Heterogeneous Skill Growth across College Majors*

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Abstract

There is a growing literature on differential wages across college majors, but few studies focus on skill growth by major. Differences in course taking by major will result in students accumulating different types and amounts of skills, and this heterogeneous skill growth will lead to differences in wage returns. This paper estimates skill growth during college by major using the NLSY97 and the O*NET. To capture both the type and quantity of accumulated skills, I assume that each major increases a general cognitive skill and a major-specific skill. I further allow for individual heterogeneity in skill growth. I take a task-based approach and use occupation choice to estimate skill growth in general cognitive skill. To deal with noisy skill measurements and endogeneity, a dynamic factor model is constructed. The results show substantial growth of general cognitive skill in all majors, but with large differences across majors. I find different effects of pre-college skill levels on skill growth by major, but the differences are not large. The contribution of major-specific skill growth to wage growth is small compared to that of general cognitive skill growth.

1 Introduction

Wage inequality across college majors has recently been attracting attention. Carnevale et al. (2015) report that the average difference in the lifetime earnings in the US between the highest-paying major, engineering, and the lowest-paying major, education, is $3.4 million while the average difference between college and high school graduates is $1 million. Inequality research has mostly focused on disparities

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between high school and college graduates, but given this earnings gap across majors, college majors are an equally important determinant of future career prospects.

Since college majors are not randomly chosen by students, sorting may contribute to these large income differences among majors. Students who choose a "high-paying" major might earn a lot even if they chose a "low-paying" major.[1] Many previous papers examine whether wages are different across college majors even when controlling for sorting or self-selection into major (see, e.g., Arcidiacono (2004); Hamermesh and Donald (2008); Arcidiacono et al. (2016), and Kirkeboen et al. (2016)). Their consensus is that there exist differences in wage returns across college majors and that math-intensive majors show high returns.

Wage is of course an important variable, but differences in the one-dimensional variable provides us information only on differences in quantity. Focusing on differences in quantity may be appropriate in studying differences between high school and college graduates because a crucial difference between these two groups is years of education. Since college graduates spent more time on studying, they will possess a larger amount of skill, which will contribute to wage differential between college and high school graduates.

However, focusing on differences in quantity may not be enough in studying differences across majors. Skill accumulation processes could be heterogeneous across college majors in both quantity and type. Courses college students take vary significantly by their college major. For example, engineering majors take more science courses than humanities courses, while the reverse occurs for education majors. These course differences will result in students accumulating different types and amounts of skills. In addition, college majors are different in how demanding they are in terms of course burden, the number of credits required to graduate, and so on. This implies that, in some majors, students can accumulate skills more than in other majors.

In this paper, I estimate skill growth during college by major in a multi-dimensional skill framework. Each major represents a different skill production function; skills students start with will evolve differently depending on their major. In order to capture skill growth differences in both quantity and type, I assume that each major increases two types of skills: a general cognitive skill and a major-specific skill. General cognitive skill can increase in all college majors, but the amount of growth can vary by

[1] For example, the average SAT scores vary by college major. According to a report in 2015 by College Board (2015), the average scores of individuals who intended to major in engineering are 526 (Critical Reading), 575 (Mathematics), and 509 (Writing) while those of individuals who intended to major in education are 481 (Critical Reading), 480 (Mathematics), and 473 (Writing).

major. In addition to general cognitive skill, each major can produce a specific skill.[2] General cognitive skill growth captures the similarity of skills produced in different college majors, while major-specific skill growth captures the uniqueness. I allow for individual heterogeneity in the growth of both types of skills. The growth in general cognitive skill can be affected by the pre-college level of general cognitive skill.

Wage returns are closely related to skill growth. If every major increases a common single type of skill and there is no heterogeneity in wage returns across occupations, then wage differences will directly reflect skill growth differences across major. However, given that college majors may increase different types of skills and that occupations matter to wage returns (see, e.g., Kinsler and Pavan (2015)), wage return differences will not directly identify skill growth differences by major.

Studying skill growth in both quantity and type will be helpful to understand various important phenomena. For example, high wage return to Science, Technology, Engineering, and Mathematics (STEM) majors might come from that their major-specific skill is highly rewarded in the labour market. If this is is the case, reducing wage difference between STEM majors and the others may not be easy. In contrast, it is also possible that higher wages among STEM majors mainly come from that they increase general cognitive skill more than the other majors. It is documented that STEM majors tend to spend more time in studying and to earn more credits compared to the other majors. If growth in general cognitive skill is the main cause, the wage differences may be able to reduce substantially by making non-STEM students study more.

Another example is college major choice. Despite the fact that STEM majors tend to earn high wages, many students avoid majoring in STEM. The governments in many countries are trying to create policies to increase the number of students in STEM majors. Kinsler and Pavan (2015) argue that skill specificity can play an important role in students' major choice. If STEM majors mainly increase skills that are specific to the major, some students might avoid STEM majors due to the risk of not working in related fields in the future. If they do not have a job related to their major, the wage return to the major might be much smaller. A multi-dimensional skills framework will explain students' college major choice better.

If there are test scores of each type of skill in both pre- and post-college periods, skill growth by major could be easily measured. However, although students take cognitive tests before entering college, such as high school graduation exams or college entrance exams, that are observed in various

[2] As described later, I aggregate majors into three types, hence, there are three types of major-specific skills in total.

data sets, people usually do not take such tests after college. Hence, the approach of comparing test scores at different periods, common in the literature on elementary schooling, cannot be applied in this case. Instead, I use an approach that exploits post-education occupation choice, which reflects post-education skill levels, in estimating general cognitive skill growth. On the other hand, although I have measures of post-college major-specific skills, I do not have measures of pre-college major-specific skills. Hence, I make some assumptions to specify pre-college major-specific skills.

I use the National Longitudinal Survey of Youth 1997 (NLSY97) for individual level data. The NLSY97 provides data on Armed Service Vocational Aptitude Battery (ASVAB) test scores, college majors, college Grade Point Average (GPA), wages, and occupations. Most respondents, including both those who eventually went to college and those who did not, took the ASVAB test while in junior high or high school. These test scores are assumed to reflect pre-college general cognitive skill levels. High school graduates who do not take further education are assumed to enter the labour market with these skill levels. The effects of general cognitive skill on occupation choice can therefore be estimated using high school graduates. Using these estimated skill effects, growth in general cognitive skill is implied by the differences in occupation choice of college graduates from high school graduates for each college major.

Of course, high school graduates and college graduates tend to take different jobs. Furthermore, as Ransom (2014) and Altonji et al. (2012) show, college majors and post-college occupations are closely related. If occupation or industry categories are used to control for jobs, the categories can be very coarse. Hence, I take a task-based approach using the Occupational Information Network (O*NET) and characterize each occupation by a low-dimensional task portfolio. Following Acemoglu and Autor (2011), I define task as a unit of work activity that produces output while skill is defined as a worker's endowment of capabilities for performing various tasks. Individuals possess skills and apply the skills in tasks to produce outputs. The task-based approach enables me to relate occupations to each other.

ASVAB test scores and college GPA in the NLSY97 are used as pre-college general cognitive skill measurements and post-college major-specific skill measurements, respectively.[3] Since they will not perfectly measure skills, I use a dynamic factor model to deal with the measurement errors.

The results show that all majors significantly increase a student's general cognitive skill and that there are also large differences in the growth across majors. This is reflected in the wage premium for college graduates and the large wage differences across majors. For those having a population average

[3]GPA is allowed to be affected by general cognitive skill as well.

level of pre-college skill, the skill growth in STEM majors is 16 points higher than that in Humanities & Social Sciences majors using a log wage point metric. Although the skill growth varies somewhat with pre-college general cognitive skill levels, STEM majors still have a larger production of skill and associated wage returns in the labour market. Wage returns to a major-specific skill are allowed for depending on whether the occupations are related to the corresponding major. The estimates suggest that only Business & Economics and STEM majors have a positive wage benefit from major-specific skill growth. The wage effect is the largest among STEM majors, but the effect is still only about one quarter of that from general cognitive skill growth. Overall, growth in general cognitive skill is most important during college and also plays an important role in wage differentials across majors.

Although there is a growing number of papers estimating wage returns to college majors, relatively few papers in the previous literature examine skill growth across majors. Lemieux (2014) is the closest to my study. He considers three channels to increase wages: general skill growth, occupation, and match between college major and occupation. The major-occupation match matters because of major-specific skill growth. He uses Canadian datasets and decomposes wage increases by major into the three channels. However, there is no sorting into majors or into occupation, and there is no individual heterogeneity in skill growth. My model allows for both sorting and individual heterogeneity. In addition, my task-based approach avoids the relatively coarse occupation classification used in Lemieux (2014), in which people classified in the same occupation category might be doing very different work.

The model developed by Kinsler and Pavan (2015) is similar to my model in that they assume students bring low-dimensional skills into college and the skills change depending on their college major. Unlike my model, their model assumes two types of general cognitive skills, verbal and math, and there is no major-specific skill. In their model, different wage returns to the general cognitive skills represent major specificity. Since their main interest is whether wage returns vary by job, they do not estimate skill changes. Furthermore, since they classify jobs based on only one dimension, related or not related, for each college major, and since related and unrelated jobs will be different depending on major, they cannot compare wage differences across majors controlling for occupations. Arcidiacono et al. (2017) use data on subjective expectations that were collected at Duke University and argue that there exist sizable complementarities between some college majors and occupations.

My paper is also related to studies showing positive effects of schooling on cognitive test scores, such as Gormley and Gayer (2005); Aaronson and Mazumder (2011), or Fitzpatrick et al. (2011). Hansen et al. (2004) and Cascio and Lewis (2006) analyze the effects on ASVAB scores. Although their focus is on an earlier level of schooling than post-secondary education, they show that schooling has positive impacts on cognitive test scores. Aucejo and James (2016) examine math and verbal skills changes during primary and secondary education using UK data.

The rest of this paper is organized as follows: section 2 describes my data. My model is explained in section 3. Section 4 explains the identification of skill growth parameters. Section 5 discusses the estimation. Results are reported in section 6. Section 7 concludes.

2 Data

I use two US datasets, the NLSY97 and the O*NET. The NLSY97 provides individual level data on test scores, college majors, college GPA, wages, and occupations. The O*NET is used to characterize occupations by task portfolios.

2.1 NLSY97

The NLSY97 is a panel survey conducted by the US Bureau of Labor Statistics. It started in 1997, was conducted annually up to 2011, and has been conducted biannually since then. In the first round, 8,984 males and females, who were between 12 and 17 years old at that time, were interviewed.

Test scores

Most of the respondents of the NLSY97 took an ASVAB test during the first round, between summer 1997 and spring 1998, when they were between 12 and 18 years old. I use ASVAB scores as noisy measurements of pre-college general cognitive skill instead of perfectly accurate measurements. Since there may be an age effect, I adjust the test scores for age using the method of Altonji et al. (2012).[4]

The ASVAB test is composed of many sections, eight of which are used in this paper: Word Knowledge, Paragraph Comprehension, Arithmetic Reasoning, Mathematics Knowledge, Numerical Operation, Mechanical Comprehension, Auto & Shop Information, and Electronics Information. Word Knowledge and Paragraph Comprehension are considered as verbal tests and Arithmetic Reasoning, Mathematics Knowledge, and Numerical Operation are considered as math tests. As Armed Forces

[4] In this method, ASVAB score at the qth percentile in the distribution of the taker's age is assigned to the ASVAB score at the qth percentile in the age 16 distribution. This method implicitly assumes that age effects do not change the rank in ASVAB scores although age effects can be nonconstant.

Qualification Test (AFQT) scores are calculated based on these five test scores, they are often used to construct cognitive skill. Mechanical Comprehension, Auto & Shop Information, and Electronics Information are used to construct mechanical skill (see Prada and Urzúa (2017) and Speer (2017)).

Education

Education levels are categorized into high school, some college, and college. High school includes GED. Some college includes those with an Associate degree and those who went to college for two years or more but who did not earn Bachelor's degree. College is further categorized into three college majors. The definition of college majors in this paper is explained below.

Education group is defined by the individual's highest education with some exceptions. First, college graduates are defined as those who obtained a Bachelor's degree before 2010. This is because of a large change in the college major classification recorded in the NLSY97. Second, since my focus is on undergraduates, I drop those with a more advanced degree, although I keep those whose highest degree is a Master's if they have full-time work experience after their Bachelor's degree but before their Master's degree.[5] I restrict my sample to those who have at least a high school diploma.

College majors

In the NLSY97, students are asked to report their college major each term. Among the reported majors, I take the final one as their college major. As mentioned above, the classification of college majors changed substantially in 2010. Hence, I use Bachelor's degree major earned before 2010. I aggregate college majors into three majors based on the extent to which they are math intensive: Humanities & Social Sciences, Business & Economics, and STEM majors.[6] A similar classification is used in Kinsler and Pavan (2015).

College GPA

College GPA is reported each term, and I calculate annual GPA as the average GPA across all semesters in the year. I use the last two years of reported GPA as noisy measures of major-specific skill.[7] I choose

[5]Those who are dropped account for only around 2% of people whose education is classified as high school or higher.

[6]STEM majors include Agriculture & natural resource sciences, Biological sciences, Architecture/environmental design, Computer/information science, Engineering, Mathematics, Physical sciences, and Nutrition/Dietics/Food science. Business & Economics majors include Business management, Economics, and Hotel/Hospitality management. Humanities & Social Sciences majors include all the other majors.

[7]I assume GPA is affected by evolved general cognitive skill as well. However, as seen in the estimation section later, GPA is not used to identify the distribution of pre-college general cognitive skill.

the last two years because major-specific skill will be acquired mainly in these years. American college students typically take general academic subjects in their first two years and take specialized subjects after that.

Occupation and wages

I consider only full-time jobs, which are defined as equal to or more than 35 hours worked per week. Part-time jobs are not used because the wage structure may be different from that of full-time jobs. I use the job information of the first year. I construct occupation variables on an annual basis. If a worker had several jobs within a year, the one with the most weeks worked is taken as the occupation in that year. Using the occupation codes, the NLSY97 and the O*NET are connected.[8] In the following analysis, I use hourly compensation rates as wages. The wages are adjusted to dollars in year 2000. I restrict to the rates between \$1 and \$100 by assuming the others are misreported wages.

2.2 O*NET

I use the O*NET to construct task portfolios, which represent how intensely each type of skill is required in work. The O*NET is sponsored by the US Department of Labor/Employment and Training Administration and started as a successor to the Dictionary of Occupational Titles (DOT). Both the O*NET and the DOT characterize occupations by standardized measures and have been used in many papers taking a task-based approach (see, e.g., Poletaev and Robinson (2008) and Guvenen et al. (2016)). The O*NET contains a number of standardized measures describing the day-to-day aspects of the job and qualifications and interests of the typical workers in the occupations.[9]

The measures chosen from the O*NET are reported in Table 1. The selection of general cognitive measures is mainly based on a technical report by the ASVAB Career Exploration Program (ASVAB Career Exploration Program (2011)) and that of mechanical measures is based on Speer (2017). For each type of element, I employ Principal Component Analysis (PCA) and take the first component as

[8] The occupation codes in the NLSY97 are based on Census 2002 occupation codes, while those in the O*NET I use are based on Standard Occupation Classification (SOC) 2010. I use crosswalks to connect these two types codes. The crosswalks between SOC 2010 and 2009, between SOC 2009 and 2006, and between SOC 2006 and 2000 are provided by O*NET resource center. The crosswalk between SOC 2000 and Census 2002 occupation codes is distributed by the National Crosswalk Service Center.

[9] For each element, Importance and Level are recorded. I use Level for general cognitive task and mechanical task, which is recorded with a range of 0-7 based on ratings by analysts or job incumbents. I use Importance, which is recorded with a range of 1 (Not important) to 5 (Extremely important), for major-specific task. I use Importance for major-specific task because I want to capture the type of jobs, but, since Importance and Level are strongly correlated, using Importance instead of Level will not change my results much.

task intensity of that type. I assume that the constructed task intensity is an accurate measurement.[10] Each type of task intensity is standardized to have mean 0 and standard deviation 1 over all full-time job observations in the NLSY97.

