

Liquidity Approach to Financial Crises I

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1 Question

How does economy fluctuate with shocks to productivity and liquidity?

→ Want to develop a canonical model of monetary economy in which money is essential for smooth running of the economy

What are the roles of monetary policy?

Approach: Real business cycles model + limited commitment

present goods

original lender



borrower

resell ↘ claim

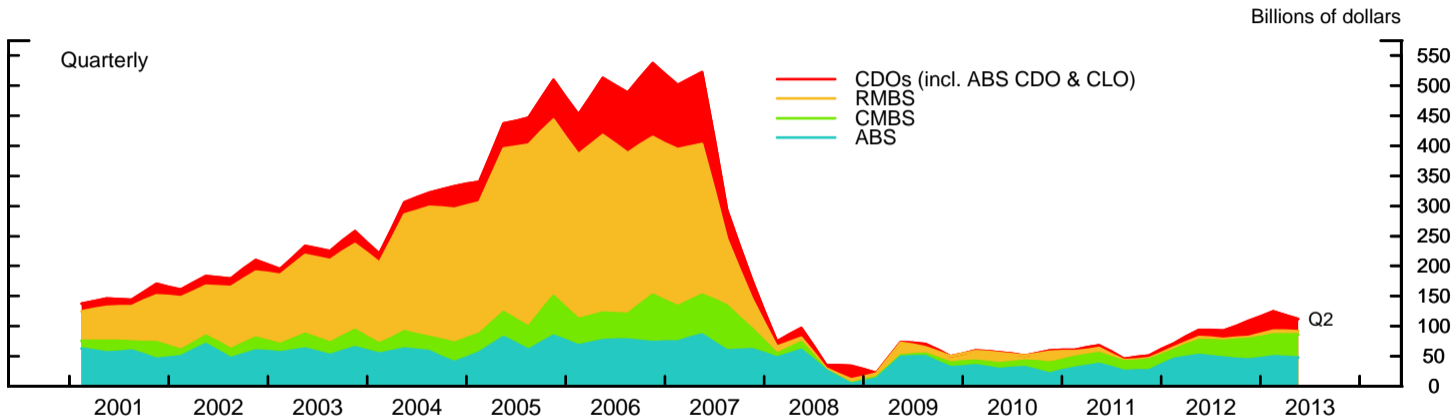
claim to future goods

new lenders

How much can the original lender enforce the borrower to repay? → borrowing constraint

How much can new lenders enforce the borrower to repay? → resaleability constraint

Chart 13
U.S. Securitization Issuance



Note: CLO refers to all securities backed by loans or bonds issued by businesses. CMBS and RMBS refer respectively to securities backed by commercial and residential mortgages. ABS refers to securities backed by consumer loans.

Source: Asset-backed Alert, Commercial Mortgage Alert from Harrison Scott Publications, Inc. (downloaded May 8, 2013).

2 Model

homogeneous output Y_t , capital K_t and fiat money M_t at each date

agents, measure 1: $E_t \sum_{s=t}^{\infty} \beta^{s-t} \log c_t$

All agent use their capital to produce goods:

$$\begin{array}{l} k_t \text{ capital} \\ \text{start of date } t \end{array} \rightarrow \begin{cases} r_t k_t \text{ goods} \\ \lambda k_t \text{ capital} \end{cases} \begin{array}{l} \\ \text{end of date } t \end{array}$$

individually constant returns & decreasing returns in aggregate

$$\begin{aligned} r_t &= a_t K_t^{\alpha-1}, \\ Y_t &= r_t K_t = a_t K_t^{\alpha} \end{aligned}$$

Fraction π of agents can invest in producing new capital:

i_t goods \rightarrow i_t new capital

start of date t \dashrightarrow end of date t

investment opportunities are i.i.d., across people, through time

no insurance market against arrival of investment opportunity

Equity:

capital is specific to the agent who produces it, but he can mortgage future returns by issuing equity

one unit of equity issued at date t promises

$$r_{t+1}, \lambda r_{t+2}, \lambda^2 r_{t+3}, \dots$$

Borrowing Constraint: an investing agent can mortgage at most θ fraction of the future returns from his new capital production

Resaleability Constraint: at each date, an agent can resell at most ϕ_t fraction of his equity holdings $\rightarrow (a_t, \phi_t)$ follows a stationary Markov process

balance sheet at the end of date t	
money: $p_t m_{t+1}$	own equity issued: $q_t^i n_{t+1}^i$
equity of others: $q_t^o n_{t+1}^o$	
own capital stock: $q_t^k k_{t+1}$	net worth

Simplification: at every date, an agent can mortgage up to a fraction ϕ_t of his unmortgaged capital stock

→ equity of the others and unmortgaged capital stock become perfect substitutes: $q_t^o = q_t^k = q_t^i = q_t$ & $n_t^o + k_t - n_t^i = n_t$

Flow-of-funds and liquidity constraints:

$$c_t + i_t + q_t(n_{t+1} - i_t) + p_t m_{t+1} = (r_t + \lambda q_t)n_t + p_t m_t$$

$$n_{t+1} \geq (1 - \theta)i_t + (1 - \phi_t)\lambda n_t$$

$$m_{t+1} \geq 0$$

Government chooses M_{t+1} (money supply), N_{t+1}^g (government paper holding) subject to the budget constraint:

$$q_t(N_{t+1}^g - \lambda N_t^g) = r_t N_t^g + p_t(M_{t+1} - M_t)$$

$$\frac{N_{t+1}^g}{K} = \psi_a \frac{a_t - a}{a} + \psi_\phi \frac{\phi_t - \phi}{\phi}$$

Claim 1: $(1 - \lambda)\theta + \pi\lambda\phi \geq (1 - \lambda)(1 - \pi) \leftrightarrow$ unconstrained ($q_t = 1$), first best allocation, no money ($p_t = 0$)

$E_t MPK =$ rate of return on equity \simeq time preference rate

$(1 - \lambda)\theta + \pi\lambda\phi < (\beta - \lambda)(1 - \pi) \rightarrow$ liquidity constrained ($q_t > 1$), monetary equilibrium exists ($p_t > 0$)

Aggregate Recursive Equilibrium: $(p_t, q_t, I_t, K_{t+1}, N_{t+1}^g)$ as functions of aggregate state $(K_t, N_t^g, a_t, \phi_t)$ satisfying:

$$a_t K_t^\alpha = I_t + (1 - \beta).$$

$$\{[r_t + (1 - \pi + \pi\phi_t)\lambda q_t + \pi(1 - \phi_t)\lambda q_t^R]N_t + p_t M_t\}$$

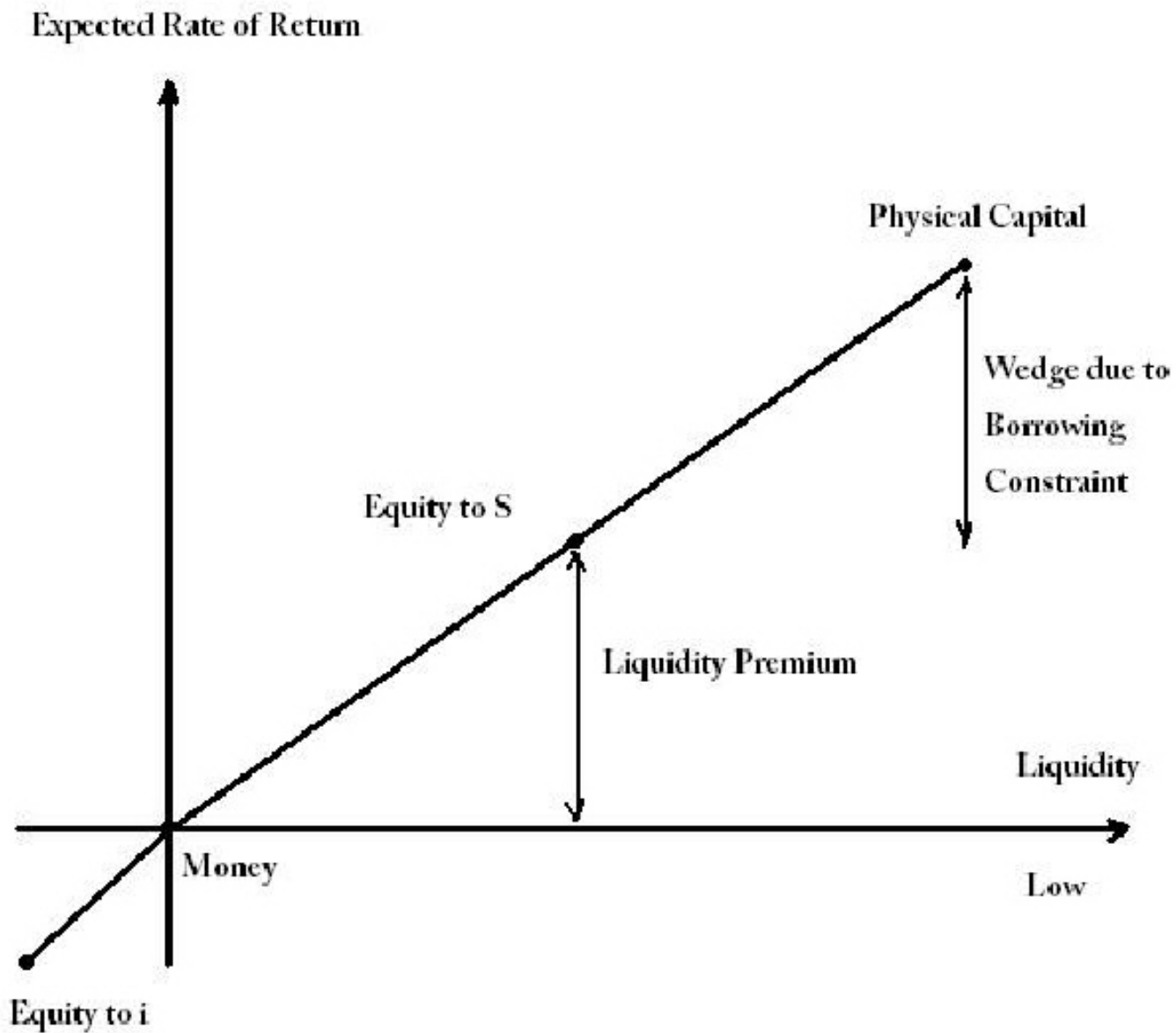
$$I_t = \pi \frac{\beta[(r_t + \lambda\phi_t q_t)N_t + p_t M_t] - (1 - \beta)(1 - \phi_t)\lambda q_t^R N_t}{1 - \theta q_t}$$

$$(1 - \pi)E_t \left[\frac{(r_{t+1} + \lambda q_{t+1})/q_t - p_{t+1}/p_t}{C_{t+1}^{ss}} \right]$$

