&\quad < p\binom{n-2}{\frac{n-w}{2}-1}p'^{\frac{n-w}{2}-1}(1-p')^{\frac{n+w}{2}-1} + (1-p)\binom{n-2}{\frac{n+w}{2}-1}p'^{\frac{n+w}{2}-1}(1-p')^{\frac{n-w}{2}-1} \\
&\Leftrightarrow \binom{n-2}{\frac{n+w}{2}}\left\{pp'^{\frac{n-w}{2}-2}(1-p')^{\frac{n+w}{2}} + (1-p)p'^{\frac{n+w}{2}}(1-p')^{\frac{n-w}{2}-2}\right\} \\
&\quad < \binom{n-2}{\frac{n+w}{2}-1}\left\{pp'^{\frac{n-w}{2}-1}(1-p')^{\frac{n+w}{2}-1} + (1-p)p'^{\frac{n+w}{2}-1}(1-p')^{\frac{n-w}{2}-1}\right\}
\end{aligned}
$$

Multiply by $(1-p)^{-1}p'^{\frac{-n+w}{2}+1}(1-p')^{-\frac{n+w}{2}+1}$,

$$
\Leftrightarrow (n-w-2)\cdot\left\{\frac{p}{1-p}\cdot\frac{1-p'}{p'} + \left(\frac{p'}{1-p'}\right)^{w+1}\right\} < (n+w)\cdot\left\{\frac{p}{1-p} + \left(\frac{p'}{1-p'}\right)^{w}\right\}
$$

Let $P := \frac{p}{1-p}$, $P' := \frac{p'}{1-p'}$,

$$
\begin{aligned}
&\Leftrightarrow (n-w-2)\cdot\left(P\cdot P'^{-1} + P'^{w+1}\right) < (n+w)\cdot(P + P'^{w}) \\
&\Leftrightarrow n\left(P\cdot P'^{-1} + P'^{w+1} - P - P'^{w}\right) < w\left(P + P'^{w}\right) + (w+2)\left(P\cdot P'^{-1} + P'^{w+1}\right) \\
&\Leftrightarrow n(1-P'^{-1})(P'^{w+1} - P) < w(1+P'^{-1})(P'^{w+1}+P) + 2\left(P\cdot P'^{-1} + P'^{w+1}\right) \\
&\Leftrightarrow \begin{cases} n < w\cdot\frac{(1+P'^{-1})(P'^{w+1}+P)}{(1-P'^{-1})(P'^{w+1}-P)} + 2\cdot\frac{P\cdot P'^{-1}+P'^{w+1}}{(1-P'^{-1})(P'^{w+1}-P)} & \text{if } P'^{w+1} > P \\ \text{holds for any n, w} & \text{if } P'^{w+1} \leq P \end{cases}
\end{aligned}
$$

By $N(P, P', w) := w\cdot\frac{(1+P'^{-1})(P'^{w+1}+P)}{(1-P'^{-1})(P'^{w+1}-P)} + 2\cdot\frac{P\cdot P'^{-1}+P'^{w+1}}{(1-P'^{-1})(P'^{w+1}-P)}$,

$$
\Leftrightarrow \begin{cases} n < N(P, P', w) & \text{if } P'^{w+1} > P \\ \text{holds for any n, w} & \text{if } P'^{w+1} \leq P \end{cases} \tag{4}
$$

Thus, the probabilities that unweighted jurors become pivotal are:

1. if $P'^{w+1} > P$,
   (a) decreasing in w for $n < N(P, P', w)$
   (b) independent of w for $n = N(P, P', w)$
   (c) increasing in w for $n > N(P, P', w)$

2. if $P'^{w+1} \leq P$, decreasing in w for all n

B Proof of Proposition 1

Recall that a juror j has an incentive to invest in her information if and only if $c \leq f_j(w, \mathbf{t}_{-j})$. Define $w_j(c, \mathbf{t})$ as a function $w_j : C \times \mathbf{T} \to W \cup \{-\infty, \infty\}$ as follows.

$$w_1(c, \mathbf{t}) := \begin{cases} -\infty & \text{if } c < f_1(1, \mathbf{t}) \\ w^*(c, \mathbf{t}) & \text{if } c \in [f_1(1, \mathbf{t}), f_1(n-2, \mathbf{t})] \\ \infty & \text{if } c > f_1(n-2, \mathbf{t}) \end{cases}$$

where $w^*(c, \mathbf{t})$ is an integer such that $f_1(w^* - 2, \mathbf{t}) < c \leq f_1(w^*, \mathbf{t})$. [36]

$$w_j(c, \mathbf{t}) := \begin{cases} \infty & \text{if } c < f_j(n-2, \mathbf{t}) \\ w^*(c, \mathbf{t}) & \text{if } c \in [f_j(n-2, \mathbf{t}), f_j(1, \mathbf{t})] \\ -\infty & \text{if } c > f_j(1, \mathbf{t}) \end{cases}$$

for $j \neq 1$ where $w^{**}(c, \mathbf{t})$ is an integer such that $f_j(w^{**} + 2, \mathbf{t}) < c \leq f_j(w^{**}, \mathbf{t})$. [37]

The function $w_1(c, \mathbf{t})$ describes the minimum weight for the weighted juror 1 to have an incentive to invest when the cost is c and the strategy profile is $\mathbf{t}$.[38] Thus, the weighted juror 1 has an incentive to invest upon$(w, c, \mathbf{t})$ if and only if $w \geq w_1(c, \mathbf{t})$. On the other hand, $w_j(c, \mathbf{t})$ describes the maximum weight for the unweighted juror $j \neq 1$ to have an incentive to invest when the cost is c and the strategy profile is $\mathbf{t}$. Thus, the unweighted juror j has an incentive to invest upon$(w, c, \mathbf{t})$ if and only if $w \leq w_j(c, \mathbf{t})$.[39] The function

[36] If $c \in [f_1(1, \mathbf{t}), f_1(n-2, \mathbf{t})]$, there is always an unique integer w^* that satisfies

$$f_1(w^* - 2, \mathbf{t}) < c \leq f_1(w^*, \mathbf{t}).$$

[37] If $c \in [f_j(n-2, \mathbf{t}), f_j(1, \mathbf{t})]$, there is always an unique integer w^{**} that satisfies

$$f_j(w^* + 2, \mathbf{t}) < c \leq f_j(w^{**}, \mathbf{t}).$$

[38] Note that $w \in \{1, 3, \ldots, n-2\}$.