Although many papers on the wage penalty of working in unrelated jobs use a worker's self-assessed relatedness measure, not many datasets include one and my dataset does not have one. Hence, I define relatedness using O*NET measures. The selection of measures for major-specific task follows Freeman and Hirsch (2008). In Freeman and Hirsch (2008), each college major is connected with one measure of knowledge in the O*NET. Since college majors are aggregated into three types in my study, I construct major-specific task intensity as follows. I select O*NET knowledge measures related to detailed majors contained in each aggregated college major category. Then, I employ PCA to the selected measures and take the first component as the major-specific task intensity.

Using the constructed task intensity, I categorize jobs into related jobs and unrelated jobs. I define jobs as related or unrelated instead of using the constructed major-specific task intensity for two reasons. One is comparability with the previous papers. Most of the previous papers use two or three relatedness categories. Another reason is a comparison of major-specific skill growth between majors. Since major-specific task intensity are measured differently by majors, they cannot really be compared between majors directly and so the results are difficult to interpret.[11] The histograms of the constructed task intensity are shown in Figure 1. Each major-specific task intensity is standardized to have mean 0 and standard deviation 1 over all full-time job observations in the NLSY97. They show that college graduates tend to take a higher task intensity corresponding to their college major. Since college graduates are assumed not to increase major-specific skill other than that of their own major, I expect that most of them take jobs unrelated to the other majors. Based on Figure 1, I consider jobs as related jobs if the corresponding task intensity is equal to or higher than 1, while I consider jobs as unrelated jobs if the corresponding task intensity is less than 1. For example, the major specific task intensities of the job of Accountants are -0.65 (Humanities & Social Sciences), 1.46 (Business & Economics), and -0.74 (STEM). Hence, the job of Accountants is related to Business & Economics majors, but unrelated to Humanities & Social Sciences majors and to STEM majors.[12]

[10] If the constructed task intensity is a noisy measure of the "true" task intensity and if the measurement errors are classical, my estimates of skill growth parameters are still consistent, because the effects of general cognitive skill on occupation choice are consistently estimated.

[11] The O*NET measures and a worker's self-assessed relatedness are positively correlated in the 1993 National Survey of College Graduates.

[12] Jobs can be related to more than one major in the data. This might explain why some people are in jobs that are related to another major as observed in Figure 1.

Table 1: O*NET elements used to construct task intensities

General cognitive	
Oral comprehension	Oral expression
Written comprehension	Written expression
English language	Reading comprehension
Speaking	Writing
Mathematical reasoning	Number facility
Mathematics	Mathematics skill
Deductive reasoning	Inductive reasoning
Analyzing data or information	
Mechanical	
Handling and moving objects	
Controlling machines and processes	
Repairing and maintaining mechanical equipment	
Repairing and maintaining electrical equipment	
Inspecting equipment, structures, or material	
Operating vehicles, mechanized devices, or equipment	
Equipment maintenance skill	
Mechanical knowledge	
Humanities & Social Sciences	
Communications and media	English language
Sociology and anthropology	Geography
Therapy and counseling	Foreign language
Public safety and security	Fine arts
History and archeology	Psychology
Philosophy and theology	Education and training
Business & Economics	
Administration and management	Sales and marketing
Economics and accounting	Customer and personal service
Personnel and human resources	
STEM	
Computers and electronics	Design
Engineering and technology	Mathematics
Physics	Chemistry
Biology	

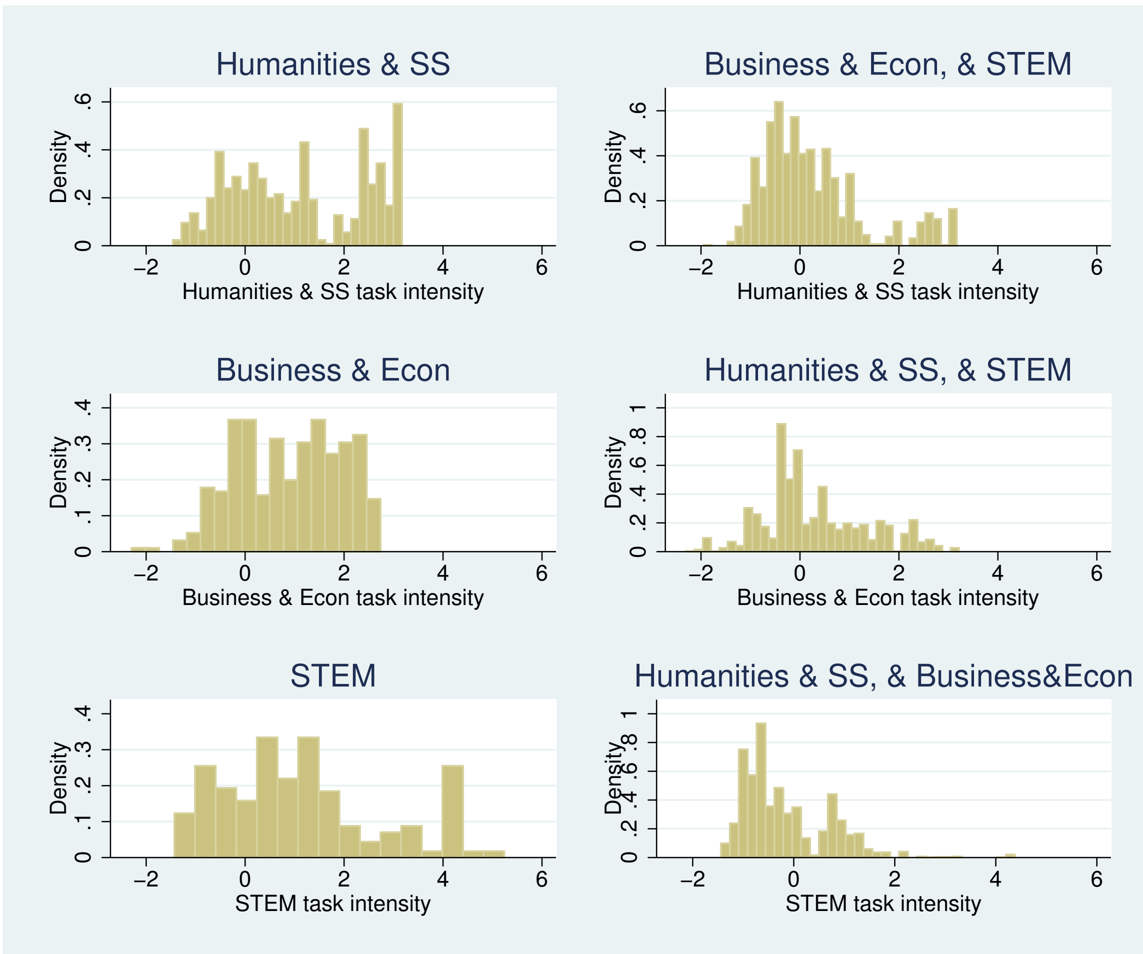

Figure 1: Histograms of major-specific task intensity by college majors

Note: Each type of task intensity is standardized to have mean 0 and standard deviation 1 over all full-time job observations in the NLSY97.

I assume that high school and some college graduates take a job unrelated to any major. In addition, I assume that college graduates can choose job relatedness only regarding their own major and have a job unrelated to majors other than that. The first assumption is made because high school and some college graduates do not have any "major". The second assumption is made because papers on the match quality between college majors and work only examine a student's major. They do not consider whether the work is related to outside of the student's major. I could potentially make a model, in which individuals can choose job relatedness regarding each major, but I do not do that for simplicity. Although there are some individuals who have jobs that are defined as "related" in the data, I assume that their employers do not care their major-specific skill levels because the skills are too low.

2.2.1 Mechanical skill and task

Mechanical skill is introduced to allow high school graduates to choose an occupation based not only on cognitive skill. A recent paper by Prada and Urzúa (2017) shows that a higher level of mechanical skill reduces the probability of attending college given cognitive skill and also shows that wage returns to mechanical skill are large for high school graduates.[13] Figure 2 shows histograms of mechanical task intensity by education level. Remember that task intensity is standardized to have mean 0 and standard deviation 1 over all full-time job observations in the NLSY97. High school shows two humps. One is between -1 and 0, and another is between 1 and 2. In the case of some college, there is a second hump between 1 and 2, but it is small and not as obvious as high school. College shows only one hump between -1.5 and 0. This implies that the mechanical dimension is not important to college graduates. Given these observations, I define jobs as mechanical if the mechanical task intensity is higher than 0.5 for high school and some college graduates. All other jobs are defined as cognitive type jobs. Mechanical skill and task intensity do not matter in cognitive type jobs.[14]

An advantage of dividing jobs into mechanical and cognitive types is to make the model easy to interpret since mechanical skill does not matter anymore given cognitive type jobs. This reduces the computation burden as well. Furthermore, it makes explicit that the mechanical dimension is not

[13] In earlier related research, Yamaguchi (2012) documents that less educated people tend to take a job involving intense physical tasks. Physical task measures and mechanical task measures in the O*NET are highly correlated. Since my dataset does not have a good measure of physical skill, I introduce mechanical skill instead of physical skill. Given the close relatedness between physical and mechanical task measures, I expect that a selection between physical and mechanical skills will not matter much.

[14] If log wages are regressed on test scores and task intensity of cognitive and mechanical, and other controls for high school graduates in cognitive type jobs, the coefficient of mechanical test scores is negative and that of mechanical task intensity is insignificant.

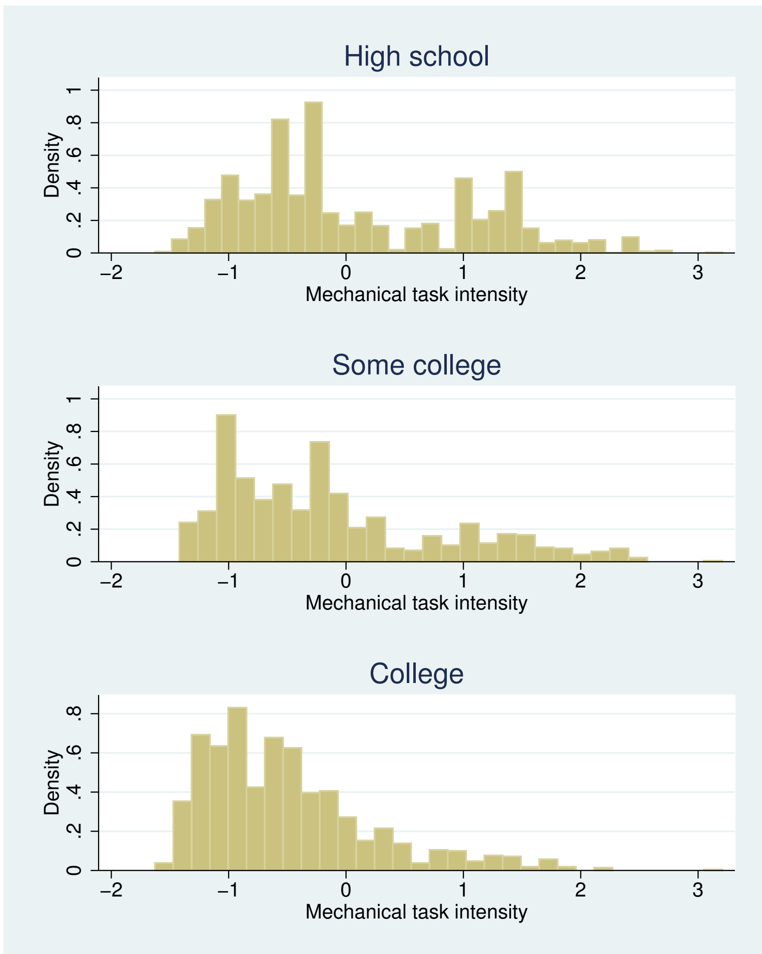

Figure 2: Histograms of mechanical task intensity in first year by education level

Note: Each type of task intensity is standardized to have mean 0 and standard deviation 1 over all full-time job observations in the NLSY97.

Table 2: Summary statistics

	High school	Some college	Humanities&SS	Business&Econ	STEM
ASVAB test scores					
Cognitive	-0.3255	0.0524	0.5306	0.5794	0.7804
	(0.8082)	(0.7093)	(0.6523)	(0.5918)	(0.5851)
Mechanical	-0.1852	0.0437	0.2157	0.3253	0.6869
	(0.8578)	(0.8246)	(0.6755)	(0.6833)	(0.7929)
Task intensity					
Cognitive	-0.4085	0.1906	0.6700	0.9107	1.1192
	(0.7933)	(0.8832)	(0.8097)	(0.8696)	(1.0027)
Mechanical	0.0869	-0.1439	-0.6213	-0.6419	-0.0369
	(0.9723)	(0.9503)	(0.5903)	(0.6881)	(0.8627)
Related job			0.5103	0.4809	0.4652
			(0.5002)	(0.5004)	(0.4997)
Wages	8.8881	11.9696	12.4170	14.3962	15.4904
	(4.8242)	(7.4956)	(6.8789)	(7.0064)	(7.9891)
N	3155	988	729	341	273

Notes: ASVAB test score is the average test score: Word Knowledge and Paragraph Comprehension, Arithmetic Reasoning, Mathematics Knowledge, and Numerical Operation are for general cognitive, and Mechanical Comprehension, Auto & Shop Information, and Electronics Information are for mechanical.

relevant for college graduates.

2.3 Summary statistics

Table 2 shows summary statistics. Both cognitive and mechanical test scores are standardized to have mean 0 and standard deviation 1 over the population. High school shows the lowest and STEM majors show the highest average score, and people with a higher level of education tend to have higher test scores. Similarly, general cognitive task intensity is higher for a higher level of education, with high school the lowest and STEM majors the highest. Interestingly, this is not the case for mechanical task. In mechanical task intensity, high school shows the highest, while Humanities & Social Sciences and Business & Economics majors show the lowest. Even STEM majors, who show much higher mechanical test scores, show lower mechanical task intensity than high school graduates. This suggests that mechanical skill is not important to them after college. About 50% of people took an occupation related to their major. Wages are higher for higher levels of education, and as expected, STEM majors show higher wages than the other majors.

3 Model

I first present an outline of the model. Then, I explain details of the empirical implementation.

3.1 Two period model outline

The model has two periods. It starts at high school graduation and there are multiple decision stages. The time frame is as follows.

1. Figure 3 shows the decision flow in period 1. In the beginning of period 1, high school graduates are endowed with five-dimensional skills: general cognitive, mechanical, Humanities & Social Sciences major specific, Business & Economics major specific, and STEM major specific skills. In the beginning of this period, they choose one from work, some college, and college. If they choose work, they first choose a job type, either cognitive or mechanical. In cognitive type jobs, mechanical skill does not matter.[15] After choosing a type, they choose task intensity corresponding to their chosen type of jobs; if they choose a cognitive type, they choose cognitive task intensity.[16] High school graduates are assumed to have a job unrelated to any major. Their wages depend on their chosen job type, task intensity, and their skill levels. If they choose some college, they do not have a choice anymore in this period. If students decide to go to college, they learn how much major-specific skills they will have at college graduation for each major. I call them potential post-college major-specific skills. Based on their skills and potential post-college major-specific skills, they choose a college major from Humanities & Social Sciences, Business & Economics, and STEM majors. Skills of students choosing post-secondary education will evolve depending on their choice of education level and major.

2. Figure 4 presents the decision flow in period 2. In period 2, high school graduates continue working. Those who took post-secondary education finish their education and enter the labour market. Their skill levels change from pre-college skill levels depending on their education choice in the previous period. Some college could increase general cognitive and mechanical skills. As with high school graduates, those who went to some college first choose either a cognitive type or mechanical type job and then choose task intensity. They are assumed to have a job unrelated

[15] Since I do not estimate the model part regarding mechanical type jobs, I do not assume which type of skill matters in mechanical type jobs.