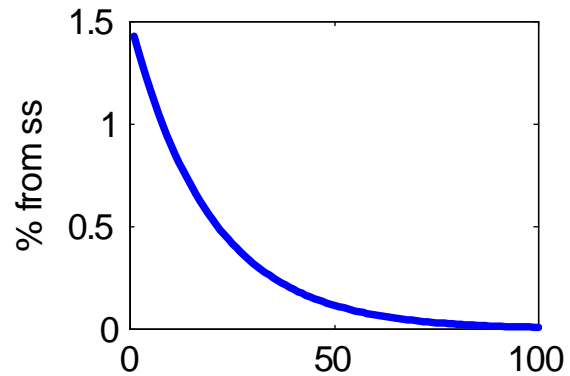
$$= \pi E_t \left[\frac{p_{t+1}/p_t - [r_{t+1} + \lambda\phi_{t+1}q_{t+1} + \lambda(1 - \phi_{t+1})q_{t+1}^R]/q_t}{C_{t+1}^{si}} \right]$$

$$K_{t+1} = \lambda K_t + I_t = N_{t+1} + N_{t+1}^g$$

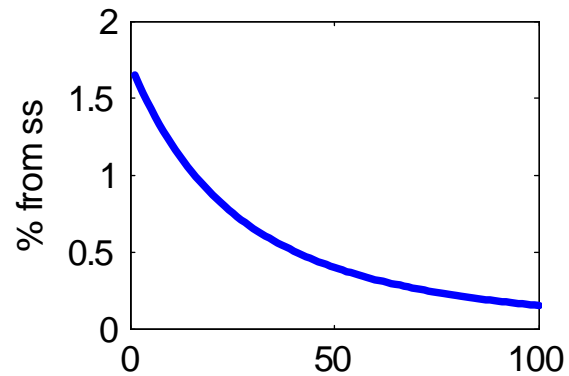
$$q_t^R = \frac{1 - \theta q_t}{1 - \theta}$$