[39] For simplicity, I assume that jurors invest if they are indifferent between investing and not investing.

$w^*(c, \mathbf{t})$ is increasing in c and $w^{**}(c, \mathbf{t})$ is decreasing in c.[40][41]

There are four possible strategy profiles, $\mathbf{t} = (1, 1, \ldots, 1), (1, 0, \ldots, 0), (0, 1, \ldots, 1), (0, 0, \ldots, 0)$. In the following, I find (c, w) for which each strategy profile forms equilibrium. For convenience, I assume $j \neq 1$, i.e., juror j is an unweighted juror in the following part of this proof.

A) A strategy profile $\mathbf{t} = (1, 1, \ldots, 1)$ is an equilibrium if $(c, w) \in \{(c, w) | w_1(c, \mathbf{t}) \leq w \leq w_j(c, \mathbf{t})\}$ where

$$w_1(c, \mathbf{t}) = \begin{cases} -\infty & \text{if } c < c_2 \\ w^*(c, \mathbf{t}) & \text{if } c \in [c_2, c_3] \\ \infty & \text{if } c > c_3 \end{cases}$$

$$w_j(c, \mathbf{t}) = \begin{cases} \infty & \text{if } c < c_1 \\ w^{**}(c, \mathbf{t}) & \text{if } c \in [c_1, c_2] \\ -\infty & \text{if } c > c_2 \end{cases}$$

for $j \neq 1$ and $\mathbf{t} = (1, 1, \ldots, 1)$.

In Figure 4 (resp. Figure 5), the red area represents (w, c) for which the weighted (resp. unweighted) juror has an incentive to invest in her information while the blue area represents those for which she has no incentive to do so. As a result, the black trapezoid in Figure 6 describes $\{c, w\}$ for which $\mathbf{t} = (1, 1, \ldots, 1)$ is an equilibrium.

[40]The function $w^{**}(c, \mathbf{t})$ is decreasing in c because I focus on the cases where unweighted jurors are less likely to be pivotal with w.

[41]Note that $w^{**}(c, \mathbf{t}) \leq w^*(c, \mathbf{t})$

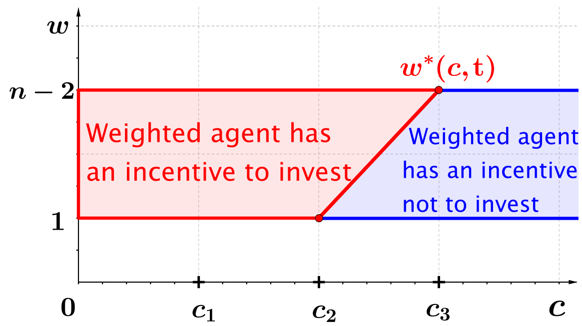

Figure 4: $w_1(c, \mathbf{t})$ for $\mathbf{t} = (1, 1, \ldots, 1)$

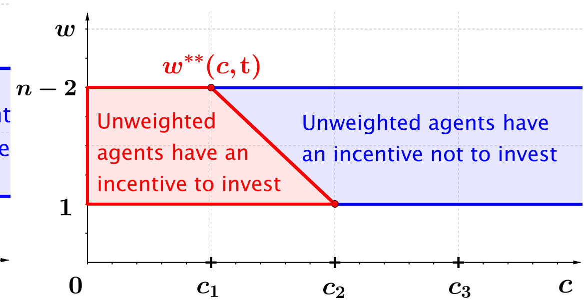

Figure 5: $w_j(c, \mathbf{t})$ for $\mathbf{t} = (1, 1, \ldots, 1)$

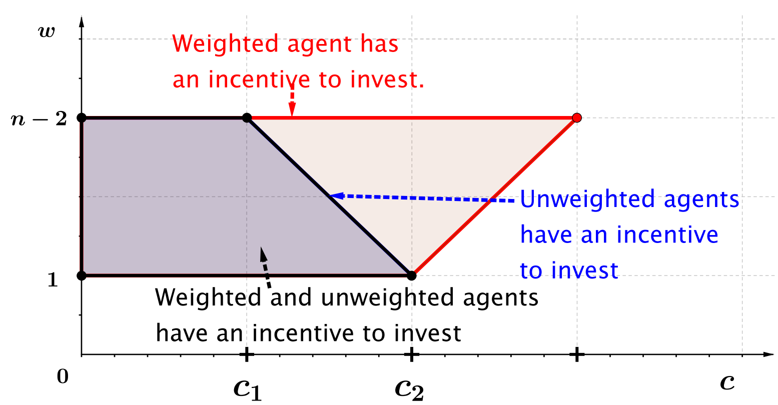

Figure 6: $\{(c, w) | w_1(c, \mathbf{t}) \leq w \leq w_j(c, \mathbf{t})\}$ for $\mathbf{t} = (1, 1, \ldots, 1)$

B) A strategy profile $\mathbf{t} = (1, 0, \ldots, 0)$ is an equilibrium if $(c, w) \in \{(c, w)|w \geq w_1(c, \mathbf{t})\ \&\ w > w_j(c, \mathbf{t})\}$ where

$$w_1(c, \mathbf{t}) = \begin{cases} -\infty & \text{if } c < c_5 \\ w^*(c, \mathbf{t}) & \text{if } c \in [c_5, c_6] \\ \infty & \text{if } c > c_6 \end{cases}$$

$$w_j(c, \mathbf{t}) = \begin{cases} \infty & \text{if } c < c_3 \\ w^{**}(c, \mathbf{t}) & \text{if } c \in [c_3, c_4] \\ -\infty & \text{if } c > c_4 \end{cases}$$

for $j \neq 1$ and $\mathbf{t} = (1, 0, \ldots, 0)$.