[16] Workers with higher general cognitive skill will choose higher cognitive task intensity because the wage return may be higher or they may be able to do the task more easily than workers with lower skill.

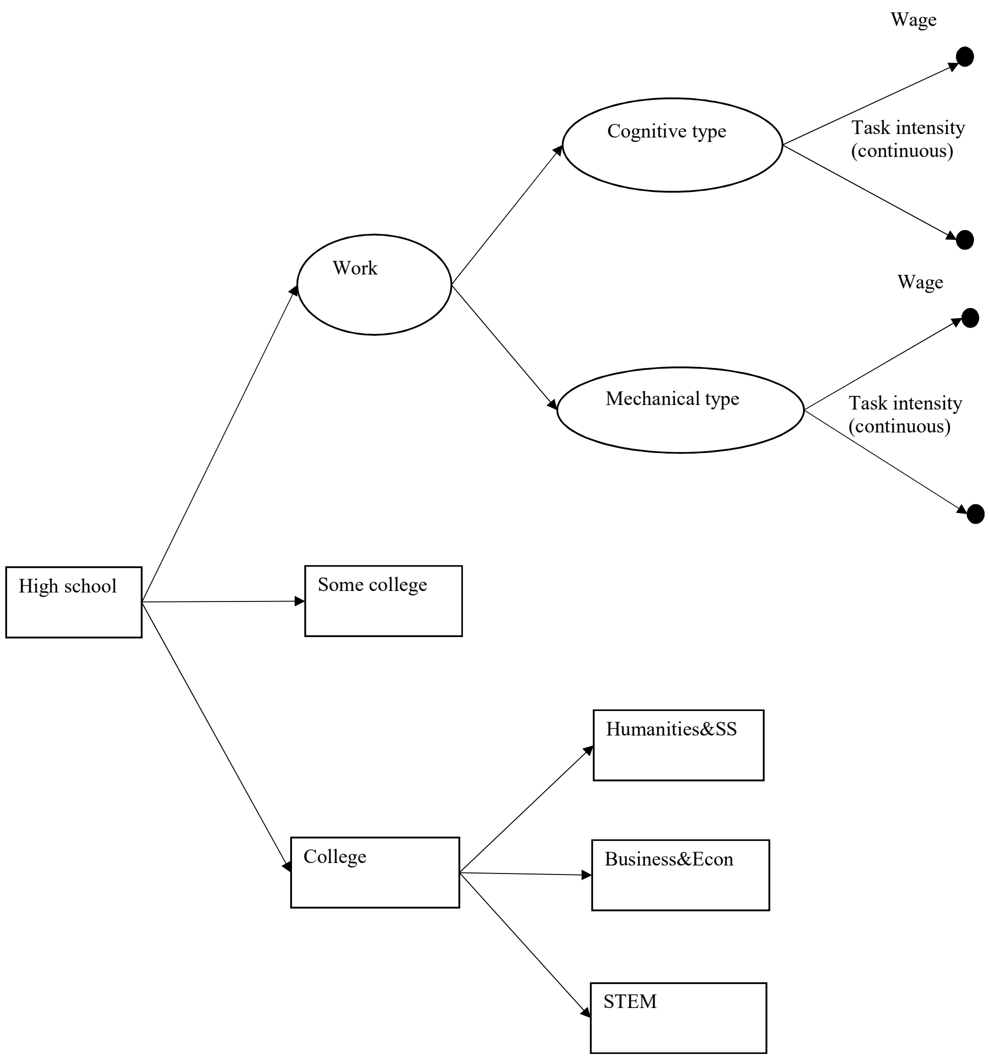

Figure 3: Decision flow; period 1

to any major. Their wages depend on their chosen job type, task intensity, and their skill levels. Each college major increases general cognitive skill and its relevant major-specific skill. All college graduates are assumed to choose a cognitive type job. Hence, they choose cognitive task intensity. They are assumed to have a job unrelated to majors other than their own major, but they can choose job relatedness to their major, either related or unrelated. Returns to major-specific skill can depend on the relatedness. Their wages depend on their chosen general cognitive task intensity, job relatedness to their major, and post-college skill levels.

It is well known that cognitive skill matters to whether people go to college or not. Furthermore, students are significantly sorted into college major based on their cognitive skill levels. I assume that

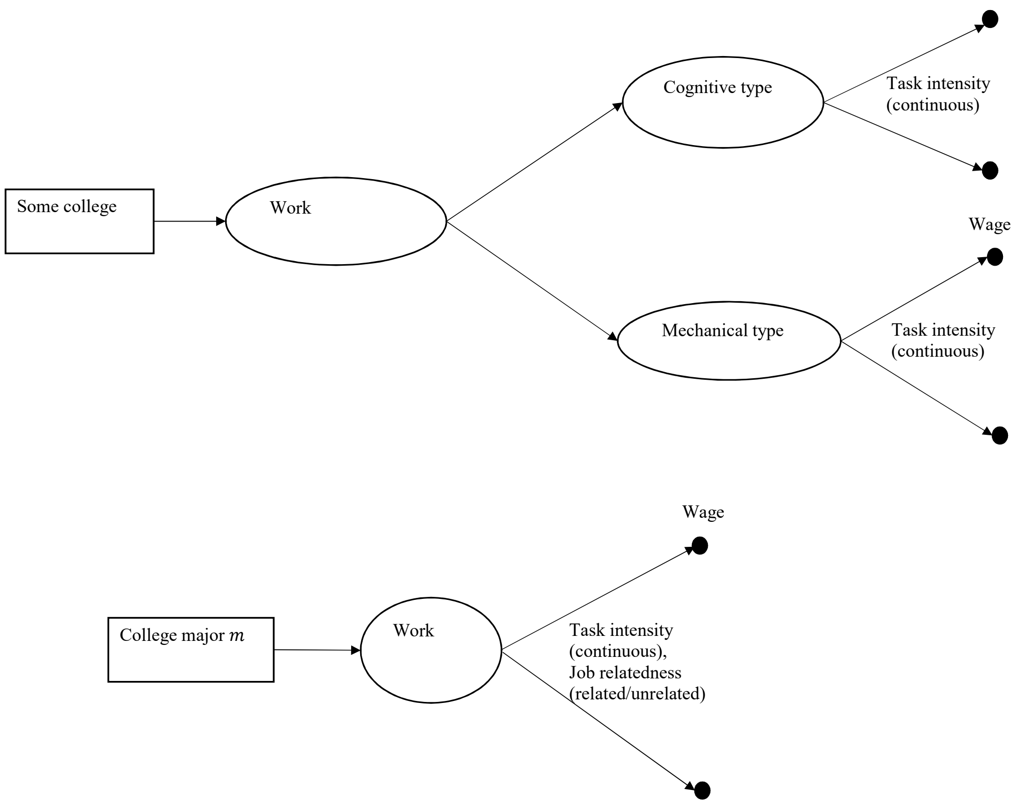

Figure 4: Decision flow; period 2

students accumulate cognitive skills in college. College students take courses, most of which require cognitive skill. Course taking varies significantly by college major and so majors may differ in the accumulation of cognitive skills may vary by college major.

I divide cognitive skill into two types of skills, general cognitive and major-specific skills. General cognitive skill can increase in any major, while major-specific skill can increase only in its relevant major. Hence, general cognitive skill captures the similarity of skills accumulated in different majors, and major-specific skill reflects the uniqueness of the skill accumulated in each major. For example, business majors will study business cases to learn business models of companies. They will increase general cognitive skill through understanding and interpreting the cases. However, knowledge on the business cases per se, such as when and what company introduced the business model or the business history, is business major specific. In my model, further education represents a set of skill production functions, each taking pre-college general cognitive and major-specific skills as an input and evolved general cognitive and major-specific skills as outputs.

3.2 Empirical model

This subsection explains the details of my empirical model. In several decision stages, I assume a linear latent utility form instead of fully specifying a dynamic discrete choice model. A linear latent utility form is used in papers applying a factor model in a dynamic treatment effect model (see, e.g., Heckman et al. (2016a,b), and Fruehwirth et al. (2016)). This specification is used mainly for simplicity, and one cannot say anything about the full underlying decision model. However, it has other advantages over fully specified dynamic discrete choice models. Fully specified dynamic discrete choice models require strong assumptions on agent preferences, constraints, and information sets. On the other hand, this simplified specification captures some essential features of dynamic discrete choice models without imposing the strong assumptions.

3.2.1 General cognitive and mechanical skills

High school graduates are assumed to have five types of skills, general cognitive, mechanical, and three types of major-specific skills. I here describe general cognitive and mechanical skills. Let $s_{1i} = (s^c_{1i}, s^{mech}_{1i})$ denote individual i's pre-college general cognitive and mechanical skill levels. The skills are modeled as follows:

$$s_{1i}^{c} = x_{si}'\alpha^{c} + \theta_i^{c} \tag{1}$$

$$s_{1i}^{mech} = x_{si}'\alpha^{mech} + \theta_i^{mech}. \tag{2}$$

This implies that the skills are the result of characteristics observed by the econometrician, x_s, and an unobserved component, $\theta = (\theta^c, \theta^{mech})$. Unobserved components θ are orthogonal with observed components x_s. The vector x_s contains a constant, a female dummy, race dummies, father's education, mother's education, dummies for regions of residence in 1997, a dummy for living in an urban area in 1997, a dummy for broken home in 1997, household income in 1997, and the number of siblings. Parent's education is categorized into high school, some college, college, or graduate degree. Household income is divided into quartile groups. This skill specification is similar to Aucejo and James (2016).[17]

The unobserved components are joint normally distributed and may be correlated with each other as in Prada and Urzúa (2017):

$$\begin{pmatrix} \theta^c \\ \theta^{mech} \end{pmatrix} \sim N\left(\mathbf{0}, \Sigma\right).$$

Theoretically, the distributions of θ's can be identified nonparametrically, and many previous papers using a factor model assume a mixture of normals instead of a normal distribution. However, since my model has five unobserved factors in total, including major-specific skills discussed below, and it is already computationally intensive, I assume normality.

Skill measurement system

Pre-college skills s_1 are not directly observed, but ASVAB test scores are assumed to be noisy measures of skills. Word Knowledge, Paragraph Comprehension, Arithmetic Reasoning, Mathematics Knowledge, and Numerical Operation are used to construct a single cognitive skill measure in many papers. I assume that general cognitive skill affects all of the five test scores. Furthermore, I use Mechanical Comprehension, Auto & Shop information, and Electronics Information as noisy measures of mechanical skill. These three are also used as mechanical tests in Prada and Urzúa (2017) and Speer (2017). General

[17] They do not assume any structure on corerlation between θ's. I allow θ^c and θ^{mech} to be correlated. The approach is different, because I do not have many skill measurements.

cognitive skill is assumed to have effects on Mechanical Comprehension and Electronics Information test scores as well. Students need to read and understand questions, and many questions also require basic knowledge on calculation.

I assume the following measurement system:

$$WordKnowledge_i = s_{1i}^c + e_{1i} \tag{3}$$

$$ParagraphComprehension_i = \delta_{12} s_{1i}^c + e_{2i} \tag{4}$$

$$ArithmeticReasoning_i = \delta_{13} s_{1i}^c + e_{3i} \tag{5}$$

$$MathematicsKnowledge_i = \delta_{14} s_{1i}^c + e_{4i} \tag{6}$$

$$NumericalOperation_i = \delta_{15} s_{1i}^c + e_{5i} \tag{7}$$

$$MechanicalComprehension_i = \delta_{16} s_{1i}^c + \delta_{26} s_{1i}^{mech} + e_{6i} \tag{8}$$

$$Auto\&ShopInformation_i = s_{1i}^{mech} + e_{7i} \tag{9}$$

$$ElectronicsInformation_i = \delta_{18} s_{1i}^c + \delta_{28} s_{1i}^{mech} + e_{8i}, \tag{10}$$

Each measure is standardized to have mean 0 and standard deviation 1, and measurement error e_s, $s = 1, 2, \cdots, 8$, is idiosyncratic with $E(e_s) = 0$, following a normal distribution. Hence, the population average of each type of skill is normalized to 0. The factor loadings on s_1^c in Word Knowledge and s_1^{mech} in Auto & Shop Information are normalized to one. This normalization is necessary for identification. Moreover, one of the mechanical tests has to be assumed to be affected only by the mechanical skill because θ^c and θ^{mech} may be correlated. I choose Auto & Shop Information following Prada and Urzúa (2017). Details on the identification of the measurement system are described in the appendix.[18]

Skill changes through post-secondary education

Skills will change through post-secondary education. The increment to skills will vary by education level and college major. As mentioned above, courses students have to take depend significantly on their college majors. This means that the amount of time students invest on skill accumulation may vary by their college major. Furthermore, I allow skill changes to depend on pre-college skill. For

[18]In order to make the skills intuitive, general cognitive and mechanical skills are defined to be affected by observed characteristics x_s in equations (1) and (2). Instead, general cognitive skill and mechanical skill could be defined as θ^c and θ^{mech}, respectively, and each type of ASVAB test scores could be assumed to be affected by x_s. The parameter estimates will not change much. This is because θ and the observed characteristics in each decision stage are assumed to be orthogonal.

example, even if students take the same course, those with higher pre-college skill might be able to understand the contents more deeply than those with lower pre-college skill. In addition, those who understand the contents better might take advanced level courses further. In these cases, students with higher pre-college skill accumulate more skills than those with lower pre-college skill. On the other hand, if a curriculum focuses on making students achieve a certain common level, then students with lower pre-college skill may have to study harder, and so their skill growth might be larger than those with higher pre-college skill.

Let *Some* denote some college, H denote Humanities & Social Sciences majors, E denote Business & Economics majors, and S denote STEM majors. For individual i from post-secondary education group $m_+ = Some, H, E, S$, I specify post-college general cognitive skill level as

$$s^c_{2m_+,i} = \lambda^c_{0m_+} + \lambda^c_{1m_+} s^c_{1i}. \tag{11}$$

Since s^c_1 is normalized to have mean 0 over the population, parameter $\lambda^c_{0m_+}$ represents an average growth in education m_+. On the other hand, $\lambda^c_{1m_+}$ shows the effects of pre-college skill level on skill growth.[19] Skill growth from periods 1 to 2 is written as

$$s^c_{2m_+,i} - s^c_{1i} = \lambda^c_{0m_+} + (\lambda^c_{1m_+} - 1)s^c_{1i}.$$

Hence, if $\lambda^c_{1m_+} > 1$, students with higher pre-college skill will accumulate more. On the other hand, if $\lambda^c_{1m_+} < 1$, students with lower pre-college skill will accumulate more.[20]

3.2.2 Major-specific skills

If they go to college, individuals can increase a major-specific skill. The increment to skill varies by individuals. I assume that individuals are endowed with potential post-college major-specific skill regarding each major, which represents post-college major-specific skill level if they choose the relevant

[19] Although post-education mechanical skill will be specified as $s^{mech}_{2i} = \lambda^{mech}_{0,Some} + \lambda^{mech}_{1,Some} s^{mech}_{1i}$ for some college, mechanical skill growth parameters are not estimated below to focus on general cognitive skill growth. I do not consider mechanical skill for those who go to college graduates. As suggested above, the mechanical dimension does not seem important to college graduates.

[20] Skill change equation (11) implies that there is no individual heterogeneity in skill changes after conditioning on pre-college skill. Suppose instead that the skill change equation is given by $s^c_{2m_+,i} = \lambda^c_{0m_+} + \lambda^c_{1m_+} s^c_{1i} + \omega_{im_+}$, where ω_{m_+} is orthogonal with s_1 and $E(\omega_{m_+}) = 0$. As long as ω_{m_+} is unknown to the individual in education choices, λ^c's are consistently estimated in my approach. As explained below, I use task intensity equation to identify λ^c's and the parameters in the equation can be estimated consistently.

major. Potential post-college major-specific skills are assumed to be unknown to the individuals until they decide to go to college. Although college students will be sorted in majors based on their potential post-college major-specific skill levels, that sorting does not affect education level choice. This assumption makes computation easier. At the same time, this assumption is reasonable given that previous papers show that the sorting occurs as college students learn their ability through GPA (see, e.g., Arcidiacono (2004) and Stinebrickner and Stinebrickner (2014)).