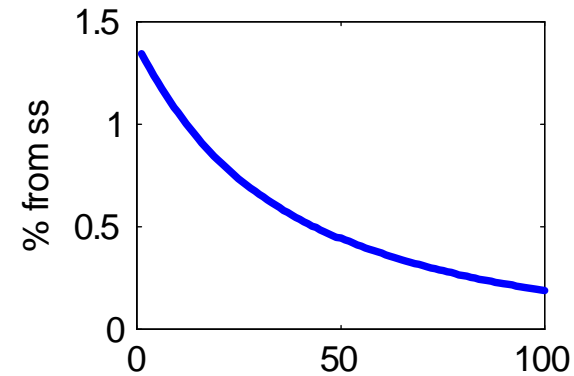
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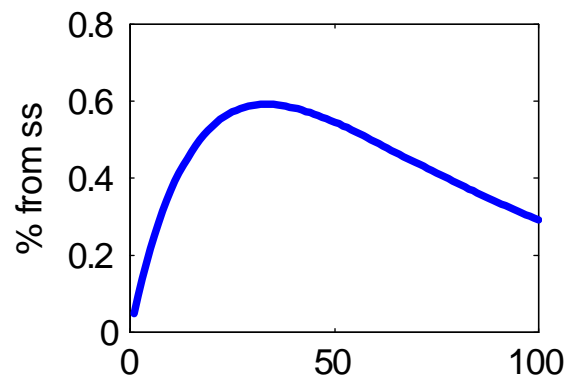
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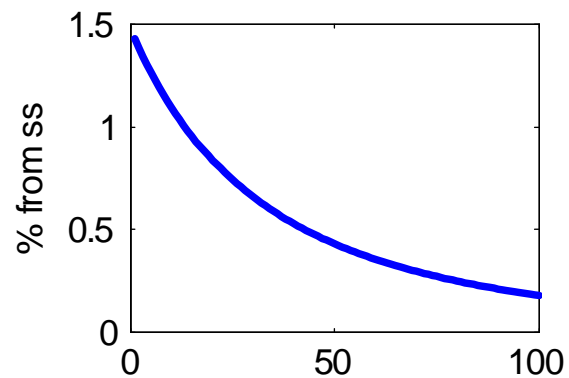
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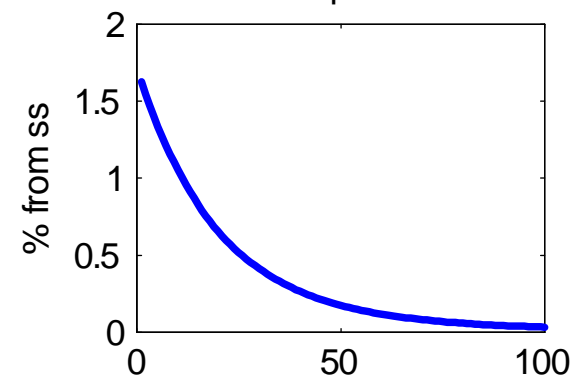
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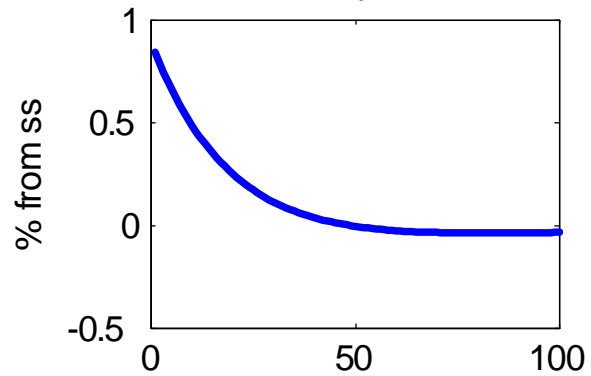
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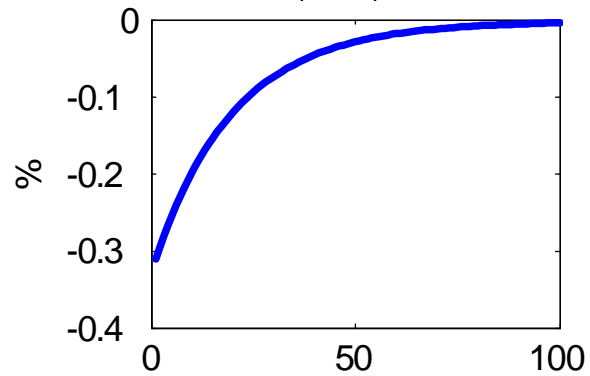
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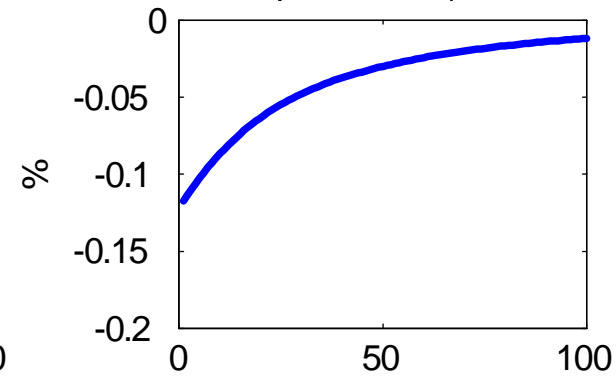
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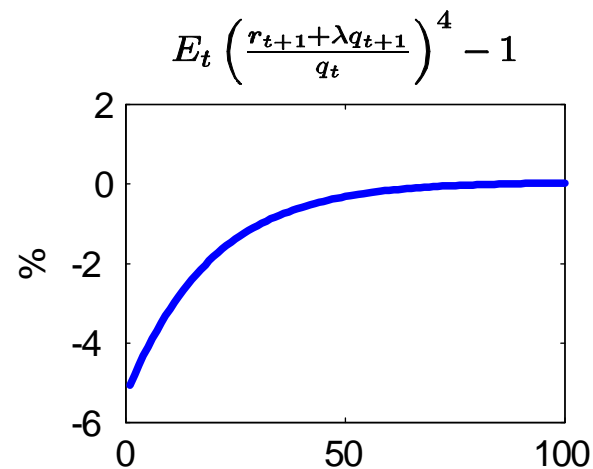
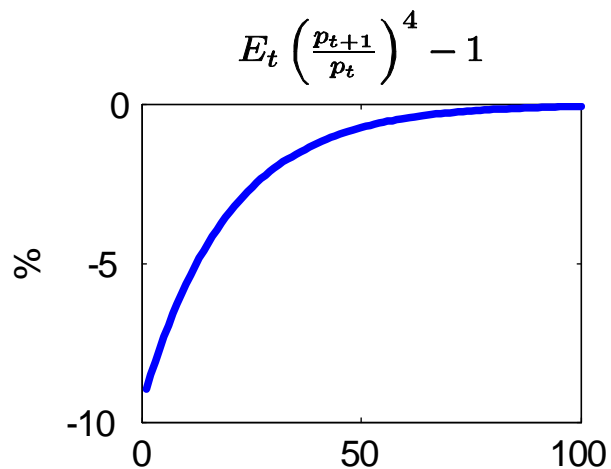
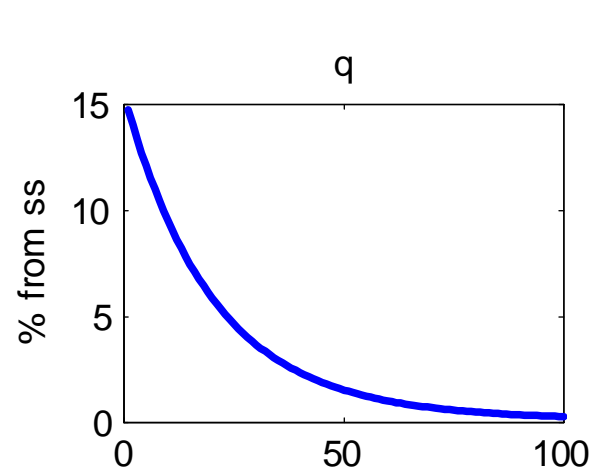
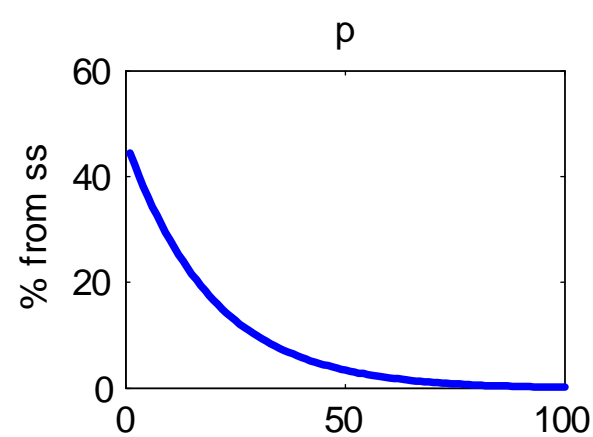
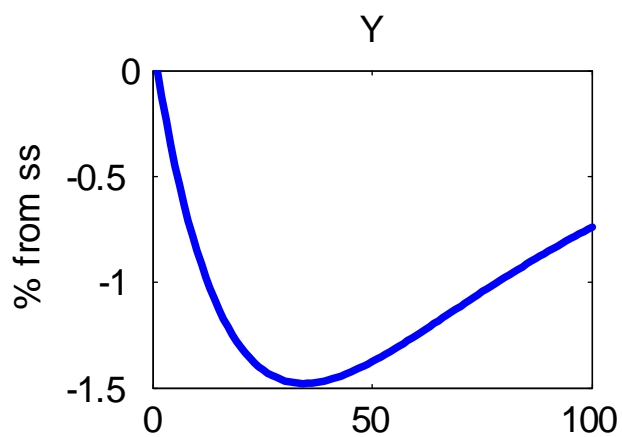
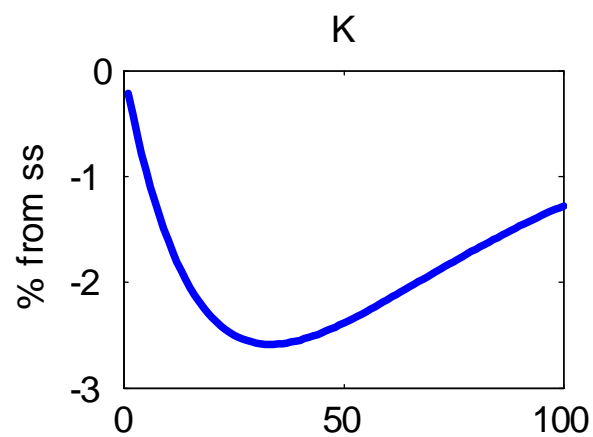
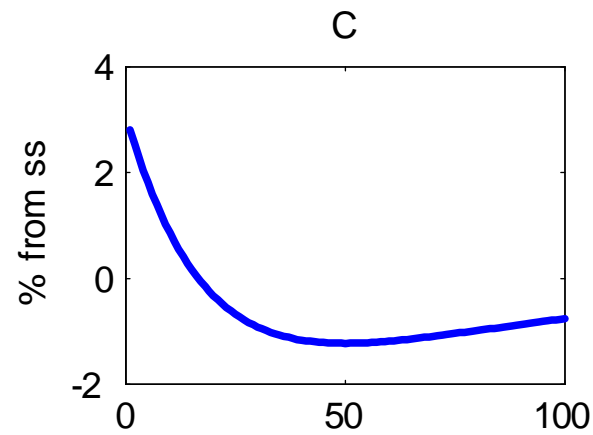
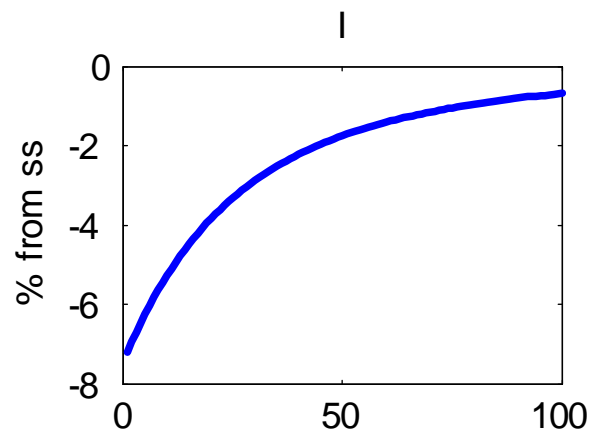
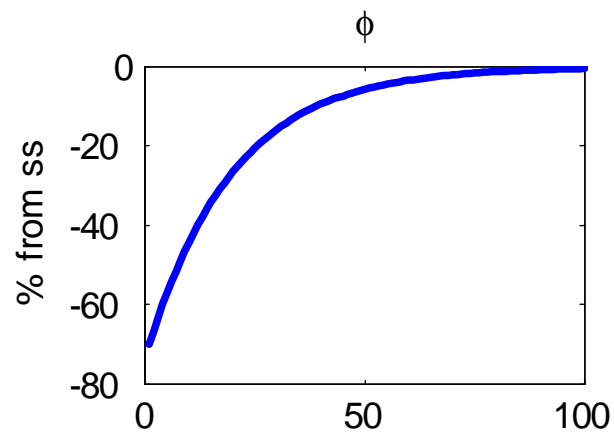


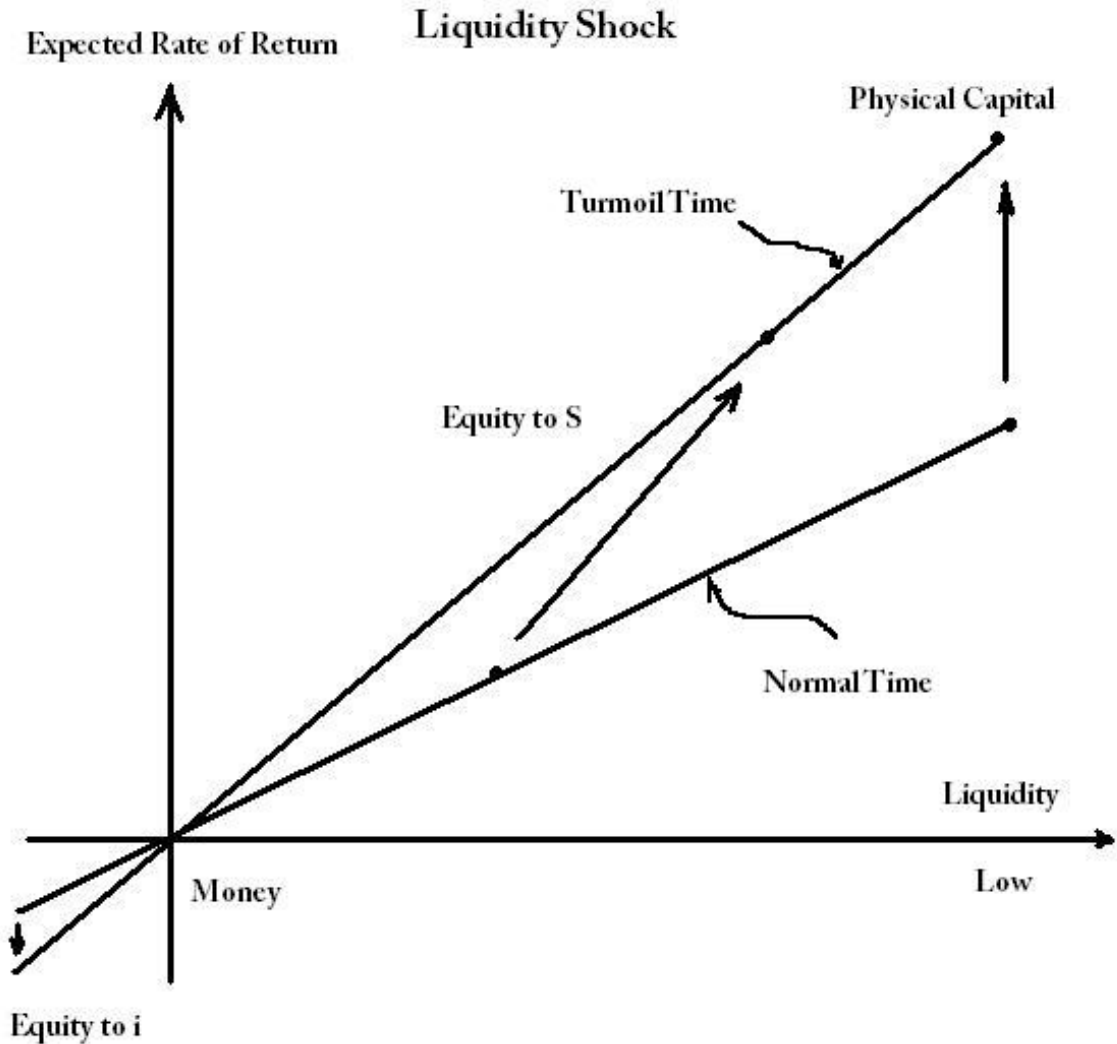
$$E_t \left(\frac{p_{t+1}}{p_t} \right)^4 - 1$$