In Figure 7 (resp. Figure 8), the red area represents (w, c) for which the weighted (resp. unweighted) juror has an incentive to invest in her information while the blue area represents those for which she has no incentive to do so and the green line represents those for which she is indifferent. As a result, the black trapezoid in Figure 9 describes $\{c, w\}$ for which $\mathbf{t} = (1, 0, \ldots, 0)$ is an equilibrium.

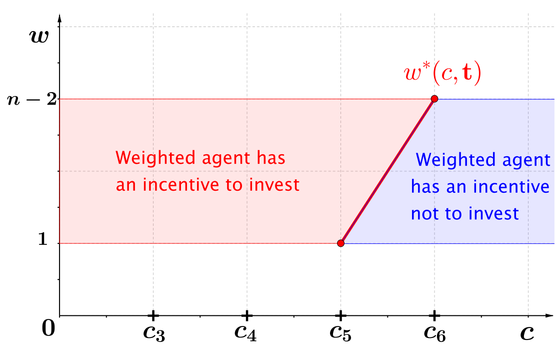

Figure 7: $w_1(c, \mathbf{t})$ for $\mathbf{t} = (1, 0, \ldots, 0)$

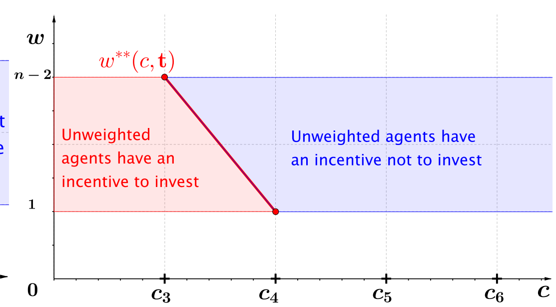

Figure 8: $w_{-1}(c, \mathbf{t})$ for $\mathbf{t} = (1, 0, \ldots, 0)$

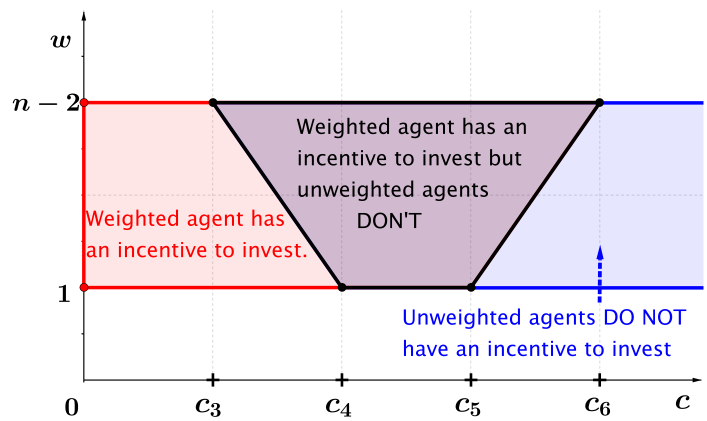

Figure 9: $\{(c,w)|w \geq w_1(c,\mathbf{t})\ \&\ w \geq w_j(c,\mathbf{t})\}$ for $\mathbf{t}=(1,0,\ldots,0)$

C) A strategy profile $\mathbf{t}=(0,0\ldots,0)$ is an equilibrium if and only if $(c,w) \in \{(c,w)|w_j(c,\mathbf{t}) < w < w_1(c,\mathbf{t})\}$. Similarly, the black trapezoid in Figure 10 describes $\{c,w\}$ for which $\mathbf{t}=(0,0,\ldots,0)$ is an equilibrium.

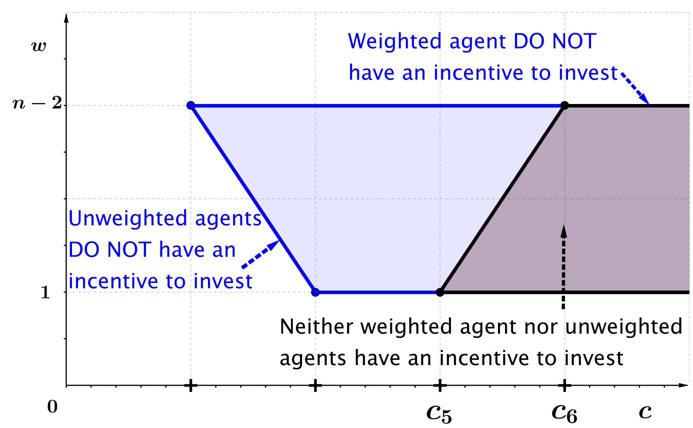

Figure 10: $\{(c,w)|w_j(c,\mathbf{t}) \leq w \leq w_1(c,\mathbf{t})\}$ for $\mathbf{t}=(0,0,\ldots,0)$

D) A strategy profile $\mathbf{t}=(0,1,\ldots,1)$ is an equilibrium if and only if

$$(c,w) \in \{(c,w)|w < w_1(c,\mathbf{t})\ \&\ w \leq w_j(c,\mathbf{t})\},$$

which is an empty set.