For major $m = H, E, S$, let s_{2i}^m denote individual i's potential post-college major m specific skill, that is, major m specific skill level in period 2 if they choose major m. I assume that post-college major-specific skills are orthogonal to s_1^c. I further assume that they are orthogonal to each other and normally distributed:

$$\begin{pmatrix} s_2^H \\ s_2^E \\ s_2^S \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_H^2 & 0 & 0 \\ 0 & \sigma_E^2 & 0 \\ 0 & 0 & \sigma_S^2 \end{pmatrix} \right).$$

The normality is assumed for simplicity. Each type of potential post-college major-specific skill is standardized to have mean 0 over the population. Hence, $s_{i2}^m = 0$ denotes that individual i's potential post-college major m specific skill is the same as the population average. If individual i chooses major m, their post-college major m specific skill is equal to s_{2i}^m, while the other types of major-specific skills do not increase.

Skill measurement system

I assume that the last two years of college GPA are noisy measures of post-college major-specific skill. Since around 10% of the college students in my data received the maximum GPA score in at least one of the last two years, I allow for ceiling effects following Hansen et al. (2004), who examine the effects of schooling on cognitive test scores. For college major m, let GPA_{1i}^{*m} and GPA_{2i}^{*m} denote individual i's latent college GPA in major m in their last year and their second last year. I assume that

$$GPA_{1i}^{*m} = \gamma_{01}^m + \gamma_{11}^m s_{2mi}^c + s_{2i}^m + x_{gi}' \beta_{g1}^m + e_{1i}^m \tag{12}$$

$$GPA_{2i}^{*m} = \gamma_{02}^m + \gamma_{12}^m s_{2mi}^c + \gamma_{22}^m s_{2i}^m + x_{gi}' \beta_{g2}^m + e_{2i}^m. \tag{13}$$

Measurement error e_t^m, $t = 1, 2$, is idiosyncratic with $E(e_t^m) = 0$ and follows a normal distribution.[21] The factor loading on s_2^m in the last year of GPA is set to one for identification. Observed variables x_g include a female dummy and region dummies. A female dummy is added, because females tend to earn better GPA than males. Region dummies are included, because college quality might be different by regions and college GPA might be standardized within college. For each year t, observed GPA is assumed to be

$$GPA_{ti}^m = \overline{GPA_t^m} \;\; if \; GPA_{ti}^{*m} \geq \overline{GPA_t^m} \tag{14}$$

$$GPA_{ti}^m = GPA_{ti}^{*m} \;\; if \; GPA_{ti}^{*m} < \overline{GPA_t^m}. \tag{15}$$

Each observed GPA is standardized to have mean 0 and standard deviation 1 over those who actually chose major m and $\overline{GPA_t^m}$ is the upper limit of GPA.

College GPA GPA_t^m, $t = 1, 2$, are observed in the data only if students actually chose major m, and the GPA equations provide only two noisy measures of each type of major-specific skill. Hence, the GPA equations are not enough to identify the distribution of (s_2^H, s_2^E, s_2^S) . The distribution can be identified by additionally using college major choice, which is described later, as another "noisy measure" of major-specific skills. See Hansen et al. (2004) for details on the identification.

Pre-college major-specific skills

In the beginning of period 1, individual i is endowed with pre-college majors specific skills $(s_{1i}^H, s_{1i}^E, s_{1i}^S)$. For each major m, if they choose high school, some college, or major m', $m' \neq m$, then major m specific skill in period 2 is s_{1i}^m. If they choose major m, then major m specific skill evolves from s_{1i}^m to s_{2i}^m. For each major m, if there is a measure of pre-college major-specific skill s_1^m, an approach of comparing s_{1i}^m and s_{2i}^m to identify major m specific skill growth of individual i from major m would be relatively straightforward. However, my data do not have good measures of pre-college major-specific skills. In section 4.2, I discuss how I make some assumptions to specify s_{1i}^m in the absence of good direct measures. However, for expositional purposes, I assume that I have a good measure of s_{1i}^m in the same scale as s_{2i}^m in the rest of this section and describe the rest of my model.

[21] Students may increase the major-specific skill between these two periods. I assume that individual i's major m specific skill in the second last year is expressed in the form of $c_0^m + c_1^m s_{2i}^m$, where c_0^m and c_1^m are constants. With regard to general cognitive skill, I assume either a similar specification holds or the skill does not increase between these periods.

3.2.3 Log wage equations

As mentioned above, skills and tasks are not the same in my model; workers possess skills and apply them to tasks. Occupations involving more intense general cognitive tasks will ask workers to use their cognitive skill intensely, and workers will produce more or higher quality goods in the occupation. Hence, log wage equations include both general cognitive skill and task intensity. Cognitive jobs are further divided into either related or unrelated to each major. The return to major-specific skill can depend on whether the job is related to the relevant major. Let r denote related and nr denote not related.

Let s_i^c denote individual i's general cognitive skill level in entering the labour market. That is, $s_i^c = s_{1i}^c$ if individual i is a high school graduate, while $s_i^c = s_{2m_+,i,}^c$ if individual i is from post-secondary education $m_+ = Some, H, E, S$. Let τ^c denote general cognitive task intensity. I assume the following log wage function for individual i from education $m_- = HS, Some$, who chooses a cognitive type job:

$$\log w_{m_-,i} = \pi_0 + \pi_1 s_i^c + \pi_{21}\tau_i^c + \pi_{22}(\tau_i^c)^2 + \sum_{m' \in \{H,E,S\}} \pi_{3,nr}^{m'} s_{1i}^{m'} + x_{wi}'\beta_{m_-}, \tag{16}$$

Remember that both high school and some college graduates are assumed to have jobs unrelated to any college major.

College graduates can choose job relatedness regarding their college major. For job relatedness $R = r, nr$, I assume the following log wage equation for college graduates from major m:

$$\begin{aligned}\log w_{mRi} &= \pi_0 + \pi_{0r}^m \cdot 1(R=r) + \pi_1 s_{2mi}^c + \pi_{21}\tau_i^c + \pi_{22}(\tau_i^c)^2 + (\pi_{3,nr}^m + \pi_{3mr} \cdot 1(R=r))s_{2i}^m \\ &\qquad + \sum_{m' \neq m} \pi_{3,nr}^{m'} s_{1i}^{m'} + x_{wi}'\beta_m \\ &= \pi_{0mR} + \pi_1 s_{2mi}^c + \pi_{21}\tau_i^c + \pi_{22}(\tau_i^c)^2 + \pi_{3,R}^m s_{2i}^m + \sum_{m' \neq m} \pi_{3,nr}^{m'} s_{1i}^{m'} + x_{wi}'\beta_m, \end{aligned} \tag{17}$$

where $\pi_{0mR} = \pi_0 + \pi_{0r}^m \cdot 1(R=r)$, $\pi_{3R}^m = \pi_{3,nr}^m + \pi_{3mr} \cdot 1(R=r)$, and $1(\cdot)$ is an indicator function. Log wages are observed with an additive idiosyncratic error ε_w, following a normal distribution. Control variables, x_w, include a female dummy, race dummies, region dummies, an urban dummy, and cohort dummies.[22] If $\pi_{3,nr}^m < \pi_{3r}^m$, the log wage return to major m specific skill is larger in related jobs than in

[22] I allow the coefficients of a female dummy and race dummies to be different across the education groups. I also assume that the coefficients of cohort dummies are common across the college majors.

unrelated jobs. Still, the major-specific skill is useful in unrelated jobs as long as $\pi^m_{3,nr} > 0$.

3.2.4 Job type choice, task intensity choice, and job relatedness choice

As mentioned above, high school graduates and some college graduates decide whether to take a cognitive or mechanical type of job when they enter the labour market. For $m_- = HS, Some$, individual i's latent utility of choosing a cognitive type job is

$$I^c_{m_-,i} = \xi^c_0 + \xi^c_1 s^c_i + \xi^c_2 s^{mech}_i + \sum_{m' \in \{H,E,S\}} \xi^c_{3m'} s^{m'}_{1i} + x'_{ci}\beta^c_{m_-} + \varepsilon^c_{m_-,i}. \tag{18}$$

Observed variables x_c include a female dummy, race dummies, region dummies, an urban dummy, household income at 17, and cohort dummies. The latent utility of choosing a mechanical type of job is normalized to 0. I assume that $\varepsilon^c_{m_-}$ follows a Type-I extreme value distribution for simplicity. This assumption gives a standard logit model.

High school and some college graduates who decide to take a cognitive type of job choose cognitive task intensity. College graduates who enter the labour market do so as well. As mentioned above, the mechanical dimension does not matter in cognitive type jobs. Thus, they only choose cognitive task intensity. Those with higher general cognitive skill will choose higher cognitive task intensity because the wage return may be higher or they may be able to do the task more easily than those with lower skill.

For individual i who possesses general cognitive skill s^c_i and whose education is $m_{++} = HS, Some, H, E, S$, optimal cognitive task intensity is assumed to be:

$$\tau^c_{m_{++},i} = \zeta_0 + \zeta_1 s^c_i + x'_{ci}\beta_{\tau m_{++}} + \varepsilon_{\tau m_{++},i}. \tag{19}$$

In addition to the observed variables, there is an idiosyncratic shock, $\varepsilon_{\tau m_{++}}$, that follows a normal distribution with mean of 0.[23] This shock is unknown to the individual when they choose their education. This linear specification can be considered as an approximation to a more general form. Or, although I do not estimate the full model, this linear form can be derived from a utility maximization problem (see the appendix).

College graduates choose job relatedness to their college major. Both major-specific and general

[23] The coefficients of region dummies, an urban dummy, and household income at 17 are assumed to be common across the education groups and the coefficients of cohort dummies are assumed to be common across college majors.

cognitive skills can affect the relatedness choice. Latent utility of choosing a related job for individual i from major m is given by

$$I_{r,m,i} = \xi_{0,r,m} + \xi_{1,r,m} s^{c}_{2mi} + \xi_{2,r,m} s^{m}_{2i} + x'_{wi}\beta_{rm} + \varepsilon_{rmi}, \tag{20}$$

where ε_{rm} is an idiosyncratic shock and assumed for simplicity to follow a Type-I extreme value distribution. The latent utility of choosing an unrelated job is normalized to 0.

In this specification, I cannot examine whether individuals are in an unrelated job because they want it or because they cannot find a related job. This distinction however does not matter for skill growth estimation.

3.2.5 Education level choice and major choice

High school graduates choose one of three options: work, some college, or college. If they choose to work, they enter the labour market with skills $(s^c_1, s^{mech}_1, s^H_1, s^E_1, s^S_1)$. If they decide to go to college, they then choose a college major. In this two-stage education choice framework, some information, such as potential major-specific skill growth during college and exogenous shocks affecting college major choice, is assumed to be revealed after students decide to go to college.

As suggested in the data section, there is sorting into education level and college major based on pre-college general cognitive skill. The cognitive test scores imply that those with a more advanced degree tend to have higher pre-college general cognitive skill; STEM majors tend to have high pre-college general cognitive skill among college graduates. There are two channels that affect sorting. One is that skill development might depend on pre-college skill level. For example, if the increment to the skill in STEM majors increases with pre-college skills, then students with high pre-college skill will be more likely to choose STEM majors. Another is something other than through skill, such as study cost. Even if low-skilled students know that they will accumulate more skill in STEM majors, keeping up with the classes or their peers may require them to work very hard. In this case, students with low pre-college skill may prefer to choose an easier major with a smaller skill increase.

Education level choice

High school graduates have three options: no further education, some college, or college. Let Col denote college. Individual i's latent utility of choosing education level $l = HS, Some, Col$, is

$$I_{li} = \eta_{0l} + \eta_{1l} s_{1i}^{c} + \eta_{2l} s_{1i}^{mech} + \sum_{m' \in \{H,E,S\}} \eta_{3,l,m'} s_{1i}^{m'} + x'_{di}\beta_l + z_{dli}\varphi_{dl} + \varepsilon_{dli}. \tag{21}$$

Observed variables x_d include the same variables in x_s and cohort dummies. In addition, a local unemployment rate at age 17 enters as an exclusion variable, z_{dl}.[24] For simplicity, I assume that ε_{dl} follows a Type-I extreme value distribution, which gives a standard multinomial logit model. The probability of choosing education level l given the skills and the observed variables is given by

$$\frac{\exp(\eta_{0l} + \eta_{1l} s_{1i}^{c} + \eta_{2l} s_{1i}^{mech} + \sum_{m'} \eta_{3,l,m'} s_{1i}^{m'} + x'_{di}\beta_l + z_{dli}\varphi_{dl})}{\sum_{L \in \{HS,Some,Col\}} \exp(\eta_{0L} + \eta_{1L} s_{1i}^{c} + \eta_{2L} s_{1i}^{mech} + \sum_{m'} \eta_{3,L,m'} s_{1i}^{m'} + x'_{di}\beta_L + z_{dLi}\varphi_{dL})}.$$

The base group is high school.

College major choice

If high school graduates choose to go to college, they then choose a college major. In addition to pre-college general cognitive and major-specific skills, potential major-specific skill growth, which is revealed after deciding to go to college, can affect college major choice. Since the mechanical dimension does not matter to college graduates, mechanical skill does not appear in the college major choice equation.

As in education level choice, I assume linear latent utility with an idiosyncratic shock following a Type-I extreme value distribution.[25] Individual i's latent utility of choosing major m is given by

$$I_{mi} = \eta_{0m} + \eta_{1m} s_{1i}^{c} + \eta_{2m} s_{2i}^{m} + \eta_{3m} s_{1i}^{m} + x'_{di}\beta_m + z'_{mi}\varphi_m + \varepsilon_{mi}. \tag{22}$$

Exclusion variables z_m include a foreign born parents dummy, math test score relative to cognitive test score, and mechanical test score relative to cognitive test score.[26] As shown in the summary statistics

[24] The local unemployment rates by education level are constructed from the CPS. The local unit is defined by the combination of regions and MSA residency.

[25] I assume ε_{rm} in equation (20) and ε_m are orthogonal. They could be allowed to be correlated, but, in that case, I want to reduce the number of majors to two because of the computation burden.

[26] A foreign born parents dummy is also used in Kinsler and Pavan (2015). The math test score relative to cognitive test score is defined as residuals from regressing average math test scores on a constant and cognitive test scores among college graduates. The constructed score is included in preferences. I assume that, given general cognitive skill, the composition of the skill does not affect wages or skill growth. Similarly, the mechanical test score relative to cognitive test score is

above, STEM majors tend to have higher pre-college mechanical test scores among college graduates.[27] Also, if cognitive tests are divided into verbal tests and math tests, Humanities & Social Sciences majors, on average, have high verbal scores compared to math scores.[28] The probability of choosing major m given the observed variables, pre-college skills, and potential post-college major-specific skills is given by

$$\frac{\exp(\eta_{0m} + \eta_{1m} s^c_{1i} + \eta_{2m} s^m_{2i} + \eta_{3m} s^m_{1i} + x'_{di}\beta_m + z'_{mi}\varphi_m)}{\sum_{n\in\{H,E,S\}} \exp(\eta_{0n} + \eta_{1n} s^c_{1i} + \eta_{2n} s^n_{2i} + \eta_{3n} s^n_{1i} + x'_{di}\beta_n + z'_{ni}\varphi_n)}.$$

The base group is Humanities & Social Sciences majors.

4 Identification of skill growth

4.1 General cognitive skill

As mentioned above, the distribution of s^c_1 can be identified from the skill measurement system, equations (3) to (10). The parameters of general cognitive skill growth are identified from general cognitive task intensity choices. Task intensity choice equation (19) is written in terms of general cognitive skill brought to the labour market. For high school graduates, the equation is

$$\tau^c_{HS,i} = \zeta_0 + \zeta_1 s^c_{1i} + x'_{ci}\beta_{\tau,HS} + \varepsilon_{\tau,HS,i}. \tag{23}$$

Hence, ζ_0, ζ_1, and $\beta_{\tau,HS}$ can be estimated from task intensity choice of high school graduates.