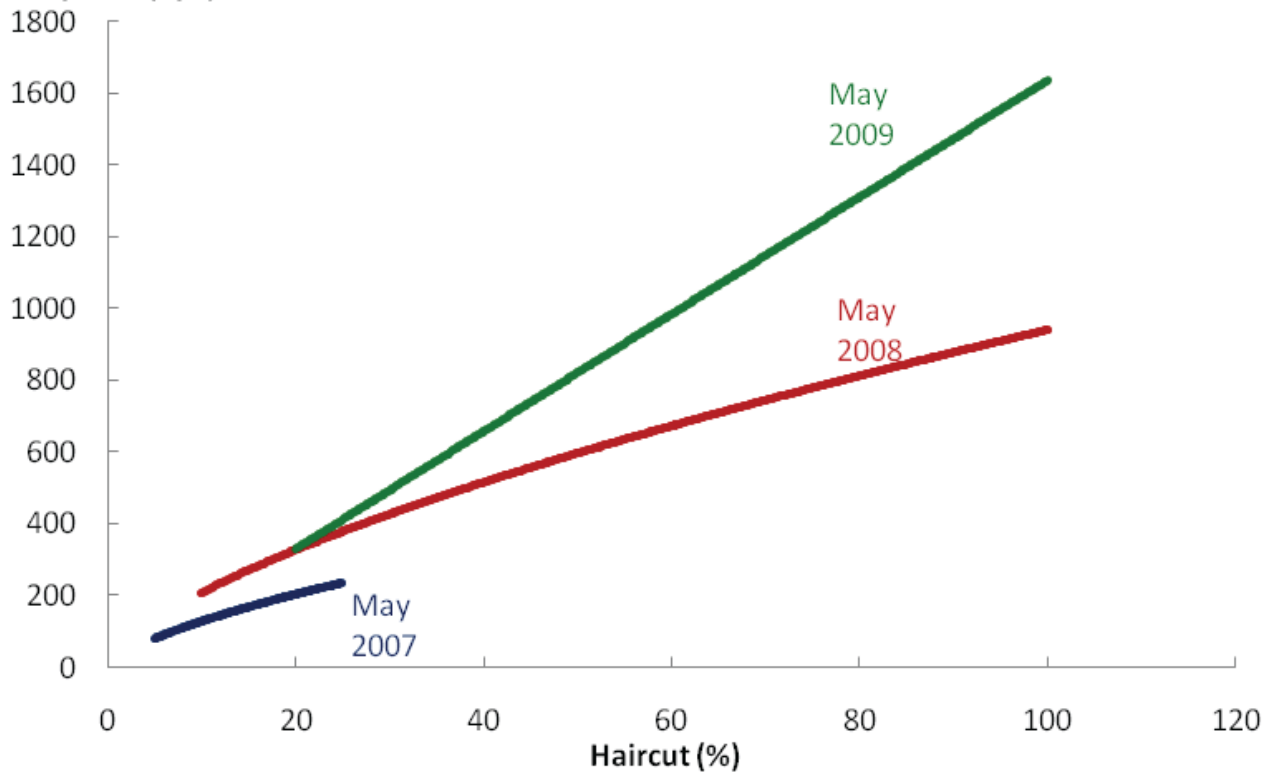
$$E_t \left(\frac{r_{t+1} + \lambda q_{t+1}}{q_t} \right)^4 - 1$$

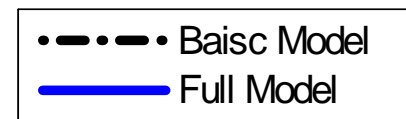
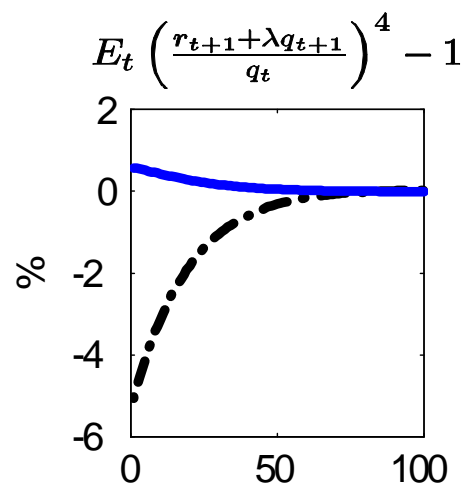
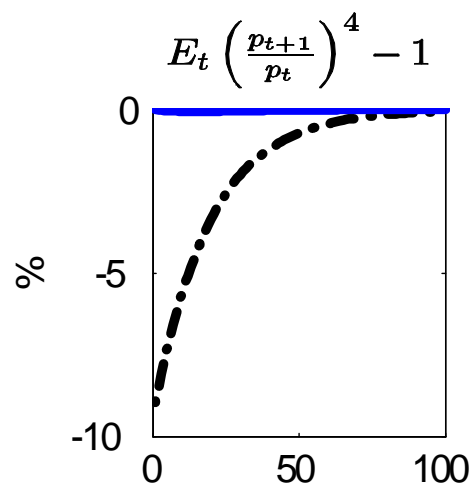
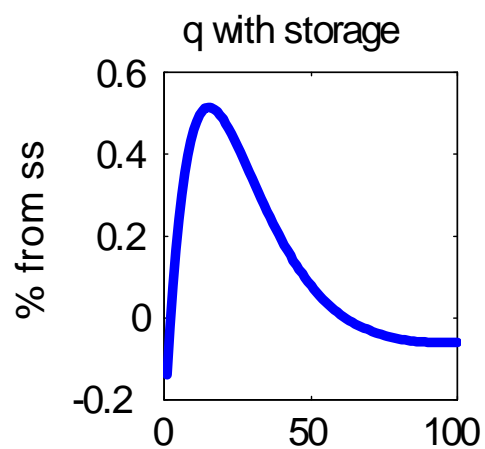
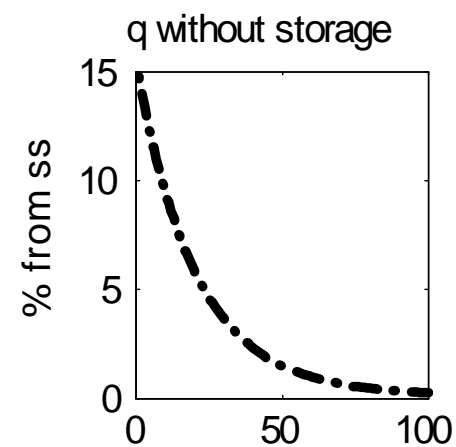
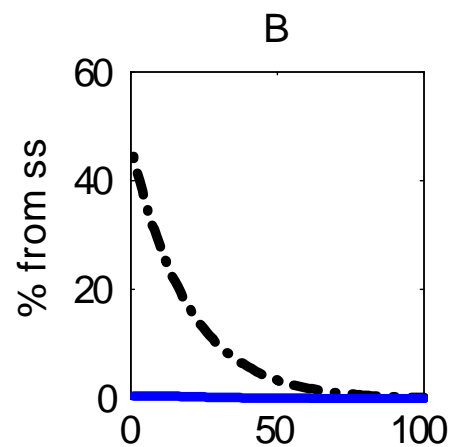
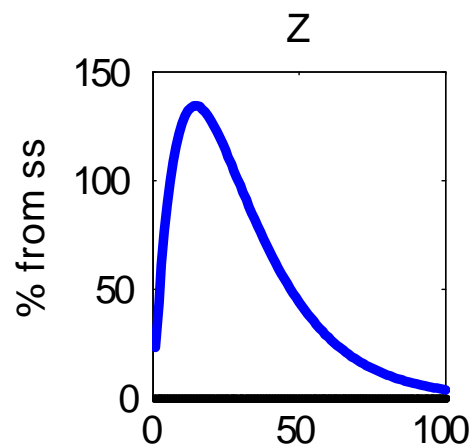
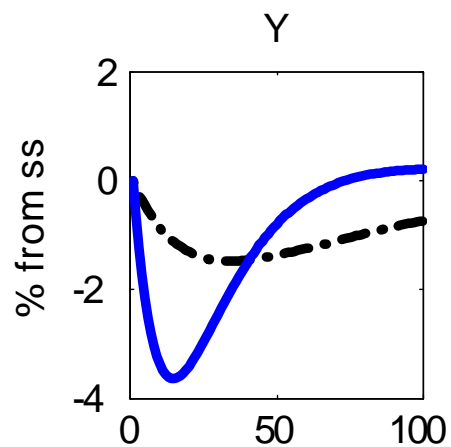
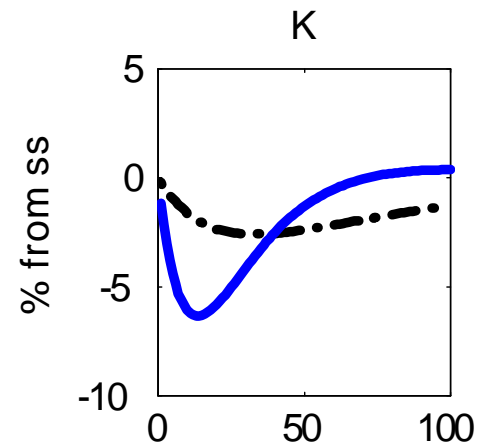
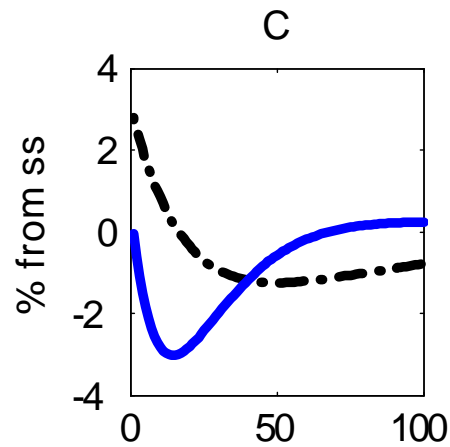
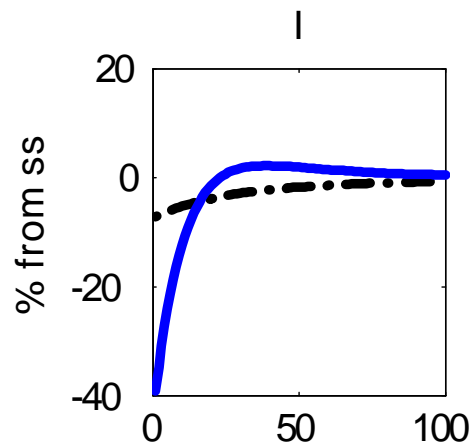
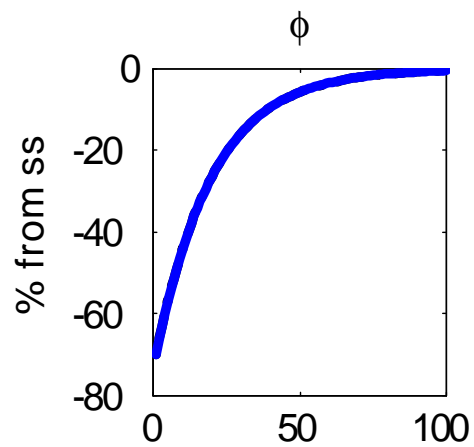