Therefore, Figure 11 represents the equilibria where $\mathbf{t} = (1, 1, \ldots, 1), (1, 0, \ldots, 0), (0, 0, \ldots, 0)$.

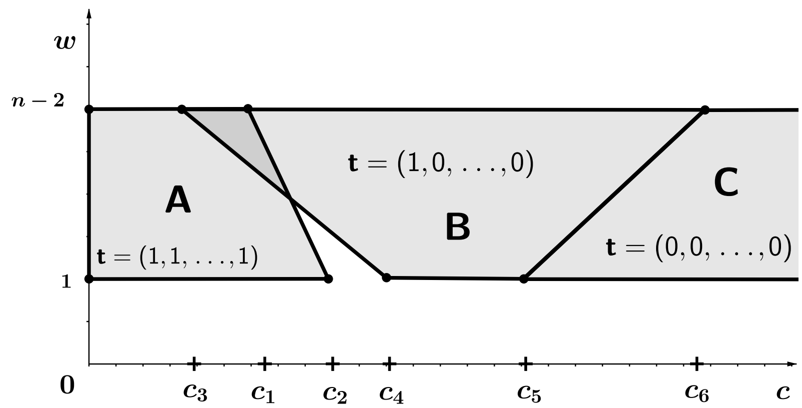

Figure 11: $\{(c, w) | w_1(c, \mathbf{t}) \leq w \leq w_j(c, \mathbf{t})\}$ for $\mathbf{t} = (1, 1, \ldots, 1)$

C Proof of Proposition 2

In Step 1, I show that the probability of making a correct decision is decreasing in w and compute the optimal weight for $c \in [0, c_5] \cup (c_6, \infty)$. Next, I compute the optimal weight for $c \in (c_5, c_6]$ in Step 2.

Step 1: $\mathbf{V(w+2,t)-V(w,t)<0}$

First, I show that the probability of making a correct decision is decreasing in w given any investment behavior.

$$
\begin{aligned}
&V(w+2,\mathbf{t})-V(w,\mathbf{t})\\
&= p\sum_{x\geq\frac{n-w}{2}-1}\binom{n-1}{x}p'^{x}(1-p')^{n-1-x}+(1-p)\sum_{x\geq\frac{n+w}{2}+1}\binom{n-1}{x}p'^{x}(1-p')^{n-1-x}\\
&\quad-\left(p\sum_{x\geq\frac{n-w}{2}}\binom{n-1}{x}p'^{x}(1-p')^{n-1-x}+(1-p)\sum_{x\geq\frac{n+w}{2}}\binom{n-1}{x}p'^{x}(1-p')^{n-1-x}\right)\\
&=\binom{n-1}{\frac{n+w}{2}}p'^{\frac{n-w}{2}-1}(1-p')^{\frac{n-w}{2}-1}\left\{p(1-p')^{w+1}-(1-p)p'^{w+1}\right\},
\end{aligned}
$$

which is less than 0 for all $(p,p')\in\{p_H,p_L\}\times\{p_H,p_L\}$ because $p_H<\frac{\left(\frac{p_L}{1-p_L^{w+1}}\right)^{w+1}}{1+\left(\frac{p_L}{1-p_L^{w+1}}\right)^{w+1}}$. Thus,

$$
V(w+2,\mathbf{t})-V(w,\mathbf{t})<0 \tag{5}
$$

for all $\mathbf{t}\in\{(1,\ldots,1),(1,0,\ldots,0),(0,\ldots,0),(0,1\ldots,1)\}$. It follows that regardless of the qualities of signals that jurors receive, the probability of making a correct decision is decreasing in w.

Therefore, the optimal weight is $w=1$ for $c\in[0,c_2]\cup[c_4,c_5]\cup(c_6,\infty)$ and $w=w^{**}(c,(1,0,\ldots,0))$ for $c\in(c_2,c_4)$. For $c\in(c_5,c_6]$, the optimal weight is either $w^{*}(c,(1,0,\ldots,0))$ where the equilibrium is $\mathbf{t}=(1,\ldots,0)$ or 1 where the equilibrium is $\mathbf{t}=(0,0,\ldots,0)$.

Step 2: The optimal weight for $c\in(c_5,c_6]$

Step 1 shows the optimal weight for $c\in[0,c_5]\cup(c_6,\infty)$. Next, I compute the optimal weight for $c\in(c_5,c_6]$, which is either $w^{*}(c,(1,0,\ldots,0)$ where the equilibrium is $\mathbf{t}=(1,\ldots,0)$ or 1 where the equilibrium is $\mathbf{t}=(0,0,\ldots,0)$. The optimal weight is $w^{*}(c,(1,0,\ldots,0)$ if

$U(c) > 0$ and 1 if $U(c) < 0$ where

$$U(c) := V(w^*(c,(1,0,\dots,0)),(1,\dots,0)) - V(1,(0,\dots,0)).$$

1) $U(c_5) > 0$

Since $w^*(c_5,(1,0,\dots,0) = 1$ by construction,

$$\begin{aligned}
&U(c_5)\\
&= V(1,(1,\dots,0)) - V(1,(0,\dots,0))\\
&= (p_H - p_L)\sum_{x\geq\frac{n-1}{2}}\binom{n-1}{x}p_L^x(1-p_L)^{n-1-x} + (p_L - p_H)\sum_{x\geq\frac{n+1}{2}}\binom{n-1}{x}p_L^x(1-p_L)^{n-1-x}\\
&= (p_H - p_L)\binom{n-1}{\frac{n-1}{2}}p_L^{\frac{n-1}{2}}(1-p_L)^{\frac{n-1}{2}} \quad > 0.
\end{aligned}$$

2) $U(c)$ **is decreasing in** c

By (5), $V(w,(1,\dots,0))$ is decreasing in w. Since $w^*(c,(1,0,\dots,0))$ is decreasing in c, $U(c)$ is also decreasing in c.