For those choosing some college or college, s^c in task intensity choice equation (19) is different from the pre-college skill level. Using the skill change equation (11), the equation can be rewritten in terms of pre-college general cognitive skill s^c_1. For post-secondary education group $m_+ = Some, H, E, S$, the equation can be rewritten as

defined as residuals from regressing average mechanical test scores on a constant and cognitive test scores among college graduates.

[27] One might think that mechanical skill increases in STEM majors. However, as shown above, the average mechanical task intensity among STEM majors is smaller than that among high school graduates and it does not seem mechanical skill is not important for STEM majors. Hence, pre-college mechanical skill is included in preferences here.

[28] Among Humanities & Social Sciences majors, the mean of the average score of the verbal tests is 0.64 and that of the math tests is 0.53. The means are 0.51 and 0.68 among Business & Economics majors, and 0.73 and 0.89 among STEM majors.

$$
\begin{aligned}
\tau^c_{m_+,i} &= (\zeta_0 + \zeta_1 \lambda^c_{0m_+}) + \zeta_1 \lambda^c_{1m_+} s^c_{1i} + x'_{ci}\beta_{\tau m_+} + \varepsilon_{\tau m_+,i} \\
&= \tilde{\zeta}_{0m_+} + \tilde{\zeta}_{1m_+} s^c_{1i} + x'_{ci}\beta_{\tau m_+} + \varepsilon_{\tau m_+,i},
\end{aligned} \tag{24}
$$

Hence, $\tilde{\zeta}_{0m_+}$, $\tilde{\zeta}_{1m_+}$, and $\beta_{\tau m_+}$ can be identified. Since $\tilde{\zeta}_{0m_+} = \zeta_0 + \zeta_1 \lambda^c_{0m_+}$ and $\tilde{\zeta}_{1m_+} = \zeta_1 \lambda^c_{1m_+}$, the skill growth parameters are written as

$$
\begin{aligned}
\lambda^c_{0m_+} &= \frac{\tilde{\zeta}_{0m_+} - \zeta_0}{\zeta_1} \\
\lambda^c_{1m_+} &= \frac{\tilde{\zeta}_{1m_+}}{\zeta_1}.
\end{aligned}
$$

Parameters ζ_0, ζ_1, $\tilde{\zeta}_{0m_+}$, and $\tilde{\zeta}_{1m_+}$ are identified as mentioned above. Hence, $\lambda^c_{0m_+}$ and $\lambda^c_{1m_+}$ in the skill change equation $s^c_{2m_+,i} = \lambda^c_{0m_+} + \lambda^c_{1m_+} s^c_{1i}$ can be identified.

Log wage point metric

Since there is no natural unit of skills, it is difficult to interpret the results without some reference point. With regard to general cognitive skill, multiplying both sides of the skill change equation (11) by π_1, which is the coefficient of general cognitive skill in log wage equation (16), gives

$$
\pi_1 s^c_{2m_+,i} = \pi_1 \lambda^c_{0m_+} + \lambda^c_{1m_+} \pi_1 s^c_{1i},
$$

for $m_+ = Some, H, E, S$. By construction, a one unit increase in $\pi_1 s^c_{1i}$ will increase log wage by 1 point. Skill growth from periods 1 to 2 is written as

$$
\pi_1 (s^c_{2m_+,i} - s^c_{1i}) = \pi_1 \lambda^c_{0m_+} + (\lambda^c_{1m_+} - 1)\pi_1 s^c_{1i}. \tag{25}
$$

In this form, skill growth is interpreted in terms of log wage points, that is, by how many points the skill growth will increase log wages. Although general cognitive skill growth can be compared across education groups without this transformation, this log wage point transformation makes interpretation easier.

4.2 Major-specific skills

The distributions of potential post-college major-specific skills are identified from the GPA equations (12) to (15) and college major choice equation (22). If I had measures of pre-college major-specific skill s_1^m in the same metric as s_2^m, major m specific skill growth could be measured by $s_2^m - s_1^m$. However, as mentioned above, I do not have measures of s_1^m, and I need to make some assumptions on s_{1i}^m.

I assume that pre-college major-specific skill levels are the same across individuals. That is, for major m, $s_{1i}^m = s_1^m$ for individual i. Under this assumption, terms of pre-college major-specific skill are absorbed in constant terms in the equations shown in the previous section. The log wage equation for education groups $m_- = HS, Some$ (16) is rewritten as

$$\begin{aligned}\log w_{m_-,i} &= \left(\pi_0 + \sum_{m' \in \{H,E,S\}} \pi_{3,nr}^{m'} s_1^{m'}\right) + \pi_1 s_i^c + \pi_{21}\tau_i^c + \pi_{22}(\tau_i^c)^2 + x'_{wi}\beta_{m_-} \\ &= \tilde{\pi}_0 + \pi_1 s_i^c + \pi_{21}\tau_i^c + \pi_{22}(\tau_i^c)^2 + x'_{wi}\beta_{m_-} \qquad (26)\end{aligned}$$

Log wage equation for major $m = H, E, S$, (17), is rewritten as

$$\begin{aligned}\log w_{mRi} &= \left(\pi_{0mR} + \sum_{m' \neq m} \pi_{3,nr}^{m'} s_1^{m'}\right) + \pi_1 s_{2mi}^c + \pi_{21}\tau_i^c + \pi_{22}(\tau_i^c)^2 + \pi_{3,R}^m s_{2i}^m + x'_{wi}\beta_m \\ &= \tilde{\pi}_{0mR} + \pi_1 s_{2mi}^c + \pi_{21}\tau_i^c + \pi_{22}(\tau_i^c)^2 + \pi_{3,R}^m s_{2i}^m + x'_{wi}\beta_m. \qquad (27)\end{aligned}$$

Parameters $\pi_{3,nr}^m$, $m = H, E, S$, cannot be identified, but that does not matter to the identification of the skill growth. Job type choice equation (18), education level choice equation (21), and college major choice equation (22) can be rewritten in the same way. The rewritten equations are described in the appendix.

Of course, s_1^m cannot be identified from these equations. I approximate s_1^m to the skill level, with which male high school graduates living in Northeast region and whose pre-college general cognitive skill level is the population average would receive zero GPA in the 0-4.0 scale for courses taken in last year of college.[29]

[29] Using another group of people, such as females or people with high pre-college skill, does not change my results on the contribution of major-specific skill growth on wage growth.

Log wage point metric

Unlike general cognitive skill, major-specific skill growth cannot be directly compared across college majors because each major increases its own type of skill, which is measured in its own scale. Major-specific skill growth, of course, cannot be directly compared with general cognitive skill growth either. As with general cognitive skill growth, I transform major-specific skill growth into a log wage point metric. However, the coefficient of major-specific skill can be different by job relatedness to the major, and which coefficient is used can significantly affect the result. I transform major-specific skill growth into log wage point metric in three ways.

Since major-specific skill will be utilized better in related jobs, I calculate log wage contribution of major-specific skill growth in related jobs, that is, $\pi_{3r}^{m}(s_2^m - s_1^m)$ for each major m. In order to see the sensitivity with the assumption on pre-college major-specific skill level, I also calculate $\pi_{3r}^{m}(s_2^m - s_{2m,10}^m)$, where $s_{2m,10}^m$ denotes the tenth percentile of s_2^m of those who choose major m.

Although the return to a major-specific skill in related jobs will be larger than that in unrelated jobs, the wage may be low if their major-specific skill level is not large enough. If their major-specific skill level is not high, college graduates can choose an unrelated job to avoid low wage in a related job. On the other side of the coin, evolved major-specific skill can be considered to provide college graduates an option to earn higher wages in related jobs than in unrelated jobs if their skill is high. Based on this perspective, I calculate the contribution of major-specific skill growth on log wage given individuals choose a job relatedness that brings them higher wages. From the point of log wage value of skill growth, this skill growth transformation will be appropriate to compare with general cognitive skill growth.

I transform the skill growth into the log wage point metric as follows. Suppose $\pi_{3r}^m > \pi_{3,nr}^m$ in log wage equation (27). In this case, there is $\bar{s}^m$ such that log wage in a related job is larger than that in an unrelated job if and only if $s^m > \bar{s}^m$. Major m specific skill growth in log wage point metric is calculated as

$$0 \;\; if\; s_1^m > s_2^m \tag{28}$$

$$\pi_{3nr}^m(s_2^m - s_1^m) \;\; if\; s_1^m \le s_2^m \le \bar{s}^m \tag{29}$$

$$\pi_{3,nr}^m(\bar{s}^m - s_1^m) + \pi_{3r}^m(s_2^m - \bar{s}^m) \;\; if\; s_1^m \le \bar{s}^m < s_2^m \tag{30}$$

$$\pi_{3r}^m(s_2^m - s_1^m) \;\; if\; \bar{s}^m < s_1^m \le s_2^m. \tag{31}$$

Equation (28) means that, if $s_1^m > s_2^m$, skill growth is calculated to be zero. Equation (29) indicates a situation in which college graduates would take an unrelated job even after graduating from college. Hence, the skill growth is calculated as log wage growth from major-specific skill growth in unrelated jobs. In equation (30), college graduates with pre-college level of major-specific skill would take an unrelated job, but, with post-college level of major-specific skill, they would take a related job. Hence, the first term indicates the log wage increase from skill growth up to $\bar{s}^m$ in an unrelated job, while the second term indicates the log wage increase because of skill growth from $\bar{s}^m$ to s_2^m in a related job. In equation (31), even college graduates with pre-college level major-specific skill would take a related job. Hence, the skill growth is calculated as log wage increase from major-specific skill growth in related jobs.

5 Estimation

I estimate my model via maximum likelihood. My model has five types of unobserved skills, general cognitive skill, mechanical skill, and three types of major-specific skills. The unobserved skills need to be integrated out in estimation. Post-secondary general cognitive skill $s_{2m_+}^c$, $m_+ = Some, H, E, S$, is unobserved, but can be rewritten in terms of s_1^c by using skill growth equation (11). The equations I estimate are written in terms of s_1^c in stead of $s_{2m_+}^c$.

For college major m, latent GPA equations (12) and (13) can be rewritten as

$$\begin{aligned} GPA_{1i}^{*m} &= (\gamma_{01}^m + \gamma_{11}^m \lambda_{0m}^c) + \gamma_{11}^m \lambda_{1m}^c s_{1i}^c + s_{2i}^m + x_{gi}' \beta_{g1}^m + e_{1i}^m \\ &= \tilde{\gamma}_{01}^m + \tilde{\gamma}_{11}^m s_{1i}^c + s_{2i}^m + x_{gi}' \beta_{g1}^m + e_{1i}^m \end{aligned} \tag{32}$$

$$\begin{aligned} GPA_{2i}^{*m} &= (\gamma_{02}^m + \gamma_{12}^m \lambda_{0m}^c) + \gamma_{12}^m \lambda_{1m}^c s_{1i}^c + \gamma_{22}^m s_{2i}^m + x_{gi}' \beta_{g2}^m + e_{2i}^m \\ &= \tilde{\gamma}_{02}^m + \tilde{\gamma}_{12}^m s_{1i}^c + \gamma_{22}^m s_{2i}^m + x_{gi}' \beta_{g2}^m + e_{2i}^m. \end{aligned} \tag{33}$$

Log wage equation (26) for some college is given by

$$\begin{aligned} \log w_{Some,i} &= (\tilde{\pi}_0 + \pi_1 \lambda_{0,Some}^c) + \pi_1 \lambda_{1,Some}^c s_{1i}^c + \pi_{21} \tau_i^c + \pi_{22} (\tau_i^c)^2 + x_{wi}' \beta_{Some} \\ &= \tilde{\pi}_{0,Some} + \pi_{1,Some} s_{1i}^c + \pi_{21} \tau_i^c + \pi_{22} (\tau_i^c)^2 + x_{wi}' \beta_{Some}, \end{aligned} \tag{34}$$

and, for major m, log wage equation (27) can be written as

$$\log w_{mRi} = (\tilde{\pi}_{0mR} + \pi_1 \lambda^c_{0m}) + \pi_1 \lambda^c_{1m} s^c_{1i} + \pi_{21}\tau^c_i + \pi_{22}(\tau^c_i)^2 + \pi^m_{3,R} s^m_{2i} + x'_{wi}\beta_m$$
$$= \dot{\pi}_{0mR} + \pi_{1m} s^c_{1i} + \pi_{21}\tau^c_i + \pi_{22}(\tau^c_i)^2 + \pi^m_{3,R} s^m_{2i} + x'_{wi}\beta_m. \quad (35)$$

The other rewritten equations, job type choice equation for some college and job relatedness choice equation, are described in the appendix.[30] Therefore, the unobserved skills that need to be integrated out are s^c_1, s^{mech}_1, s^H_2, s^E_2, and s^S_2.[31]

I estimate the model in three stages.[32] The first stage estimates the ASVAB equations, the education level choice equation, and the job type choice equations. Hence, the distributions of s^c_1 and s^{mech}_1 are estimated in this stage. Using the parameter estimates in the first stage, the second stage estimates the GPA equations and the college major choice equation. The distributions of s^H_2, s^E_2, and s^S_2 are estimated in this stage. Given the parameter estimates in the first and second stages, the general cognitive task intensity choice equations, job relatedness choice equation, and log wage equations are estimated in the third stage. This three-stages approach is less efficient than a one-stage approach. However, this approach not only makes computation easier but also makes the identification of the skills more transparent.

6 Results

The parameter estimates that are not shown below are reported in the appendix.[33]

ASVAB test score equations

Figure 5 shows the variance decomposition of ASVAB test scores. Around 70% of the variance of Word Knowledge scores, Paragraph Comprehension scores, Arithmetic Reasoning scores, and Math knowledge scores is explained by the variance of general cognitive skill, which is the sum of the variances of observed

[30] In the latent GPA equations, parameters that can be identified are $\tilde{\gamma}^m_{01}$, $\tilde{\gamma}^m_{11}$, $\tilde{\gamma}^m_{02}$, and $\tilde{\gamma}^m_{12}$. Hence, λ^c_{0m} and λ^c_{1m} cannot be identified from these equations. The latent GPA equations are not used to identify the skill growth parameters.

[31] I use Gauss-Hermeite quadrature to numerically evaluate the integral. The order of quadrature is 10.

[32] The appendix describes the estimated equations concretely.

[33] The brief summary of the job type choice and the job relatedness choice is the following: with regard to job type choice, those who have high pre-college general cognitive skill and who have low pre-college mechanical skill tend to be sorted into cognitive type jobs in both high school and some college graduates. This sorting is stronger among high school graduates, which might suggest that, even though some college graduates work in a mechanical job, some college mainly increases general cognitive skill and mechanical skill becomes less important to them compared to high school graduates.

and unobserved general cognitive components. Numerical Operation scores are noisier compared to the other cognitive test scores. Less than 40% of its variance can be explained by the variance of general cognitive skill.

With regard to the mechanical test scores, 20% of the variance of Mechanical Comprehension scores and of Electronics Information scores can be explained by the variance of general cognitive skill. The variance of mechanical skill, which is the sum of the observed and unobserved mechanical components, explains 20% to 30% of the variance of each type of test scores.