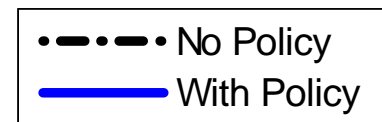
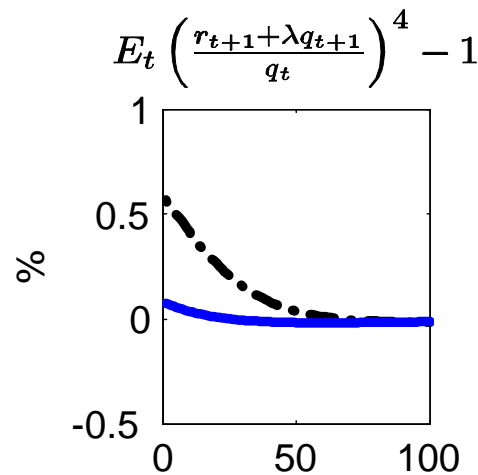
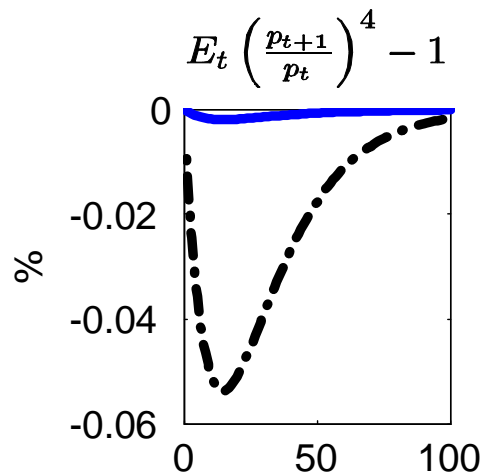
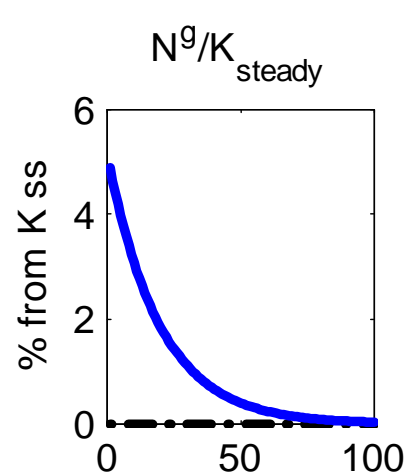
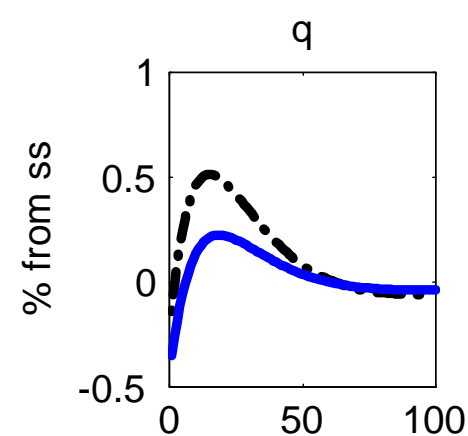
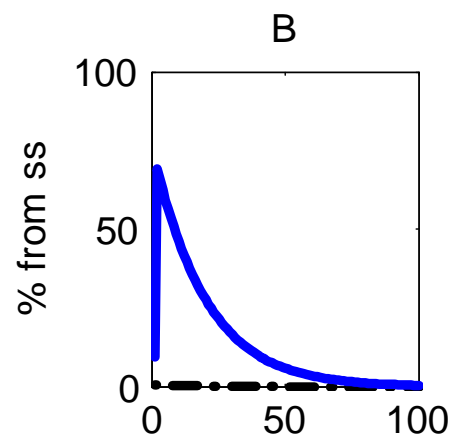
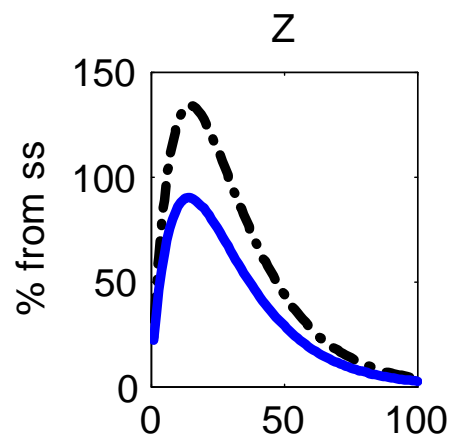
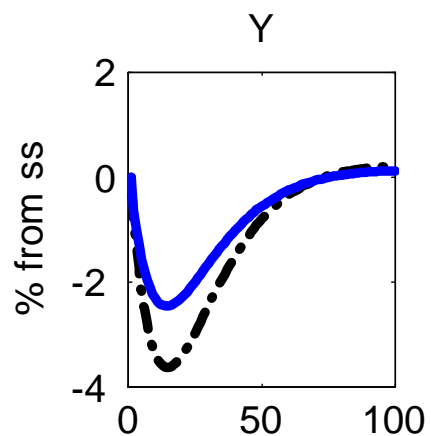
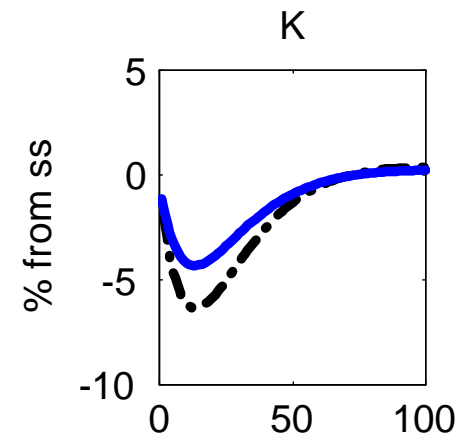
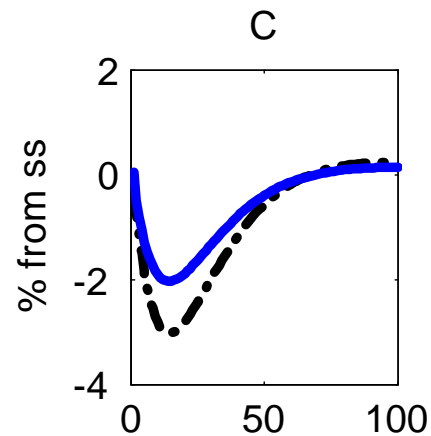
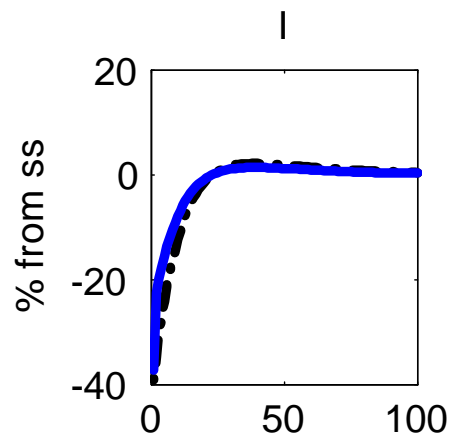
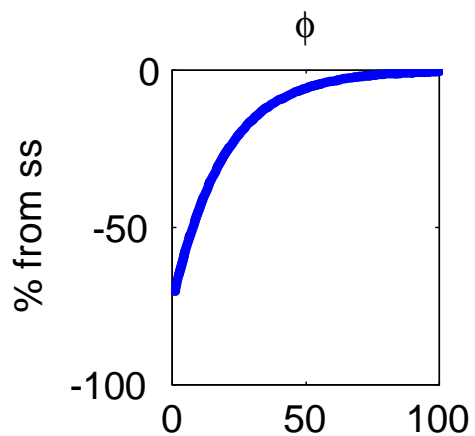