3) Optimal weight for $c \in (c_5, c_6]$

Since $w^*(c_6,(1,0,\dots,0)) = n-2$ by construction,

$$U(c_6) = V(n-2,(1,\dots,0)) - V(1,(0,\dots,0)),$$

which may take positive or negative values, depending on the parameters, p_H, p_L, and n.[42] If $U(c_6) < 0$, there exists $c_7 \in (c_5, c_6)$ such that $U(c) = 0$. In this case, $\hat{w} = w^*(c,(1,0,\dots,0))$ for $c \in (c_5, c_7]$ and $\hat{w} = 1$ for $c \in [c_7, c_6)$. If $U(c_6) \geq 0$, $\hat{w} = 1$ for

[42]For example,

$$V(n-2,(1,\dots,0)) - V(1,(0,\dots,0) = 0.3 \quad > 0$$

for $(p_H, p_L) = (0.8, 0.5)$ and $n = 1001$ while

$$V(n-2,(1,\dots,0)) - V(1,(0,\dots,0) = -0.2 \ < 0$$

for $(p_H, p_L) = (0.8, 0.7)$ and $n = 1001$.

$c \in [c_5, c_6)$.[43]

D Future extension: more than one weighted jurors

For now, I analyzed the case in which there is at most one weighted juror, $m = 1$. For the future research, I intend to analyze the case of $m > 1$ and describe the direction here. The purpose of this future section is to characterizes the equilibria in which weighted jurors invest in information and unweighted jurors do not, and compute the optimal number of weighted jurors and their weights. Those equilibria are denoted by (m, w) and I develop the "(m, w) equilibria" where $1 < m < \frac{n+1}{w+1}$. Since I restrict our attention to symmetric equilibria and assume sincere voting, the (m, w) equilibrium is simply a pair of critical points (c_1^*, c_2^*) such that any weighted jurors invest if and only if $c < c_1^*$ and any unweighted jurors invest if and only if $c < c_2^*$.

Consider the case where jurors $1, \ldots, m$ are weighted and jurors $m+1, \ldots, n$ are unweighted. A weighted juror $j \in \{1, \ldots, m\}$ has an incentive to invest conditional on the other weighted jurors investing and all the unweighted jurors not investing if and only if

$$\begin{aligned} & EU_j[t_j = 1|m, w] - EU_j[t_j = 0|m, w] \geq 0 \\ & \Leftrightarrow \Pr(G, \mathrm{piv}_j|m, w)(p_H - p_L) - c \geq 0 \\ & \Leftrightarrow g_w(m, w) \geq c \end{aligned}$$

where

$$g_w(m, w) := \Pr(G, \mathrm{piv}_j|m, w)(p_H - p_L) =$$
$$\tfrac{1}{2}\left\{\sum_{\nu=0}^{m-1} \tbinom{m-1}{\nu} p_H^{\nu}(1-p_H)^{m-1-\nu} \sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w - w}^{\frac{n+m(w-1)+1}{2}-\nu w - 1} \tbinom{n-m}{\tau} p_L^{\tau}(1-p_L)^{n-m-\tau}\right\}(p_H - p_L)$$

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$$\begin{aligned} U(c_6) &= V(n-2, (1, \ldots, 0)) - V(1, (0, \ldots, 0)) \\ &= p_H\left(1 - (1-p_L)^{n-1}\right) + (1-p_H)p_L^{n-1} - \sum_{x \geq \frac{n+1}{2}} \binom{n}{x} p_L^x (1-p_L)^{n-x} \end{aligned}$$

for $j \in \{1, \dots, m\}$ and $EU_j[s_j|m,w]$ is juror j's expected utility of s_j.

Similarly, an unweighted juror $j \in \{m+1, \dots, n\}$ has an incentive not to invest if and only if

$$\begin{aligned}
&EU_j[s_j = 1|m,w] - EU_j[s_j = 0|m,w] < 0 \\
&\Leftrightarrow \Pr(G, \mathrm{piv}_j|m,w)(p_H - p_L) < c \\
&\Leftrightarrow g_{uw}(m,w) < c
\end{aligned}$$

where

$$\begin{aligned}
&g_{uw}(m,w) := \Pr(G, \mathrm{piv}_j|m,w)(p_H - p_L) \\
&= \tfrac{1}{2}\left\{\sum_{\nu=0}^{m} \binom{m}{\nu} p_H^{\nu}(1-p_H)^{m-\nu} \binom{n-m-1}{\frac{n+m(w-1)+1}{2} - \nu w - 1} p_L^{\frac{n+m(w-1)+1}{2} - \nu w - 1}(1-p_L)^{\frac{n-m(w+1)-1}{2} + \nu w}\right\}(p_H - p_L)
\end{aligned}$$

for $j \in \{m+1, \dots, n\}$.

We define $\bar{c}(m,w)$ by the equality $g_w(m,w) = \bar{c}(m,w)$ for $j \in \{1, \dots, m\}$ and $\underline{c}(m,w)$ by the equality $g_{uw}(m,w) = \underline{c}(m,w)$ for $j \in \{m+1, \dots, n\}$. An equilibrium in our model is a set of juror decision rules, $(\bar{c}, \underline{c})$ such that weighted jurors invest in information if $c \leq \bar{c}(m,w)$ and unweighted jurors invest if $c < \underline{c}(m,w)$. Thus, the (m,w) equilibrium exists if and only if $\underline{c}(m,w) \leq c \leq \bar{c}(m,w)$. Propositions 3 and 4 show that jurors are less likely to be pivotal as the number of weighted jurors increases.