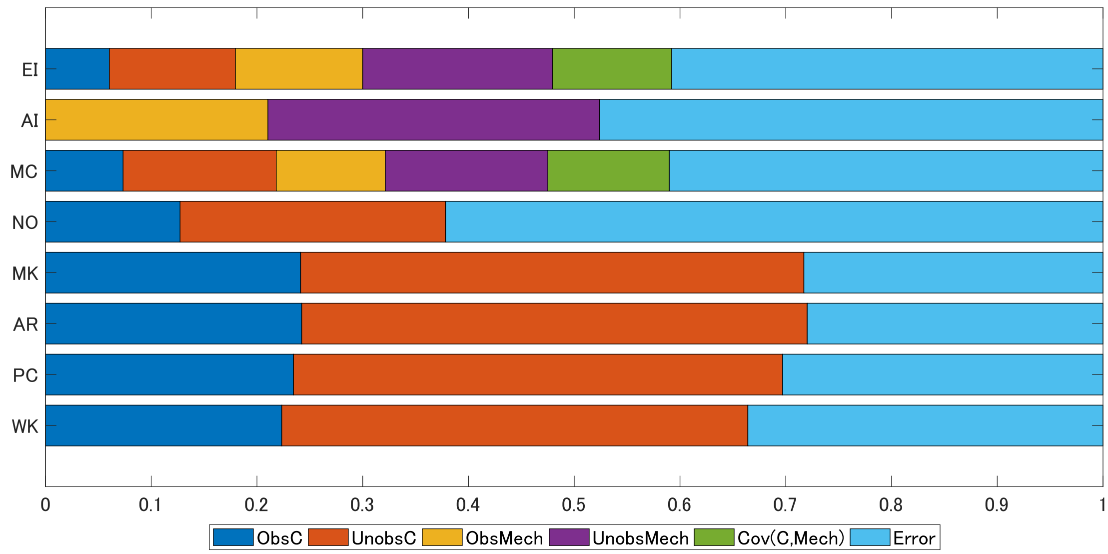

Figure 5: Variance decomposition of ASVAB test scores

Notes: Skill equations are $s^c_{1i} = x'_{si}\alpha^c + \theta^c_i$ and $s^{mech}_{1i} = x'_{si}\alpha^{mech} + \theta^{mech}_i$ and the ASVAB equations are equations (3) to (10). WK: Word Knowledge; PC: Paragraph Comprehension; AR: Arithmetic Reasoning; MK: Mathematics Knowledge; NO: Numerical Operation; MC: Mechanical Comprehension; AI: Auto & Shop Information; EI: Electronics Information. For $s = 1, 2, \cdots, 8$, let $\delta_{11} = 1$, $\delta_{17} = 0$, $\delta_{21} = \delta_{22} = \cdots = \delta_{25} = 0$, and $\delta_{27} = 1$. ObsC: $Var(\delta_{1s}x'_s\alpha^c)$; UnobsC: $Var(\delta_{1s}\theta^c)$; ObsMech: $Var(\delta_{2s}x'_s\alpha^{mech})$; UnobsMech: $Var(\delta_{2s}\theta^{mech})$; Cov(C,Mech): $Cov(\delta_{1s}s^c_1, \delta_{2s}s^{mech}_1)$; Error: $Var(e_s)$.

GPA equations

The last two years of college GPA are used as noisy measures of major-specific skills. Figure 6 shows the variance decomposition of the latent college GPA. The last year of latent GPA and the second last year of latent GPA show somewhat different results. The variance of measurement errors is relatively large in the second last year latent GPA of Humanities & Social Sciences and Business & Economics majors. In STEM majors, the two measures are similar.

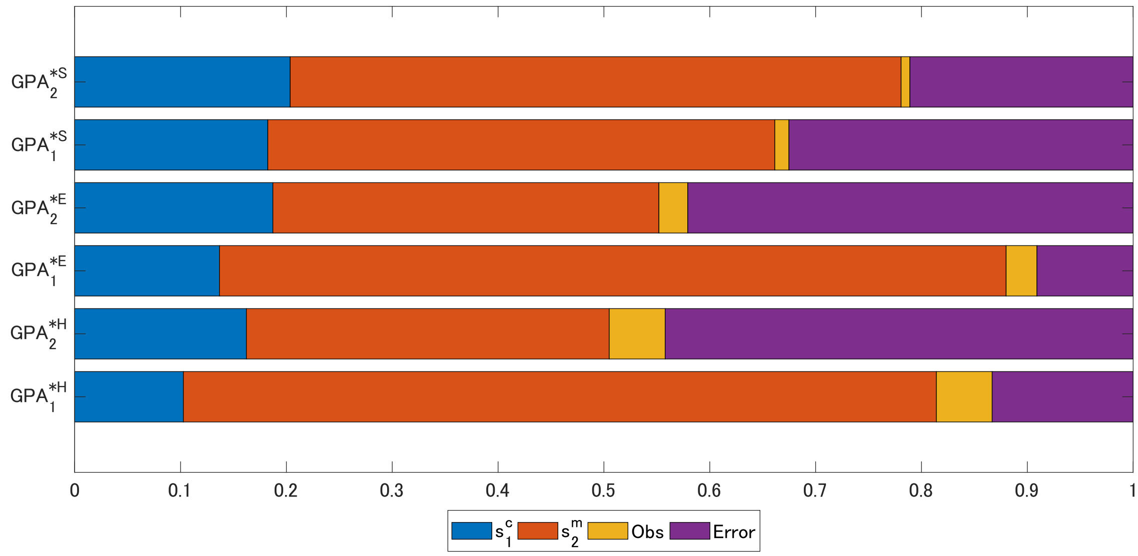

Figure 6: Variance decomposition of latent college GPA

Notes: The latent GPA equations are equations (32) and (33). For $t = 1, 2$, s_1^c: $Var(\gamma_{1t}^m s_1^c)$; s_2^m: $Var(\gamma_{2t}^m s_2^m)$, where $\gamma_{21}^m = 1$; Obs: $Var(x_g' \beta_{gt}^m)$, Error: $Var(e_t^m)$.

In any major, a large part of the variance of GPA is explained by the variance of major-specific skill. This implies that college students learn something that cannot be captured by general cognitive skill and that the skills are heavily weighed in grading. Students with higher pre-college general cognitive skill tend to earn higher GPA in any college major, but the impacts are smaller than those of major-specific skill.

Sorting into education level and major by skills

Figure 7 shows average pre-college skill levels conditional on education level. Each type of skill is standardized to have mean 0 and standard deviation 1 in this figure. Students who choose more education tend to have higher pre-college cognitive and mechanical skill levels. However, given pre-college general cognitive skill, those who have higher pre-college mechanical skill tend to choose high school. This is consistent with Prada and Urzúa (2017).

Figure 8 shows average pre-college general cognitive skill levels and average levels of potential post-college major-specific skills by college major. In this figure, each type of skill is standardized to have mean 0 and standard deviation 1 over the population. STEM majors tend to have higher pre-college general cognitive skill than the other two majors. With regard to potential post-college major-specific skills, students are positively sorted into Humanities & Social Sciences and STEM majors based on their respective major-specific skill. Especially, students choosing STEM majors tend to have much

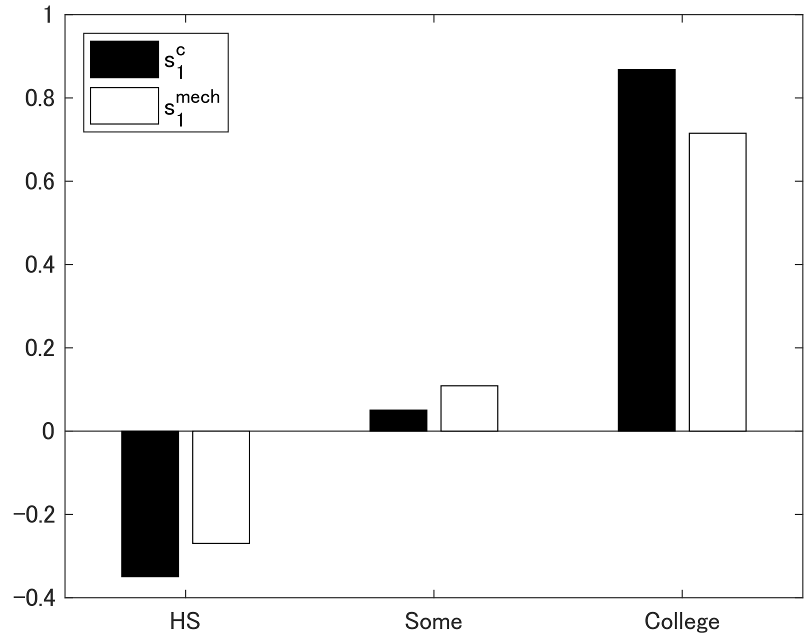

Figure 7: Sorting into education level by pre-college skills

Note: Each type of skill is standardized to have mean 0 and standard deviation 1 in this figure.

higher potential post-college STEM major specific skill than the average. In contrast, students are negatively sorted into Business & Economics majors. They tend to be below average in all types of potential post-college major-specific skills. This negative selection on major-specific skill into Business & Economics majors seems counter intuitive. However, a counterfactual analysis in Kinsler and Pavan (2015) shows that the average return to business major is smallest for those who choose business among college graduates. My result appears consistent with that.

Task intensity choice

Table 3 reports parameter estimates of the general cognitive task intensity choice equations, (23) for high school graduates and (24) for the other education groups. Constant terms represent the average level of cognitive task intensity taken by those with the population average level of pre-college general cognitive skill. The estimates of the constant are substantially different across the education groups. People with a higher level of education tend to take a job involving more intense general cognitive tasks. This suggests that a higher level of education will increase general cognitive skill more than a lower level of education. Also, the largest constant for STEM majors implies that general cognitive skill increases the most in STEM majors.

The estimate of ζ_1 is positive. This means that general cognitive skill has positive effects on general

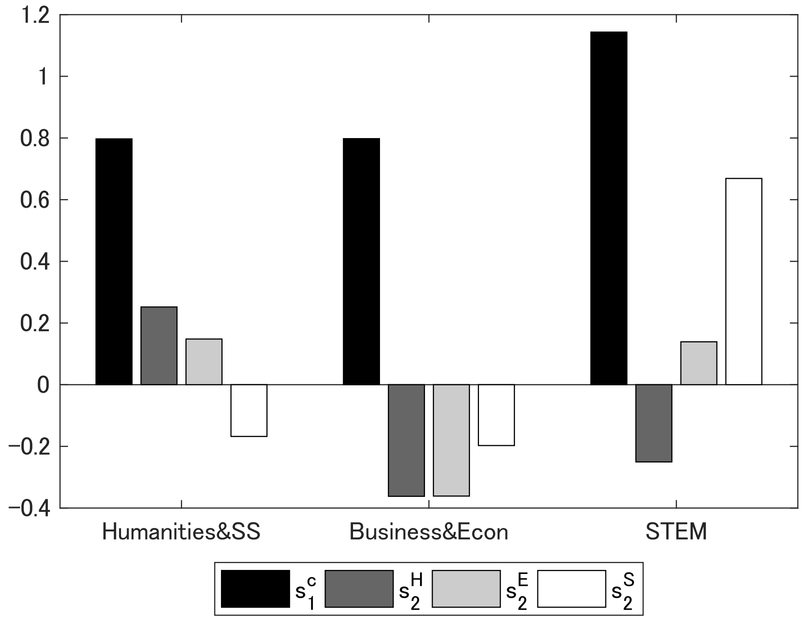

Figure 8: Sorting into major by skills

Note: Each type of skill is standardized to have mean 0 and standard deviation 1 in this figure.

cognitive task choice. In every education group, the sign of pre-college general cognitive skill is positive. This implies that those with higher pre-college general cognitive skill are more likely to end up in an occupation involving more intense cognitive tasks. There are some differences in size across the education groups and these differences reflect the differences in the effects of pre-college skill level on skill growth.

Table 3: Cognitive task intensity choice parameter estimates

	Const		s_1^c	
High school	(ζ_0)	-0.5865	(ζ_1)	0.1694
Some college	$(\tilde{\zeta}_{0Some})$	-0.1024	$(\tilde{\zeta}_{1Some})$	0.1664
Humanities&SS	$(\tilde{\zeta}_{0H})$	0.4507	$(\tilde{\zeta}_{1H})$	0.0085
Business&Econ	$(\tilde{\zeta}_{0E})$	0.7450	$(\tilde{\zeta}_{1E})$	0.1716
STEM	$(\tilde{\zeta}_{0S})$	1.1397	$(\tilde{\zeta}_{1S})$	0.2614

Notes: Equation (23) for high school and equation (24) for the other education groups. Other parameter estimates are omitted in this table.

Log wage equations

The parameter estimates of log wage equations, (26) for high school graduates, (34) for some college graduates, and (35) for college graduates, are reported in Table 4. Parameter π_1 is estimated to be positive, which means that general cognitive skill has a positive effect on log wages. The positive coefficients of task intensity terms show that wages are higher in occupations involving more intense cognitive tasks.

In all three majors, major-specific skills are unrewarded in jobs unrelated to their respective majors. In Humanities & Social Sciences majors, major-specific skill does not have a positive effect on wages even in jobs related to their majors. Skills that are specifically acquired in Humanities & Social Sciences majors do not appear to be rewarded in the labour market. In Business & Economics and STEM majors, major-specific skill has a positive effect on wages in related jobs.

Table 4: Log wage equation estimates

τ^c	(π_{21})	0.0991			
			Business&Econ		
$(\tau^c)^2$	(π_{22})	0.0388	s_1^c	(π_{1E})	0.0148
High school			Unrelated		
Const	$(\tilde{\pi}_0)$	1.9211	Const	$(\dot{\pi}_{0E,nr})$	2.3333
s_1^c	(π_1)	0.0396	s_2^E	$(\pi_{3,nr}^E)$	-0.0178
Some college			Related		
Const	$(\tilde{\pi}_{0Some})$	2.1844	Const	$(\dot{\pi}_{0E,r})$	2.2911
s_1^c	(π_{1Some})	0.0411	s_2^E	$(\pi_{3,r}^E)$	0.0369
Humanities&SS			STEM		
s_1^c	(π_{1H})	0.0025	s_1^c	(π_{1S})	0.0674
Unrelated			Unrelated		
Const	$(\dot{\pi}_{0H,nr})$	2.2768	Const	$(\dot{\pi}_{0S,nr})$	2.3343
s_2^H	$(\pi_{3,nr}^H)$	-0.0019	s_2^S	$(\pi_{3,nr}^S)$	-0.0186
Related			Related		
Const	$(\dot{\pi}_{0H,r})$	2.1494	Const	$(\dot{\pi}_{0S,r})$	2.2737
s_2^H	$(\pi_{3,r}^H)$	-0.0099	s_2^S	$(\pi_{3,r}^S)$	0.1093

Notes: Equation (26) for high school, equation (34) for some college, and equation (35) for major $m = H, E, S$.
Other parameter estimates are omitted in this table.

6.1 Skill growth

6.1.1 General cognitive skill growth

As explained in Section 5, the general cognitive task intensity choices are used to estimate the skill growth parameters in equation (25). Figure 9 shows general cognitive skill growth across pre-college skill levels. As can be expected, every major shows higher skill growth than some college. Among college majors, STEM majors show substantially larger skill growth than the other two majors regardless of pre-college skill levels. At the population average pre-college skill level, the difference between STEM majors and Humanities & Social Sciences majors is 16 log wage points. There are some differences in how much pre-college skill levels matter to skill growth. However, even though students with lower

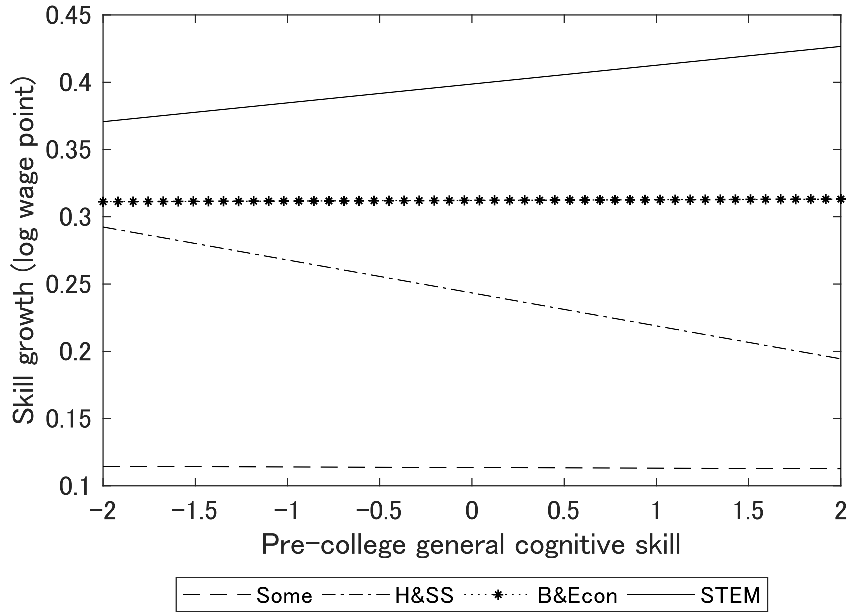

Figure 9: General cognitive skill growth by education group

Notes: Pre-college general cognitive skill in the x-axis is standardized to have mean 0 and standard deviation 1. General cognitive skill growth of individual i from education groups m_+, $m_+ = Some, H, E, S$, is given by $\pi_1(s^c_{2m_+,i} - s^c_{1i}) = \pi_1\lambda_{0m_+} + (\lambda^c_{1m_+} - 1)\pi_1 s^c_{1i}$.