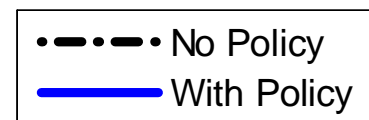
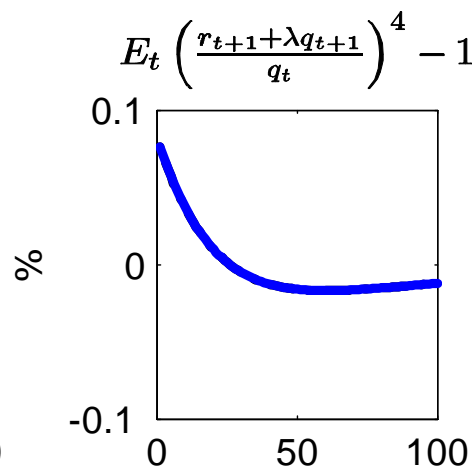
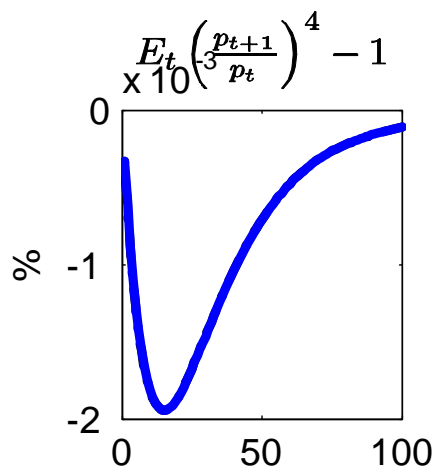
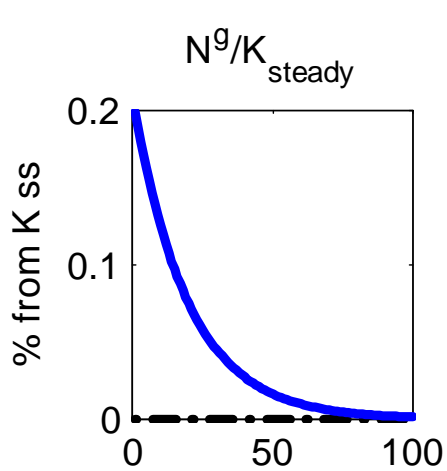
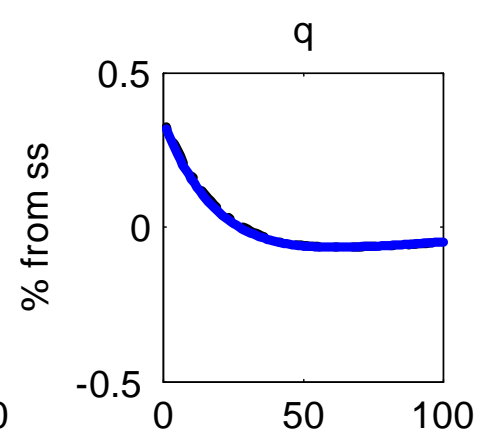
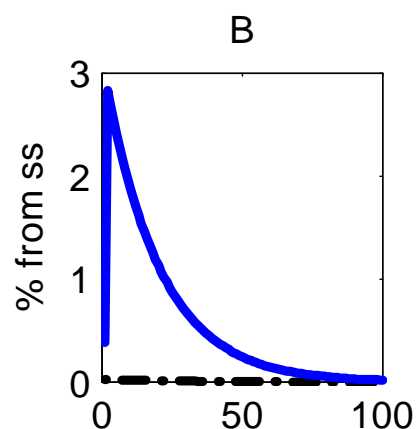
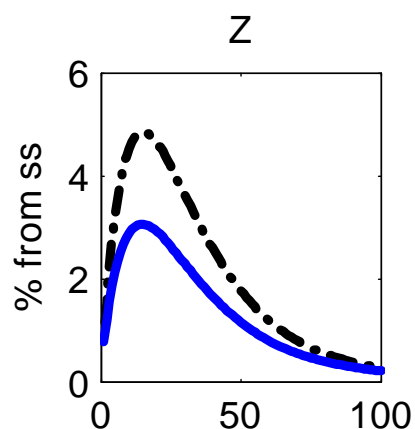
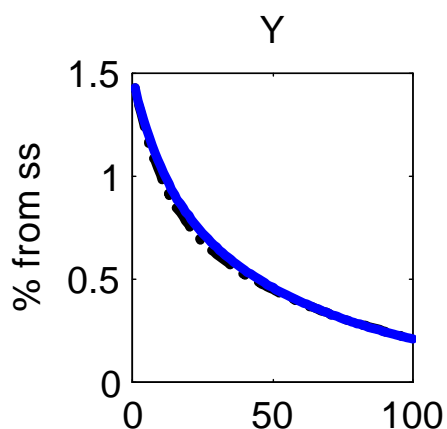
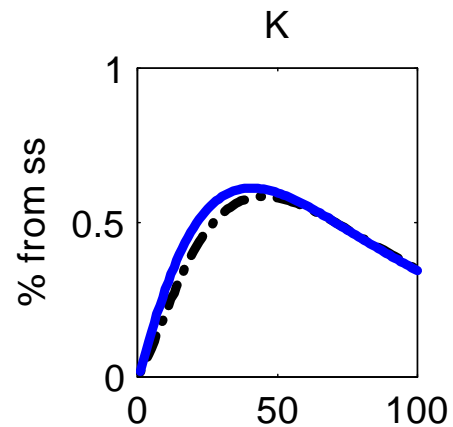
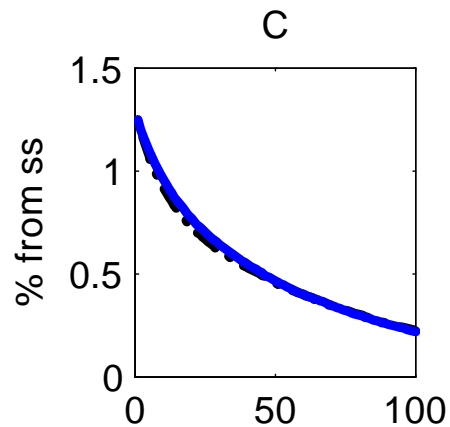
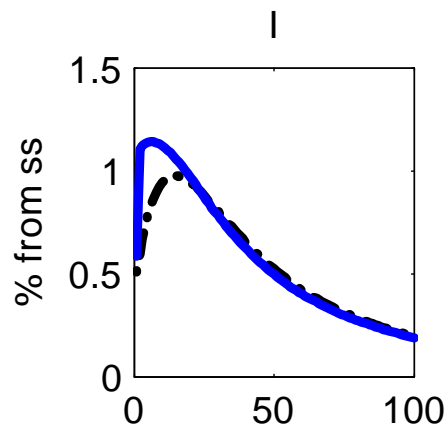
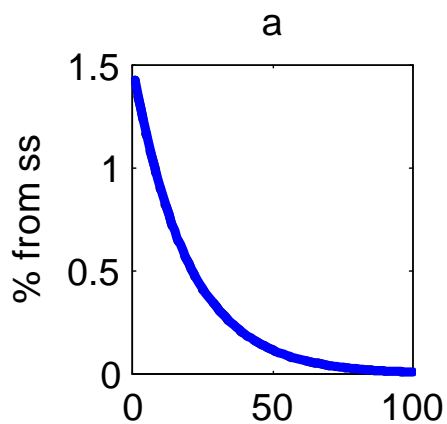


**Option Adjusted
Spread (bps)**









Calibration of Slightly Modified Model:

Homogeneous output Y_t , capital K_t and treasury bills B_t with nominal gross interest rate $R_t \geq 1$ in each period

Each household consists of $[0,1]$ continuum of members, each member receives iid draw to become an entrepreneur with probability κ , or a worker *wp.* $1 - \kappa$

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{\omega}{1+\nu} \int_{\kappa}^1 H_s(j)^{1+\nu} dj \right],$$