Proposition 3. *Suppose that n is sufficiently large, $p_H \geq \frac{2m-1}{2m+1}$ and $p_L = \frac{1}{2}$. The probability that a weighted juror j becomes pivotal, $\Pr(G, piv_j|m,w)$, is decreasing in the number of weighted jurors, m.*

Proof. See Appendix E. □

Proposition 3 gives sufficient conditions for which a weighted juror is less willing to invest in information as the number of weighted jurors increases. Similarly, Proposition 4 is intended to give sufficient conditions for which an unweighted juror is less willing to invest in information as the number of weighted jurors increases.

Proposition 4. *The probability that an unweighted juror j becomes pivotal,* $\Pr(G, piv_j|m, w)$, *is decreasing in the number of weighted jurors, m.*

Proof. The complete proof is yet to be done. Note that the following Corollary 2 and Proposition 5 are based on this unproven Proposition. □

As a corollary, Propositions 3 and 4 will give the shape of $\underline{c}(m, w)$ and $\overline{c}(m, w)$ as a function of m.

Corollary 2. *Suppose that n is sufficiently large, $p_H \geq \frac{2m-1}{2m+1}$ and $p_L = \frac{1}{2}$. $\underline{c}(m, w)$ and $\overline{c}(m, w)$ are decreasing in m.*

Corollary 2 will imply the range of cost c within which m investments can be made in equilibrium. The expected explanation is as follows. Weighted jurors have incentives to invest if and only if m is small enough to satisfy $\overline{c}(m, w) \geq c$. Denote the largest such m by $\overline{m}(c, w)$, i.e., $\overline{m}(c, w) := \max\{m|\overline{c}(m, w) \leq c\}$. Unweighted jurors have incentives not to invest if and only if m is large enough to satisfy $\underline{c}(m, w) \leq c$. Denote the smallest such m by $\underline{m}(c, w)$, i.e., $\underline{m}(c, w) := \min\{m|\underline{c}(m, w) \leq c\}$. These observations will lead to the following Proposition.

Proposition 5. *Suppose that n is sufficiently large, $p_H \geq \frac{2m-1}{2m+1}$ and $p_L = \frac{1}{2}$. Given c and w, the (m, w) equilibrium exists if and only if $m \in \{\underline{m}, \dots, \overline{m}\}$.*[44]

Proof. Take arbitrary c and w. Suppose that all of m weighted jurors invest in information and all of $(n - m)$ jurors do not invest.

The weighted jurors have incentives to invest if and only if $c \leq \overline{c}(m, w)$. Since $\overline{c}(m, w)$ is decreasing in m (Corollary 2), $\overline{c}(\overline{m}, w) \leq \overline{c}(m, w)$ for all $m \leq \overline{m}$. Hence, for given c and w, the weighted jurors have incentives to invest if and only if $m \leq \overline{m}$.

Similarly, the unweighted jurors have incentives not to invest if and only if $c \geq \underline{c}(m, w)$. Since $\underline{c}(m, w)$ is decreasing in m (Corollary 2), $\underline{c}(\underline{m}, w) \geq \underline{c}(m, w)$ for all $m \geq \underline{m}$. Hence, for given c and w, the unweighted jurors have incentives not to invest if and only if $m \geq \overline{m}$. □

[44](1) $\underline{c}(m, w) \leq \overline{c}(m, w)$ for any (m, w) and (2) $\underline{m}(c, w) \leq \overline{m}(c, w)$ for any (c, w).

In particular, if $\underline{c}(1,w) \leq c \leq \overline{c}(m^*,w)$, the (m,w) equilibrium exists for every $m \in [1,m^*]$, i.e., up to m^* investments can be made in equilibrium for given c and w.

E Proof of Proposition 3

We show that the probability that a weighted voter becomes pivotal is decreasing in m in the following. Suppose that $w \geq 2$, and let $p_L = 1/2$ for simplicity. Subsequent arguments should carry over to the other cases with $p_L \neq 1/2$ by some suitable modifications.

We derive (sufficient) conditions under which it holds that

$$\Pr\left(\text{piv}_j|m+1,w\right) < \Pr\left(\text{piv}_j|m,w\right),$$

where

$$\Pr\left(\text{piv}_j|m,w\right) := \left(\frac{1}{2}\right)^{n-m} (1-p_H)^{m-1} \sum_{\nu=0}^{m-1} \begin{pmatrix} m-1 \\ \nu \end{pmatrix} \left(\frac{p_H}{1-p_H}\right)^{\nu} \Gamma(\nu,m)$$

and

$$\Gamma(\nu,m) = \Gamma_{n,w}(\nu,m) := \sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+m(w-1)+1}{2}-\nu w-1} \begin{pmatrix} n-m \\ \tau \end{pmatrix}. \tag{6}$$

The domain of (ν,m) should be suitably defined, which may depend on (n,w), so that the right-hand side of (6) is well-defined.

Step 1: $\Gamma(\nu, m)$ is strictly decreasing in m for each ν

For our purpose, I first show that $\Gamma(\nu, m)$ is strictly decreasing in m for each ν. Consider the following decomposition:

$$\Gamma(\nu, m+1) = \sum_{\tau=\frac{n+(m+1)(w-1)+1}{2}-\nu w-w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w-1} \begin{pmatrix} n-m-1 \\ \tau \end{pmatrix}$$

$$= \left\{ \underbrace{\sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w-1}}_{(*1)} + \underbrace{\sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+m(w-1)+1}{2}-\nu w-1}}_{(*2)} - \underbrace{\sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w-w-1}}_{(*3)} \right\} \begin{pmatrix} n-m-1 \\ \tau \end{pmatrix}. \tag{7}$$

Note that the number of summands is $\frac{1}{2}(w-1)$ (each) in $(*1)$ and $(*3)$, while that in $(*2)$ is w. Note also that the sum in $(*2)$ is computed over the same values of τ as those in the sum in $\Gamma(\nu, m)$.