Table 5: Average treatment effects; general cognitive skill growth

	Treatment			
	Some college	Humanities & SS	Business & Econ	STEM
High school	0.1138	0.2541	0.3119	0.3925
Some college	0.1136	0.2419	0.3122	0.3995
Humanities & SS	0.1132	0.2189	0.3126	0.4126
Business & Econ	0.1132	0.2189	0.3126	0.4126
STEM	0.1130	0.2083	0.3129	0.4187

pre-college skill levels have a weaker monetary incentive to choose STEM majors, majoring in STEM will still bring them higher growth in general cognitive skill. Table 5 shows that, for any education group of people, average general cognitive skill growth will be the highest in STEM majors.

Although I cannot identify this from my model, the observed strong sorting based on pre-college general cognitive skill might be due to study cost differences by college major. Students may be able to accumulate a larger amount of general cognitive skill in STEM majors. However, this might mean that students have to study much harder than in the other college majors. Despite the fact that STEM majors have higher pre-college cognitive test scores on average, they tend to spend more time in studying than other majors (see, e.g., Brint et al. (2012) and Ahn et al. (2018)). Keeping up in classes and with peers may be very difficult for those who do not have enough background or preparation.

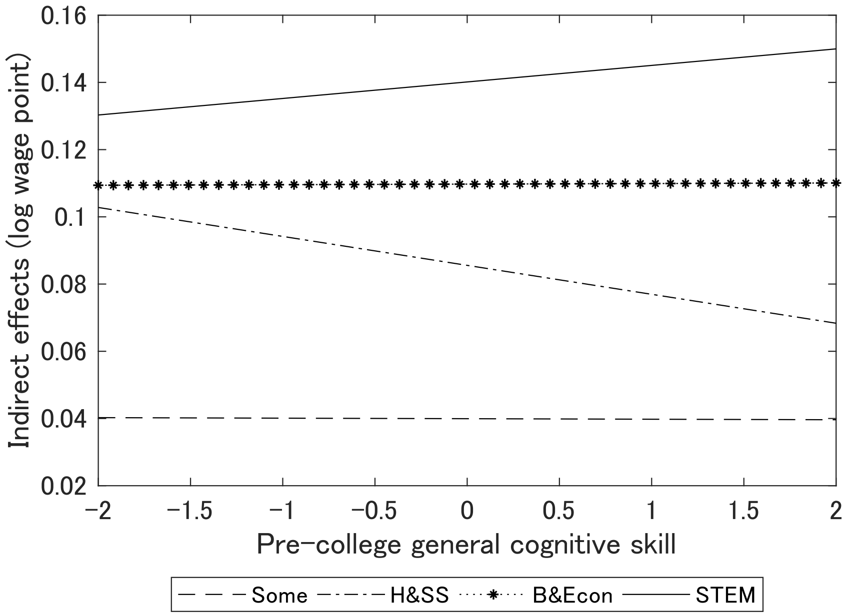

Figure 10: Indirect effects of general cognitive skill growth on wages through task intensity choice

Note: Pre-college general cognitive skill in the x-axis is standardized to have mean 0 and standard deviation 1. The effects for individual i from education groups m_+, $m_+ = Some$, H, E, S, is calculated by $\hat{\pi}_{1,\tau}(s^c_{2m_+,i} - s^c_{1i}) - \pi_1(s^c_{2m_+,i} - s^c_{1i})$, where $\hat{\pi}_{1\tau}$ is the estimate of π_1 in equation (26) excluding task intensity terms.

6.1.2 Occupation choice and wages

General cognitive skill growth in the log wage point metric indicates the direct contribution of general cognitive skill growth on log wages. There is also an indirect contribution of skill growth through occupation choice. As seen above, workers with higher general cognitive skill tend to choose a job involving more intense general cognitive tasks. Also, wages in jobs involving more intense general cognitive tasks tend to be higher.

In order to examine the size of the indirect effects, I estimate the log wage equations excluding task intensity terms. The parameter estimates are reported in the appendix. Now the estimate of π_1 in equation (26) without the task intensity terms, $\hat{\pi}_{1,-\tau}$, means the effects of general cognitive skill, including the indirect effects through task intensity choice. Since $\hat{\pi}_1$, which is the estimate of π_1 in equation (26) with the task intensity terms, only includes the direct effects of general cognitive skill, $(\hat{\pi}_{1,-\tau} - \hat{\pi}_1)(s^c_2 - s^c_1)$ measures the indirect effects of general cognitive skill. Figure 10 shows the indirect effects of general cognitive skill growth on log wages. The indirect effects are smaller than the direct effects. The direct effects are about three times larger. STEM majors show the largest indirect effects among the majors.

Table 6: Pre-college levels of major-specific skills

	Humanities&SS	Business&Econ	STEM
s_1^m	-6.94	-7.50	-5.02

Note: $s_1^m = \underline{GPA}_1^m - \hat{\gamma}_{01}^m$, where $\underline{GPA}_1^m$ corresponds to zero GPA in major m in the 0-4.0 scale and $\hat{\gamma}_{01}^m$ is the estimate of γ_{01}^m.

6.1.3 Major-specific skill growth

Using the estimate of $\pi_{3,r}^m$ and the estimated standard deviation of s_2^m, the log wage effect of one standard deviation increase in s_2^m can be calculated for each major m. Since the estimate of π_{3r}^H is almost zero, I set it to 0. Then, the effect is 0 for Humanities & Social Sciences majors, 0.0390 for Business & Economics majors, and 0.1292 for STEM majors. The log wage effect of a major-specific skill in related jobs is the largest for STEM majors. It is more than three times larger than for Business and Economics majors.

In symmetry with general cognitive skill, I have measures of post-college major-specific skill levels, but do not have a measure of pre-college skill levels. In the absence of such a measure, I approximate pre-college major-specific skills to levels, with which an average male high school graduate living in Northeast region who has the population average general cognitive skill would fail all courses taken in the last year of college.[34] Table 6 shows the calculated pre-college levels of each major-specific skill. The metric of s_1^m is the same as that of s_2^m. For each major m, potential post-college major-specific skill s_2^m is standardized to have mean 0 and standard deviation 1 over the population.

Table 7 reports major-specific skill growth measured in related jobs. It is evaluated at the population average level of potential post-college major-specific skill, that is, $s_2^m = 0$ for each major m. As a reference point, pre-college major-specific skill s_1^m is used in the first row, while the tenth percentile of post-college skill level among those who choose the major $s_{2m,10}^m$ is used in the second row. The skill growth is calculated at the population mean of s_2^m for each major m. Business & Economics and STEM majors show huge differences between the two cases. With regard to STEM majors, the major-specific skill growth is larger than that of general cognitive skill growth in the first case. On the other hand, it is much smaller in the second case and is even smaller than that of Business & Economics majors. This huge reduction is because, as seen in Figure 8, students who choose STEM majors tend to have high potential post-college STEM major specific skill and $s_{2S,10}^S$ is much larger than s_1^S.

[34] Using another groups of people, such as females, people with high pre-college skill, or different regions, does not change my results on the contribution of major-specific skill growth on wage growth because the calculated pre-college levels in any group are very small compared to post-college levels.

Table 7: Major-specific skill growth at average; related jobs

	Humanities&SS	Business&Econ	STEM
$\pi_{3r}^{m}(0 - s_1^m)$	0	0.2762	0.5488
$\pi_{3r}^{m}(0 - s_{2m,10}^m)$	0	0.0666	0.0295

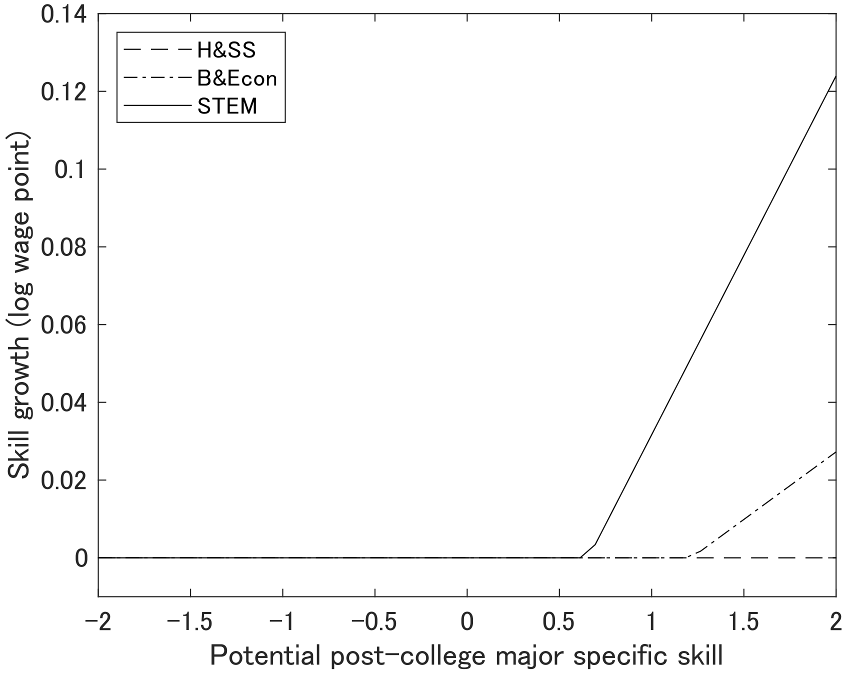

Figure 11: Major-specific skill growth across post-college major specific skill

Note: Potential post-college major specific skill in the x-axis is standardized to have mean 0 and standard deviation 1. See equations (28) to (31) for the definition of the skill growth.

I then calculate major-specific skill growth following equations (28) to (31). Although I call it major-specific skill growth, this growth represents the contribution of major-specific skill growth on log wage growth given individuals choose the higher-paying job relatedness. Hence, even if skill itself increases, that is, $s_2^m > s_1^m$, the calculated skill growth measure can be zero if s_2^m is not rewarded in the chosen job. Figure 11 shows major-specific skill growth across potential post-college major-specific skill levels. As shown in Table 4, the wage returns to major-specific skills are almost zero in unrelated jobs for all majors and in related jobs for Humanities & Social Sciences majors. Hence, I set them to 0 for the figure. In order to make it easy to compare the growth across majors, the x-axis is standardized to mean 0 and standard deviation 1. Since a large part of people would choose unrelated jobs, their major-specific skill are not utilized and the contribution on wage growth is zero among them. Therefore, the average major-specific skill growth is much smaller than general cognitive skill growth (see Table 8).

Table 8: Average treatment effects; major-specific skill growth

	Treatment		
	Humanities & SS	Business & Econ	STEM
Humanities & SS	0	0.0025	0.0091
Business & Econ	0	0.0008	0.0084
STEM	0	0.0024	0.0339

Major-specific skill growth has small positive effects on wages among Business & Economics majors. STEM majors show larger effects. If the skill level is 1.5 standard deviations above the population mean, then the growth in STEM major specific skill increases wages by about 10%. Although major-specific skill growth brings positive wage returns to some people, the returns are small relative to those from general cognitive skill growth. For example, even if STEM major specific skill is 1.5 standard deviations above the population average, growth in general cognitive skill contributes three to four times more than growth in major-specific skill. The skill growth estimates are robust with s_1^m because they do not change as long as $s_1^m \leq \bar{s}^m$ and $\bar{s}^m$ is more than 0.5 standard deviation above the average of s_2^m in any major m.

Since I take a different approach from Lemieux (2014), who takes a wage decomposition approach, my results cannot be directly compared to his results. Still, my estimates on major-specific skill growth are small compared to his. There are several reasons. Lemieux (2014) does not consider individual heterogeneity in major-specific skill growth, but people with a high level of major-specific skill may have a job related to their major. If this is true, major-specific skill growth is overestimated in his paper. Another reason is the coarse occupation classification in his paper. Related and unrelated jobs may be different in general cognitive task intensity even if they are both categorized in the same occupation in his paper. If unrelated jobs tend to involve less intensive general cognitive task, his estimated major-specific skill growth may partially reflect general cognitive skill growth and task intensity choice effects.[35]

Major-specific skill growth in log wage point metric depends on their estimated wage returns. Since skill price and skill quantity cannot be separately identified, I cannot conclude whether Humanities & Social Sciences majors learn something that is not rewarded in the labour market or that they learn little other than general cognitive skill. However, given that the variance decomposition of GPA is not much different across college majors, I suspect that Humanities & Social Sciences majors learn

[35] Other reasons are the different countries, Canada and US, and the different types of job relatedness measures, worker's self-assessed measure and job analysis measure.

something other than general cognitive skill as the other majors.[36] If students mainly develop general cognitive skill in Humanities & Social Sciences majors, a large part of their GPA variances should be explained by the variance of general cognitive skill.

7 Conclusion

Large income differences across college majors are attracting a lot of attention, and many studies examine whether there exist wage differentials across majors even when controlling for ability or skill selection into majors. College students may accumulate different types and amounts of skills by college major, and examining the similarity and the uniqueness of the accumulated skills in different majors will be helpful for understanding various important issues, including the sources of wage differentials across majors or students' major choice.

This paper estimates skill growth by college major in a multi-dimensional skill framework. I assume that each major increases two types of skills: general cognitive skill and a major-specific skill. The general cognitive skill can increase in any major. This skill captures the similarity of skills accumulated across majors. On the other hand, major-specific skill can be acquired only in its relevant major. This skill captures uniqueness of skills acquired in each major. I allow for individual heterogeneity in skill growth.

As in most datasets, my dataset has only cognitive test scores measured at a pre-college period, which can be used as measures of general cognitive skill. Hence, I use an approach to identify general cognitive skill growth by utilizing occupation choice. By assuming that high school graduates enter the labour market with a skill level measured by pre-college cognitive tests, the effects of general cognitive skill on occupation choice are estimated. Then, using these estimated effects, skill growth is implied by the differences in occupation choice of college graduates from high school graduates for each college major. Since high school graduates and college graduates tend to take different occupations and since college majors and post-college occupations are closely related, I use a task-based approach to relate occupations to each other. My dataset has measures of post-college major-specific skills, but does not have measures of pre-college major-specific skills. Hence, I specify pre-college major-specific skills with some assumptions.

[36]This type of skill, which is learned in college but is not be rewarded in the labour market, is called an academic skill in some papers such as Kinsler and Pavan (2015) and Arcidiacono et al. (2016).

I use US datasets, the NLSY97 and the O*NET. ASVAB scores and college GPA in the NLSY97 are used as pre-college general cognitive skill measures and post-college major-specific skill measures, respectively. Instead of assuming they accurately measure the skills, I assume that they are noisy measures and use a dynamic factor model to deal with the measurement errors.

The results show that all majors substantially increase general cognitive skill, but with large differences across majors. For students with a population average level of pre-college general cognitive skill, STEM majors increase general cognitive skill by 16 log wage points more than Humanities & Social Sciences majors. This comes from the fact that, given pre-college general cognitive skill levels, college graduates tend to work in jobs involving much more intense general cognitive tasks than high school graduates and that the difference in the task intensity from high school graduates varies by major. The effects of pre-college general cognitive skill levels on skill growth are somewhat different by major. Skill growth increases with pre-college skill levels in STEM majors, while it decreases in Humanities & Social Sciences. Still, the effects are not large and people who choose another major would increase their skill more in STEM majors. Indirect effects of general cognitive skill growth on wages through occupation choice are smaller than the direct effects. The direct effects are about three times larger than the indirect effects.