Investing member issues equity to finance investment

At the beginning of period

balance sheet at the start of period	
nominal bond: B_t/P_t	own equity issued: $q_t N_t^I$
equity of others: $q_t N_t^O$	
own capital stock: $q_t K_t$	net worth: $q_t N_t + B_t/P_t$

Net equity

$$N_t = N_t^O + K_t - N_t^I$$

During the period flow-of-funds of each household member j

$$\begin{aligned} & C_t(j) + p_t^I I_t(j) + q_t [N_{t+1}(j) - I_t(j)] + \frac{B_{t+1}(j)}{P_t} \\ &= \left[R_t^k + (1 - \delta)q_t \right] N_t + \frac{R_{t-1}B_t}{P_t} + \frac{W_t(j)}{P_t} H_t(j) - \tau_t \end{aligned}$$

Borrowing Constraint: an investing member can issue new equity at most θ fraction of his investment

Resaleability Constraint: in each period, an agent can resell at most ϕ_t fraction of his equity holdings

$$N_{t+1}(j) \geq (1 - \theta)I_t(j) + (1 - \phi_t)(1 - \delta)N_t$$

$$B_{t+1}(j) \geq 0$$

If $q_t > p_t^I$, entrepreneurs use all the liquidity to invest

$$I_t = \varkappa \frac{[R_t^k + (1 - \delta)\phi_t q_t] N_t + \frac{R_{t-1} B_t}{P_t} - \tau_t}{p_t^I - \theta q_t}$$

Each worker supplies differentiated labor according to the demand, buys consumption goods and chooses portfolio

$$\begin{aligned} & C_t^{-\sigma} \\ &= \beta \mathbb{E}_t \left[C_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} (\mathbf{1} + \Lambda_{t+1}) \right], \text{ where } \Lambda_{t+1} = \frac{\varkappa(q_{t+1} - p_{t+1}^I)}{p_{t+1}^I - \theta q_{t+1}} \\ &= \beta \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \left[\frac{R_{t+1}^k + (1-\delta)q_{t+1}}{q_t} + \Lambda_{t+1} \frac{R_{t+1}^k + (1-\delta)\phi_{t+1}q_{t+1}}{q_t} \right] \right\} \end{aligned}$$

At the end of period, all member get together to consume and shares the assets $C_t = \int_{\varkappa}^1 C_t(j) dj$.

Consider nominal discount bond which pays \$1 for sure at the end of period $t+1$. Euler equation implies

$$\mathbf{1} = \mathbb{E}_t \left(m_{t+1} \frac{R_t^0}{\pi_{t+1}} \right), \text{ where } m_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}$$

where R_t^0 is the gross nominal interest rate on this illiquid bond.

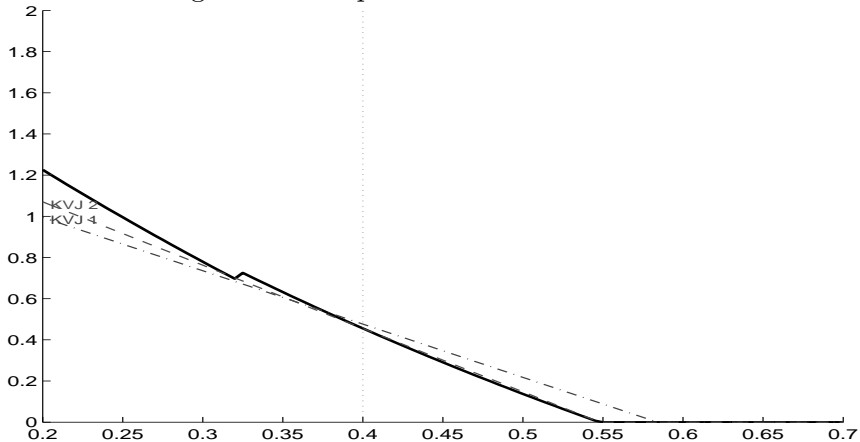
The convenience yield in theory

$$CY_t = \mathbb{E}_t (\Lambda_{t+1})$$

The convenience yield in measurement

$$\begin{aligned} \overline{CY}_t &= (R_t^0 - R_t) \mathbb{E}_t \left(\frac{\mathbf{1}}{\pi_{t+1}} \right) \\ &\approx \frac{\mathbf{1}}{\mathbb{E}_t (m_{t+1})} \cdot \frac{CY_t}{\mathbf{1} + CY_t} \\ &\approx CY_t \end{aligned}$$

Figure 2: Two-parts KVJ Demand Curve



Notes: The figure plots the regressions line $CY = b_1 \max \left\{ b_2 - \frac{B}{PY}, 0 \right\}$ where the estimates of b_1 and b_2 come from the first two columns of Table 3 of Krishnamurthy and Vissing-Jorgensen (2012) (dashed lines). The average value of $\frac{B}{PY}$ in our sample (40%) is indicated by the vertical line.

Table 2: Targets and Model-Implied Values in Loss Function-Based Calibration of Steady State Parameters

Targets	CY	$\overline{\frac{B}{PY}}$	Real rate	Liquidity Share	Labor Share	Investment/GDP Ratio
Data	0.455	0.548	2.200	12.55	0.65	0.260
Model	0.455	0.548	2.200	12.55	0.66	0.264

Notes: The table shows the empirical targets and the model-implied values in the loss function-based calibration of the six steady state parameters. The first two targets are obtained from the regressions in the second column of Table 3 of Krishnamurthy and Vissing-Jorgensen (2012). We set $CY = b_1 \max \left\{ b_2 - \frac{B}{PY}, 0 \right\}$, where $\frac{B}{PY}$ is the average value of government debt in our sample, and $\overline{\frac{B}{PY}} = b_2$. The construction of the liquidity share is described in section A.1 of the appendix, and the construction of the remaining three data counterparts—which is standard—is described in footnote 25 of section 3.1. The sample used to compute the data counterparts of the targets is 1953Q1-2008Q3.

Table 1: Parameters

Steady State Parameters						
ϕ	θ	β	\varkappa	δ	γ	$\frac{B/P}{4Y}$
Resaleability constraint	Borrowing constraint	Discount factor	Probability of investment opportunity	Depreciation rate	Capital share	Annualized s.s. liquidity
0.309	0.792	0.993	0.009	0.024	0.340	0.400
Parameters Characterizing the Dynamics						
σ	ν	$S''(1)$	ζ_p	ζ_w	λ_p	λ_w
Relative risk aversion	Inverse Frisch elasticity	Investment adjustment cost	Price Calvo probability	Wage Calvo probability	Price s.s. markup	Wage s.s. markup
1.000	1.000	0.750	0.750	0.750	0.100	0.100
ψ_π	ψ_y	ψ_τ				
Taylor rule inflation response	Taylor rule output response	Tax rule response				
1.500	0.125	0.100	Liquidity Shock and Policy Response			
$\Delta\phi$	Baseline	ψ_k	$\Delta\phi$	Great Escape	ψ_k	
Size of liquidity shock	ρ_ϕ	Policy intervention	Size of liquidity shock	ρ_ϕ	Policy intervention	
-0.218	0.953	-4.801	same	0.984	same	

Notes: The table shows the parameter values of the model for the baseline calibration. The last three rows also report the size and the persistence of the shock, and the coefficient in the government rule for purchases of private assets in the Great Escape calibration.

Let $P_t^{(T, j)}$ be the price of a long-term bond j with maturity T which pays \$1 at date $t + T$ for sure.

$$\begin{aligned} P_t^{(T, j)} &= \mathbb{E} \left[\frac{m_{t+1}}{\pi_{t+1}} \left(\mathbf{1} + \phi_{t+1}^j \Lambda_{t+1} \right) P_{t+1}^{(T-1, j)} \right] \\ &= \mathbb{E} \left[\prod_{s=1}^T \frac{m_{t+s}}{\pi_{t+s}} \left(\mathbf{1} + \phi_{t+s}^j \Lambda_{t+s} \right) \right] \end{aligned}$$

Let $\text{nytm}_t^{(T, j)}$ be gross nominal yield to maturity.

$$1 = \text{nytm}_t^{(T, j)} \left(\mathbb{E} \left[\prod_{s=1}^T \frac{m_{t+s}}{\pi_{t+s}} \left(\mathbf{1} + \phi_{t+s}^j \Lambda_{t+s} \right) \right] \right)^{1/T}$$

The yield spread of bond j over treasury is approximately

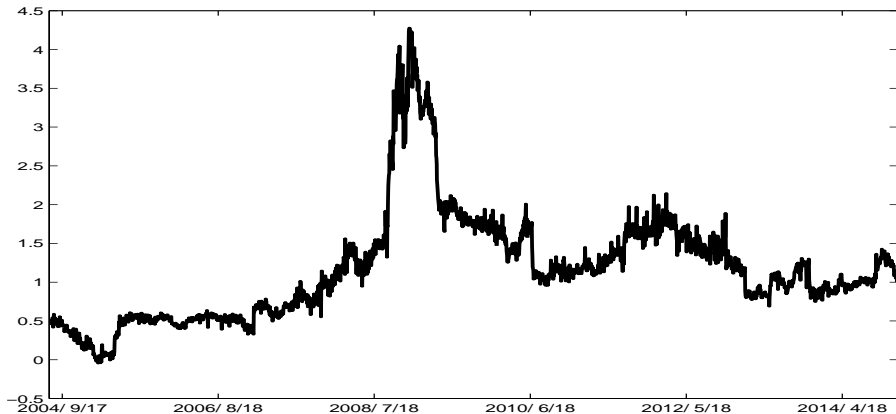
$$\begin{aligned} &\text{nytm}_t^{(T, j)} - \text{nytm}_t^{(T, l)} - \left[\text{nytm}^{(T, j)} - \text{nytm}^{(T, l)} \right] \\ &\simeq \varphi^j [CY_t - CY] + \epsilon_t^j \end{aligned}$$