Since $\binom{n-m-1}{\tau} = \underbrace{\frac{(n-m-\tau)}{(n-m)}}_{<1}\binom{n-m}{\tau}$, I can write (7) as

$$\Gamma(\nu, m+1) = \Gamma(\nu, m) + \sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+m(w-1)+1}{2}-\nu w-1} \left[\frac{n-m-\tau}{n-m} - 1\right]\binom{n-m}{\tau}$$
$$+ \left\{\underbrace{\sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w-1}}_{(*1)} - \underbrace{\sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w-w-1}}_{(*3)}\right\}\binom{n-m-1}{\tau}$$
$$= \Gamma(\nu, m) + \sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+m(w-1)+1}{2}-\nu w-1} \frac{-\tau}{n-m}\binom{n-m}{\tau}$$
$$+ \sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w-w-1} \left\{\binom{n-m-1}{\tau+w} - \binom{n-m-1}{\tau}\right\}$$
$$= \Gamma(\nu, m) + \underbrace{\sum_{\tau=\frac{n+(m+1)(w-1)+1}{2}-\nu w-w}^{\frac{n+m(w-1)+1}{2}-\nu w-1} \frac{-\tau}{n-m}\binom{n-m}{\tau}}_{<0}$$
$$+ \sum_{\tau=\frac{n+m(w-1)+1}{2}-\nu w-w}^{\frac{n+(m+1)(w-1)+1}{2}-\nu w-w-1} \underbrace{\left\{\binom{n-m-1}{\tau+w} - \binom{n-m-1}{\tau} + \frac{-\tau}{n-m}\binom{n-m}{\tau}\right\}}_{(\#\#)}.$$

To determine the sign of the last term on the right-hand side, look at

$$\binom{n-m}{\tau} = \binom{n-m-1}{\tau}\frac{n-m}{n-m-\tau}$$
$$\binom{n-m-1}{\tau+w} = \frac{(n-m-1-\tau)}{(\tau+w)} \times \cdots \times \frac{(n-m-1-\tau-w+1)}{(\tau+1)}\binom{n-m-1}{\tau}.$$

Therefore,

$$(\#\#) = \left\{\frac{(n-m-1-\tau)}{(\tau+w)} \times \cdots \times \frac{(n-m-1-\tau-w+1)}{(\tau+1)} - \frac{n-m}{n-m-\tau}\right\}\binom{n-m-1}{\tau}.$$

The inside of the curly bracket in the last line is negative for large n. To see this, for example, note that it holds that for large n,

$$\frac{(n-m-1-\tau)}{(\tau+w)} \leq \left[\frac{n-m}{(n-m-\tau)}\right]^{1/w} \Longleftrightarrow \left[\frac{(n-m-\tau-1)}{(\tau+w)}\right]^{w}(n-m-\tau) \leq (n-m)$$

where the inequalities hold since τ takes some value between $\left(\frac{n+m(w-1)+1}{2}-\nu w-w\right)$ and $\left(\frac{n+(m+1)(w-1)+1}{2}-\nu w-w-1\right)$. Suppose m and w are small relatively to n. Then, for n large enough,

$$\left[\frac{(n-m-\tau-1)}{(\tau+w)}\right]^{w} \sim \left[\frac{n-\tau}{\tau}\right]^{w} \sim \left[\frac{n/2}{n/2}\right]^{w} \sim 1.$$[45]

From these arguments, I can write

$$\Gamma(\nu, m+1) = \Gamma(\nu, m) + \underbrace{R(v, m)}_{<0},$$

i.e, $\Gamma(\nu, m)$ is strictly decreasing in m (again, it needs to be confirmed that $\Gamma(a, b)$ is well-defined at $(a, b) = (m, m)$.

Step 2: $\Pr\left(\text{piv}_j | m+1, w\right) - \Pr(\text{piv}) < 0$

Thanks to Step 1, I know that $\Gamma(\nu, m)$ is strictly decreasing in m. By using this, I have

$$\begin{aligned}
&\Pr\left(\text{piv}_j | m+1, w\right) - \Pr\left(\text{piv}_j | m, w\right) \\
&= \left(\frac{1}{2}\right)^{n-m} (1-p_H)^{m-1} \sum_{\nu=0}^{m} \left(\frac{1}{2}\right)^{-1} (1-p_H) \binom{m}{\nu} \left(\frac{p_H}{1-p_H}\right)^{\nu} \Gamma(\nu, m+1) \\
&\quad - \left(\frac{1}{2}\right)^{n-m} (1-p_H)^{m-1} \sum_{\nu=0}^{m-1} \binom{m-1}{\nu} \left(\frac{p_H}{1-p_H}\right)^{\nu} \Gamma(\nu, m)
\end{aligned}$$

$$= \left(\frac{1}{2}\right)^{n-m} (1-p_H)^{m-1} \left\{ \underbrace{\sum_{\nu=0}^{m} \left(\frac{1}{2}\right)^{-1} (1-p_H) \binom{m}{\nu} \left(\frac{p_H}{1-p_H}\right)^{\nu} R(\nu, m)}_{<0} \right.$$

$$+ \underbrace{\left(\frac{1}{2}\right)^{-1} (1-p_H) \binom{m}{m} \left(\frac{p_H}{1-p_H}\right)^{m} \Gamma(m, m)}_{=I}$$

$$\left. + \sum_{\nu=0}^{m-1} \underbrace{\left[\left(\frac{1}{2}\right)^{-1} (1-p_H) \binom{m}{\nu} \binom{m-1}{\nu}\right] \left(\frac{p_H}{1-p_H}\right)^{\nu} \Gamma(\nu, m)}_{=J(\nu)} \right\}. \tag{8}$$

The first term inside the curly bracket is strictly negative since $R(\nu, m) < 0$ (Step 1).

