

# Matching Theory – from Gale and Shapley to Trading Networks

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**Course Description.** This mini-course introduces the theory and core results of two-sided matching. We then build on those results through a series of successive generalizations, developing first *matching with contracts* and then *trading network matching* models. Throughout, we pay attention to key applications in marketplace design.

## References

### Day 1: Two-Sided Matching.

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- L. E. Dubins and D. A. Freedman. Machiavelli and the Gale-Shapley algorithm. *American Mathematical Monthly*, 88:485–494, 1981.
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### Day 2: Generalized Matching.

- Hiroyuki Adachi. On a characterization of stable matchings. *Economics Letters*, 68:43–49, 2000.
- John William Hatfield and Paul Milgrom. Matching with contracts. *American Economic Review*, 95:913–935, 2005.
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- John William Hatfield and Fuhito Kojima. Group incentive compatibility for matching with contracts. *Games and Economic Behavior*, 67:745–749, 2009.
- Tamás Fleiner. A fixed-point approach to stable matchings and some applications. *Mathematics of Operations Research*, 28:103–126, 2003.

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- Michael Ostrovsky. Stability in supply chain networks. *American Economic Review*, 98: 897–923, 2008.
- John William Hatfield and Scott Duke Kominers. Matching in networks with bilateral contracts. *American Economic Journal: Microeconomics*, 4:176–208, 2012.
- John William Hatfield, Scott Duke Kominers, Alexandru Nichifor, Michael Ostrovsky, and Alexander Westkamp. Stability and competitive equilibrium in trading networks. *Journal of Political Economy*, 121:966–1005, 2013.
- Tayfun Sönmez and Tobias B. Switzer. Matching with (branch-of-choice) contracts at United States Military Academy. *Econometrica*, 81:451–488, 2013.
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- Ravi Jagadeesan. Complementary inputs and the existence of stable outcomes in large trading networks. Harvard University Working Paper, 2018.
- Yuichiro Kamada and Fuhito Kojima. Efficient matching under distributional constraints: Theory and applications. *American Economic Review*, 105:67–99, 2015.
- Tayfun Sönmez and Tobias B. Switzer. Matching with (branch-of-choice) contracts at United States Military Academy. *Econometrica*, 81:451–488, 2013.

#### **Further Reading.**

- Fuhito Kojima. Recent developments in matching theory and its practical applications. In Bo Honoré, Ariel Pakes, Monika Piazzesi, and Larry Samuelson, editors, *Advances in Economics and Econometrics: Eleventh World Congress of the Econometric Society*, Econometric Society Monographs, pages 138–175. Cambridge, 2017.
- Scott Duke Kominers, Alexander Teytelboym, and Vincent P. Crawford. An invitation to market design. *Oxford Review of Economic Policy*, 33:541–571, 2017.