Table A-2: Average Returns and Implied ϕ_j

2004/ 7/21–2007/ 6/29			2008/10/ 1–2008/12/31		
CY: 0.46			CY: 3.42		
	ϕ	spread		ϕ	spread
1Y Refcorp	<i>1.150</i>	-0.07	20Y TIPS	<i>0.806</i>	0.65
6M Refcorp	<i>1.067</i>	-0.03	On-Off	<i>0.795</i>	0.69
2Y Refcorp	<i>0.990</i>	0.00	20Y Refcorp	<i>0.747</i>	0.85
AA CDS-Bond Basis	<i>0.985</i>	0.01	2Y Refcorp	<i>0.720</i>	0.94
A CDS-Bond Basis	<i>0.945</i>	0.02	5Y Refcorp	<i>0.702</i>	1.01
3Y Refcorp	<i>0.920</i>	0.04	7Y Refcorp	<i>0.701</i>	1.01
On-Off	<i>0.889</i>	0.05	10Y Refcorp	<i>0.692</i>	1.04
5Y Refcorp	<i>0.854</i>	0.07	1Y Refcorp	<i>0.690</i>	1.05
BBB CDS-Bond Basis	<i>0.851</i>	0.07	10Y TIPS	<i>0.682</i>	1.07
4Y Refcorp	<i>0.822</i>	0.08	3Y Refcorp	<i>0.679</i>	1.08
7Y Refcorp	<i>0.779</i>	0.10	4Y Refcorp	<i>0.665</i>	1.13
10Y Refcorp	<i>0.671</i>	0.15	5Y TIPS	<i>0.654</i>	1.17
20Y Refcorp	<i>0.659</i>	0.15	6M Refcorp	<i>0.612</i>	1.31
5Y TIPS	<i>0.371</i>	0.28	7Y TIPS	<i>0.586</i>	1.40
7Y TIPS	<i>0.311</i>	0.31	AA CDS-Bond Basis	<i>0.548</i>	1.53
20Y TIPS	<i>0.298</i>	0.31	AAA	<i>0.448</i>	1.86
10Y TIPS	<i>0.219</i>	0.35	A CDS-Bond Basis	<i>0.282</i>	2.43
AAA	<i>-0.294</i>	0.58	BBB CDS-Bond Basis	<i>0.000</i>	3.39

Notes: The two panels show the average spread for the securities listed above (see Appendix A.2 for a description) for the 2004/ 7/21–2007/ 6/29 (left) and 2008/10/ 1–2008/12/31 (right) periods, as well as the implied ϕ^j computed according to formula (25).

Figure 3: A Time-Series for the Convenience Yield



Notes: The figure plots a daily time series of the convenience yield from July 21 2004 to December 31 2014, constructed using a panel of 18 liquidity-related spreads as described in [3.3](#).

Figure 4: Response of Output, Inflation, and the Nominal Interest Rate to the Liquidity Shock

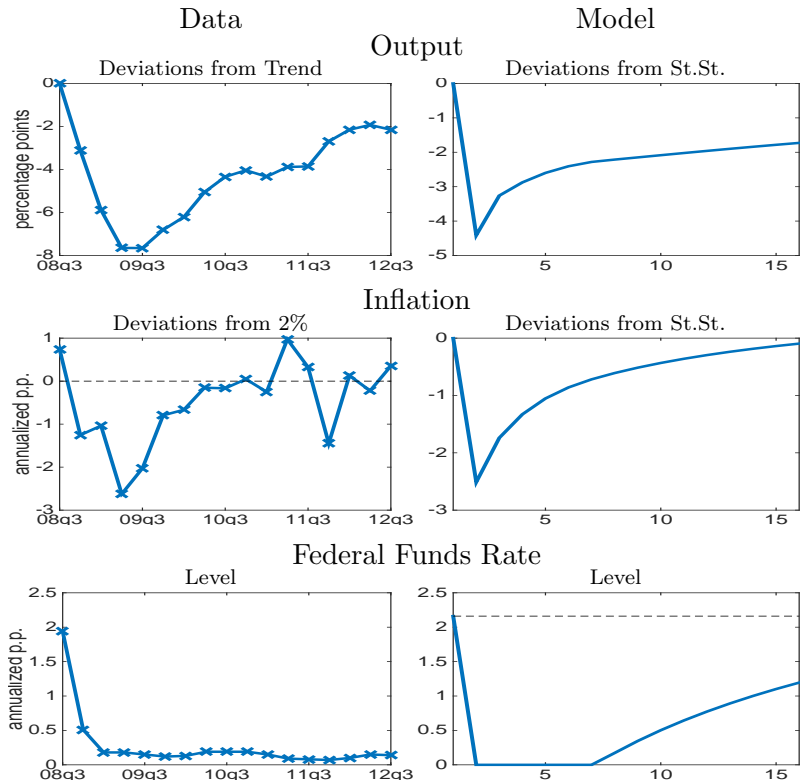


Figure 5: Response of Consumption, Investment, the Nominal Value of Capital, and Convenience Yield to the Liquidity Shock

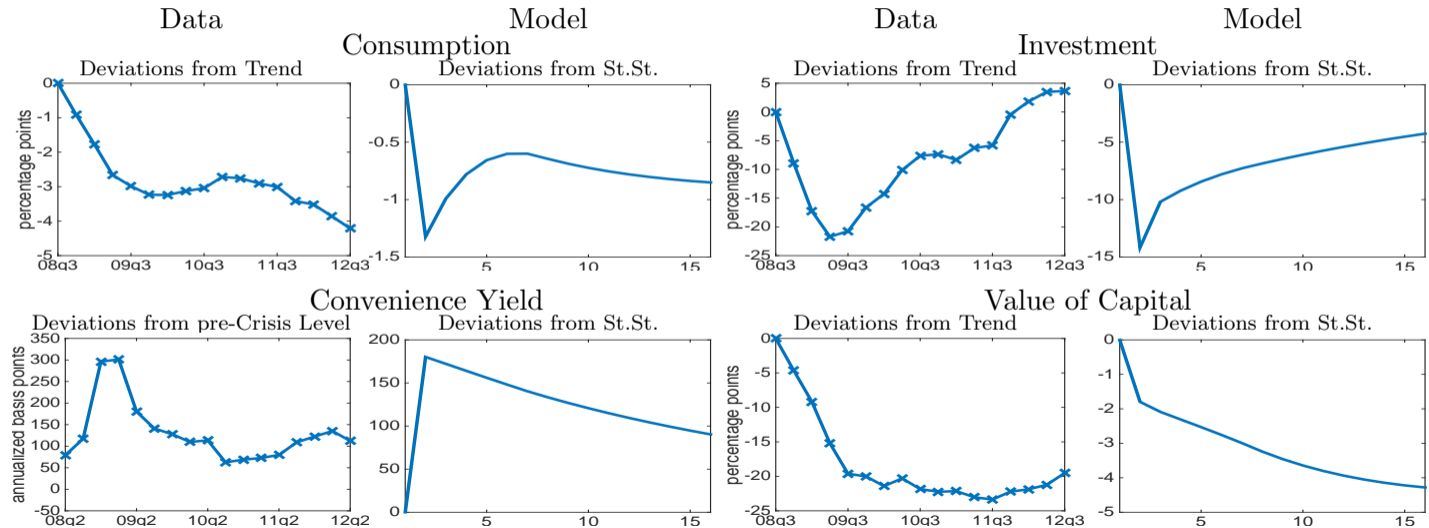
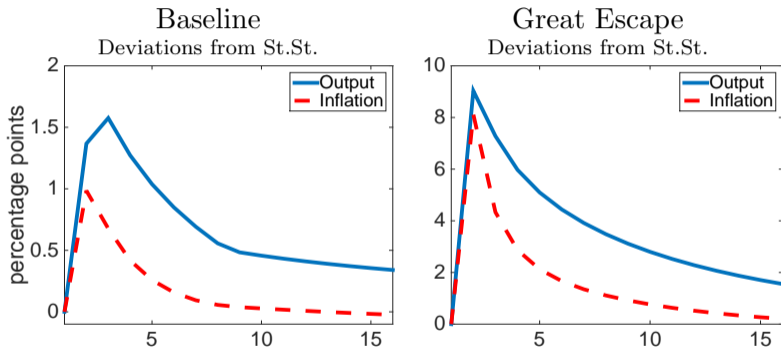


Figure 6: The Effect of the Liquidity Facilities on Output and Inflation in the Baseline and in the Great Escape Experiment



Notes: The figure shows the difference between counterfactual and actual response of output (solid blue) and inflation (dashed red) in the model in response to the calibrated liquidity shock under the baseline persistence (left panel), and with increased persistence such that the zero lower bound binds for twenty quarters (right panel).

Normal features of "monetary economy"

- interest rates spread between assets with different liquidity

rate of return on money $<$ rate of return on equity $<$ time preference rate $<$ expected marginal product of capital

- quantities and asset prices react to liquidity shock

Policy: Can use open market operation to accommodate productivity shock and to offset shocks to liquidity (resaleability)

Needs to buy (or lend against) partially resaleable assets which has liquidity premium