We now consider a condition under which the sum of the 2nd and 3rd terms inside the curly bracket, $I + \sum_{\nu=0}^{m-1} J(\nu)$, is negative. To this end, I use the following result:

$$\Gamma(m, m) < \Gamma(m-1, m),^{46} \tag{9}$$

i.e., $\Gamma(a, m)$ is strictly decreasing in a, whose proof is provided below. Then, noting

$\binom{m}{m} = 1$,

$$I < \left(\frac{1}{2}\right)^{-1} (1-p_H) \binom{m}{m-1} \left(\frac{1}{m} \frac{p_H}{1-p_H}\right) \left(\frac{p_H}{1-p_H}\right)^{m-1} \Gamma(m-1, m).$$

Therefore,

$$\begin{aligned}
& I + \sum_{\nu=0}^{m-1} J(\nu) = [I + J(m-1)] + \sum_{\nu=0}^{m-2} J(\nu) \\
& < \left[\left(\frac{1}{2}\right)^{-1} (1-p_H) \binom{m}{m-1} \left(1 + \frac{1}{m} \frac{p_H}{1-p_H}\right) - \binom{m-1}{m-1}\right] \left(\frac{p_H}{1-p_H}\right)^{m-1} \Gamma(m-1, m) + \sum_{\nu=0}^{m-2} J(\nu) \\
& = \underbrace{[2(m(1-p_H) + p_H) - 1]}_{(@)} \left(\frac{p_H}{1-p_H}\right)^{m-1} \Gamma(m-1, m) \\
& \quad + \sum_{\nu=0}^{m-2} \underbrace{\left[\left(\frac{1}{2}\right)^{-1} (1-p_H) \binom{m}{\nu} - \binom{m-1}{\nu}\right]}_{(@@)} \left(\frac{p_H}{1-p_H}\right)^{\nu} \Gamma(\nu, m),
\end{aligned}$$

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[46]**Proof of (9):** $\Gamma(m,m) < \Gamma(m-1,m)$

By the definition of Γ in (6), I can see that

$$\Gamma(\nu-1, m) := \sum_{\tau = \frac{n+m(w-1)+1}{2} - (\nu-1)w - w}^{\frac{n+m(w-1)+1}{2} - (\nu-1)w - 1} \binom{n-m}{\tau} = \sum_{\tau = \frac{n+m(w-1)+1}{2} - \nu w - w}^{\frac{n+m(w-1)+1}{2} - \nu w - 1} \binom{n-m}{\tau + w},$$

Therefore,

$$\begin{aligned}
\Gamma(m, m) - \Gamma(m-1, m) &= \sum_{\tau = \frac{n+m(w-1)+1}{2} - mw - w}^{\frac{n+m(w-1)+1}{2} - mw - 1} \left[\binom{n-m}{\tau} - \binom{n-m}{\tau + w}\right] \\
&= \sum_{\tau = \frac{n+m(w-1)+1}{2} - \nu w - w}^{\frac{n+m(w-1)+1}{2} - \nu w - 1} \left[\frac{(n-m)!}{\tau!\,(n-m-\tau)!} - \frac{(n-m)!}{(\tau+w)!\,(n-m-\tau-w)!}\right] \\
&= \sum_{\tau = \frac{n+m(w-1)+1}{2} - mw - w}^{\frac{n+m(w-1)+1}{2} - mw - 1} \frac{(n-m)!}{\tau!\,(n-m-\tau)!} \left[1 - \frac{(n-m-\tau)}{(\tau+w)} \times \cdots \times \frac{(n-m-\tau-w+1)}{(\tau+1)}\right] < 0,
\end{aligned}$$

where the last inequality holds by noting that

$$\begin{aligned}
& (\tau + w) < (n - m - \tau) \\
& \Longleftrightarrow 2\tau + w + m < n \\
& \Longleftarrow 2\left(\frac{n + m(w-1) + 1}{2} - mw - 1\right) + w + m < n \\
& \Longleftrightarrow -mw - 1 + w < 0.
\end{aligned}$$

where I below derive the conditions under which (@) and (@@) are negative. As for (@), look at

$$(@) \leq 0 \Longleftrightarrow 2m - 1 \leq (2m+1)\, p_H \Longleftrightarrow \frac{2m-1}{2m+1} \leq p_H \tag{10}$$

For the component of (@@), I only need to consider the case where $0 \leq \nu \leq m-2$. Since $\binom{m-1}{\nu} = \binom{m}{\nu} \dfrac{m-\nu}{m}$,

$$(@@) = \binom{m}{\nu} \left[\left(\frac{1}{2}\right)^{-1} (1 - p_H) - \frac{m-\nu}{m} \right] < \binom{m}{\nu} \left[2\,(1-p_H) - \frac{2}{m} \right]$$

which is negative when

$$1 - \frac{1}{m} \leq p_H. \tag{11}$$

Since (10) implies (11), the condition (10) implies that the sum of the 2nd and 3rd terms inside the curly brackets, $I + \sum_{\nu=0}^{m-1} J(\nu)$, is negative. Therefore, $\frac{2m-1}{2m+1} \leq p_H$ implies that the probability that a weighted juror j becomes pivotal is decreasing in m, i.e., $\Pr\left(\text{piv}_j | m+1, w\right) < \Pr\left(\text{piv}_j | m, w\right)$. $\square$

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