Wage returns to major-specific skills depend on whether the job is related to the corresponding major or not. The returns are zero in unrelated jobs for all majors, but they are positive in related jobs for Business & Economics and STEM majors. STEM majors show larger returns. However, the wage growth effects of major-specific skill growth are only about one quarter of those from general cognitive skill growth even for STEM majors whose potential post-college major-specific skill is 1.5 standard deviation above the population average.

These results suggest that growth in the general cognitive skill is the main contributor to the wage increase over high school graduation. The quantity of skill growth is substantially different across majors, and that is a big factor in the observed large differences in wages across majors. My results suggest that majoring in STEM will bring students a large monetary return in the labour market. I cannot identify from my paper, but its large skill increase might mean a large study cost in STEM majors. In fact, it is documented that STEM majors tend to study more hours than other majors despite their high pre-college cognitive test scores. Also, although many students switch out STEM majors to another major during college, only a small number of students switch into STEM majors from another major (see, e.g., Arcidiacono (2004) and Stinebrickner and Stinebrickner (2014)). This suggests that STEM majors are more difficult than other majors. Therefore, improving general cognitive skill and academic preparation in the pre-college stage may be an effective strategy to increase the number of STEM majors.

I show that college students accumulate a large amount of general cognitive skill, but what else they learn during college remains unclear. As shown above, the growth in major-specific skill does not contribute to wages for Humanities & Social Sciences majors although the acquired major-specific skill is an important factor of GPA. Why this is so remains an interesting question for future research. My paper examines only skill growth during college, but on-the-job skill accumulation might be different by major as well. This is another interesting topic for the future.

References

Aaronson, D. and B. Mazumder (2011). The Impact of Rosenwald Schools on Black Achievement. *Journal of Political Economy 119*(5), 821–888.

Acemoglu, D. and D. Autor (2011). *Skills, Tasks and Technologies: Implications for Employment and Earnings*, Volume 4. Elsevier B.V.

Ahn, T., P. Arcidiacono, A. Hopson, and J. Thomas (2018). Grade Inflation in General Equilibrium with Implications for Female Interest in STEM Majors. *mimeo*.

Altonji, J. G., P. Bharadwaj, and F. Lange (2012). Changes in the Characteristics of American Youth: Implications for Adult Outcomes. *Journal of Labor Economics 30*(4), 783–828.

Altonji, J. G., E. Blom, and C. Meghir (2012). Heterogeneity in Human Capital Investments: High School Curriculum, College Major, and Careers. *Annual Review of Economics 4*(1), 185–223.

Arcidiacono, P. (2004). Ability Sorting and the Returns to College Major. *Journal of Econometrics 121*(1-2), 343–375.

Arcidiacono, P., E. Aucejo, A. Maurel, and T. Ransom (2016). College Attrition and the Dynamics of Information Revelation. *NBER Working Paper*.

Arcidiacono, P., E. M. Aucejo, and V. J. Hotz (2016). University Differences in the Graduation of Minorities in STEM Fields: Evidence from California. *American Economic Review 106*(3), 525–562.

Arcidiacono, P., J. Hotz, A. Maurel, and T. Romano (2017). Ex Ante Returns and Occupational Choice. *mimeo*.

ASVAB Career Exploration Program (2011). The ASVAB Career Exploration Program: Theoretical and Technical Underpinnings of the Revised Skill Composites and OCCU-Find. Technical report.

Aucejo, E. M. and J. James (2016). The Path to College Education : Are Verbal Skills More Important than Math Skills? *mimeo*.

Brint, S., A. M. Cantwell, and P. Saxena (2012). Disciplinary Categories, Majors, and Undergraduate Academic Experiences: Rethinking Bok's "Underachieving Colleges" Thesis. *Research in Higher Education 53*(1), 1–25.

Carneiro, P., K. Hansen, and J. Heckman (2003). Estimating Distribution of Counterfactuals with an Application to the Returns to Schooling and Measurement of the Effect of Uncertainty on Schooling Choice. *International Economic Review 44*(2), 361–422.

Carnevale, A. P., B. Cheah, and A. R. Hanson (2015). The Economic Value of College Majors. *Gerogetown University*.

Cascio, E. U. and E. G. Lewis (2006). Schooling and the Armed Forces Qualifying Test Evidence from School-Entry Laws. *Journal of Human Resources 41*(2), 294–318.

College Board (2015). Total Group Profile Report. Technical report.

Fitzpatrick, M. D., D. Grissmer, and S. Hastedt (2011). What a Difference a Day Makes: Estimating Daily Learning Gains during Kindergarten and First Grade Using a Natural Experiment. *Economics of Education Review 30*(2), 269–279.

Freeman, J. A. and B. T. Hirsch (2008). College Majors and the Knowledge Content of Jobs. *Economics of Education Review 27*(5), 517–535.

Fruehwirth, J. C., S. Navarro, and Y. Takahashi (2016). How the Timing of Grade Retention Affects Outcomes : Identification and Estimation of Time-Varying Treatment Effects. *Jornal of Labor Economics 34*(4), 979–1021.

Gormley, W. T. and T. Gayer (2005). Promoting School Readiness in Oklahoma: An Evaluation of Tulsa's Pre-K Program. *Journal of Human Resources 40*(3), 533–558.

Guvenen, F., B. Kuruscu, S. Tanaka, and D. Wiczer (2016). Multidimensional Skill Mismatch. *mimeo*.

Hamermesh, D. S. and S. G. Donald (2008). The Effect of College Curriculum on Earnings: An Affinity Identifier for Non-Ignorable Non-Response Bias. *Journal of Econometrics 144*(2), 479–491.

Hansen, K. T., J. J. Heckman, and K. J. Mullen (2004). The Effect of Schooling and Ability on Achievement Test Scores. *Journal of Econometrics 121*(1-2), 39–98.

Heckman, J., J. E. Humphries, and G. Veramendi (2016a). Returns to Education: The Causal Effects of Education on Earnings, Health and Smoking. *NBER Working Paper*.

Heckman, J. J., J. E. Humphries, and G. Veramendi (2016b). Dynamic Treatment Effects. *Journal of Econometrics 191*(2), 276–292.

Kinsler, J. and R. Pavan (2015). The Specificity of General Human Capital: Evidence from College Major Choice. *Journal of Labor Economics 33*(4), 933–972.

Kirkeboen, L. J., E. Leuven, and M. Mogstad (2016). Field of Study, Earnings, and Self-Selection. *Quarterly Journal of Economics 131*(3), 1057–1111.

Lemieux, T. (2014). Occupations, Fields of Study and Returns to Education. *Canadian Journal of Economics 47*(4), 1047–1077.

Poletaev, M. and C. Robinson (2008). Human Capital Specificity: Evidence from the Dictionary of Occupational Titles and Displaced Worker Surveys, 1984–2000. *Journal of Labor Economics 26*(3), 387–420.

Prada, M. F. and S. S. Urzúa (2017). One Size Does Not Fit All: Multiple Dimensions of Ability, College Attendance and Wages. *Journal of Labor Economics 55*(4), 953–991.

Ransom, M. R. (2014). The Changing Occupational Distribution by College Major. *mimeo*.

Speer, J. D. (2017). Pre-Market Skills, Occupational Choice, and Career Progression. *Journal of Human Resources 52*(1), 187–246.

Stinebrickner, R. and T. R. Stinebrickner (2014). A Major in Science? Initial Beliefs and Final Outcomes for College Major and Dropout. *Review of Economic Studies 81*, 426–472.

Yamaguchi, S. (2012). Tasks and Heterogeneous Human Capital. *Journal of Labor Economics 30*(1), 1–53.

Appendix

Identification of measurement system of general cognitive skill

I specify $\theta^{mech} = a\theta^c + \theta^{omech}$ and assume θ^c and θ^{mech} are orthogonal to each other. Using this specification and skill equations (1) and (2), the skill measurement system, equations (3) to (10), can be rewritten as follows:

$$
\begin{aligned}
WordKnowledge_i &= x'_{si}\alpha^c + \theta_i^c + e_{1i} \\
ParagraphComprehension_i &= \delta_{12}(x'_{si}\alpha^c + \theta_i^c) + e_{2i} \\
ArithmeticReasoning_i &= \delta_{13}(x'_{si}\alpha^c + \theta_i^c) + e_{3i} \\
MathematicsKnowledge_i &= \delta_{14}(x'_{si}\alpha^c + \theta_i^c) + e_{4i} \\
NumericalOperation_i &= \delta_{15}(x'_{si}\alpha^c + \theta_i^c) + e_{5i} \\
MechanicalComprehension_i &= \delta_{16}(x'_{si}\alpha^c + \theta_i^c) + \delta_{26}(x'_{si}\alpha^{mech} + a\theta_i^c + \theta_i^{omech}) + e_{6i} \\
Auto\&ShopInformation_i &= x'_{si}\alpha^{mech} + a\theta_i^c + \theta_i^{omech} + e_{7i} \\
ElectronicsInformation_i &= \delta_{18}(x'_{si}\alpha^c + \theta_i^c) + \delta_{28}(x'_{si}\alpha^{mech} + a\theta_i^c + \theta_i^{omech}) + e_{8i}.
\end{aligned}
$$

Since x_s is orthogonal with θ^c, θ^{omech}, and e's, the parameters on x_s, that is, α^c, $\delta_{1s}\alpha^c$, α^{mech}, and $\delta_{2s}\alpha^{mech}$, $s = 1, 2, \cdots, 8$, can be identified. Moving the identified terms into the left hand side gives

$$\widetilde{WordKnowledge}_i = \theta_i^c + e_{1i} \tag{36}$$

$$
\begin{aligned}
\widetilde{ParagraphComprehension}_i &= \delta_{12}\theta_i^c + e_{2i} \\
\widetilde{ArithmeticReasoning}_i &= \delta_{13}\theta_i^c + e_{3i} \\
\widetilde{MathematicsKnowledge}_i &= \delta_{14}\theta_i^c + e_{4i} \\
\widetilde{NumericalOperation}_i &= \delta_{15}\theta_i^c + e_{5i} \\
\widetilde{MechanicalComprehension}_i &= (\delta_{16} + a\delta_{26})\theta_i^c + \delta_{26}\theta_i^{omech} + e_{6i}
\end{aligned}
$$

$$\widetilde{Auto\&ShopInformation}_i = a\theta_i^c + \theta_i^{omech} + e_{7i} \tag{37}$$

$$\widetilde{ElectronicsInformation}_i = (\delta_{18} + a\delta_{28})\theta_i^c + \delta_{28}\theta_i^{omech} + e_{8i}.$$

This is a triangular form explained in Carneiro et al. (2003). There are five measures affected only by θ^c and three measures affected by θ^c and θ^{omech}, and the factor loadings on θ^c in equation (36) and on θ^{omech} in equation (37) are normalized to 1. Therefore, the factor loadings on θ^c and on θ^{omech} and the distributions of θ^c, θ^{omech}, and e's can be identified. Since a, δ_{26}, δ_{28}, $\delta_{16} + a\delta_{26}$, and $\delta_{18} + a\delta_{28}$ are identified, δ_{16} and δ_{18} can be identified.

Derivation of linear task intensity choice

Individual i's problem in period 2 is

$$\max_{\tau} u(\tau^c, s_i^c, \varepsilon_{hi}^c),$$

where ε_h^c is a working cost shock. Utility in period 2 is modeled as

$$u(\tau_i^c, s_i^c, \varepsilon_{hi}^c) = \log w(\tau_i^c, s_i^c) - h(\tau_i^c, s_i^c, \varepsilon_{hi}^c),$$

where $h(\cdot)$ is a working cost function. For simplicity, suppose that log wage equation is

$$\log w_i = \pi_0 + \pi_1 s_{i2}^c + \pi_{21}\tau_i^c + \pi_{22}(\tau_i^c)^2.$$

Suppose working cost is

$$h(\tau_i^c, s_i^c, \varepsilon_{hi}) = h_1 s_{i2}^c + h_2\tau_i^c + h_3(\tau_i^c)^2 + h_4(s_i^c\tau_i^c) + \tau_i^c\varepsilon_{hi}^c$$

The FOC for τ^c is

$$\pi_{21} + 2\pi_{22}\tau_i^c = h_2 + 2h_3\tau_i^c + h_4 s_i^c + \varepsilon_{hi}^c.$$

Hence, the optimal math task intensity given s_2^c is

$$\tau_i^* = \frac{\pi_{21} - h_2 - h_4 s_i^c - \varepsilon_{hi}^c}{2(h_3 - \pi_{22})}.$$

which is linear in s^c.

Equations rewritten using the assumptions on pre-college major-specific skills

Job type choice equation (18) is given by

$$\begin{aligned} I_{m_-,i}^c &= \left(\xi_0^c + \sum_{m' \in \{H,E,S\}} \xi_{3m'}^c s_1^{m'}\right) + \xi_1^c s_i^c + \xi_2^c s_i^{mech} + x_{ci}' \beta_{m_-}^c + \varepsilon_{m_-,i}^c \\ &= \tilde{\xi}_0^c + \xi_1^c s_i^c + \xi_2^c s_i^{mech} + x_{ci}' \beta_{m_-}^c + \varepsilon_{m_-,i}^c. \end{aligned} \tag{38}$$

Education level choice equation (21) is rewritten as

$$\begin{aligned} I_{li} &= \left(\eta_{0l} + \sum_{m' \in \{H,E,S\}} \eta_{3,l,m'} s_1^{m'}\right) + \eta_{1l} s_{1i}^c + \eta_{2l} s_{1i}^{mech} + x_{di}' \beta_l + z_{dli} \varphi_{dl} + \varepsilon_{dli} \\ &= \tilde{\eta}_{0l} + \eta_{1l} s_{1i}^c + \eta_{2l} s_{1i}^{mech} + x_{di}' \beta_l + z_{dli} \varphi_{dl} + \varepsilon_{dli}. \end{aligned} \tag{39}$$

College major choice equation (22) is rewritten as

$$\begin{aligned} I_{mi} &= (\eta_{0m} + \eta_{3m} s_1^m) + \eta_{1m} s_{1i}^c + \eta_{2m} s_{2i}^m + x_{di}' \beta_m + z_{mi}' \varphi_m + \varepsilon_{mi} \\ &= \tilde{\eta}_{0m} + \eta_{1m} s_{1i}^c + \eta_{2m} s_{2i}^m + x_{di}' \beta_m + z_{mi}' \varphi_m + \varepsilon_{mi}. \end{aligned} \tag{40}$$

Equations rewritten in terms of pre-college general cognitive skill

I suppose $s_{2,Some,i}^{mech} = \lambda_{0,Some}^{mech} + \lambda_{1Some}^{mech} s_{1i}^{mech}$ similar to general cognitive skill change. The job type choice equation (38) for some college is rewritten as

$$\begin{aligned} I_{Some,i}^c &= (\tilde{\xi}_0^c + \xi_1^c \lambda_{0,Some}^c + \xi_2^c \lambda_{0,Some}^{mech}) + \xi_1^c \lambda_{1,Some}^c s_{1i}^c + \xi_2^c \lambda_{1,Some}^{mech} s_{1i}^{mech} + x_{ci}' \beta_{Some}^c + \varepsilon_{Some,i}^c. \\ &= \tilde{\xi}_{0,Some}^c + \tilde{\xi}_1^c s_{1i}^c + \bar{\xi}_2^{mech} s_{1i}^{mech} + x_{ci}' \beta_{Some}^c + \varepsilon_{Some,i}^c. \end{aligned} \tag{41}$$

Job relatedness choice equation (20) can be rewritten as

$$\begin{aligned} I_{rmi} &= (\xi_{0rm} + \xi_{1rm} \lambda_{0m}^c) + \xi_{1rm} \lambda_{1m}^c s_{1i}^c + \xi_{2rm} s_{2i}^m + x_{wi}' \beta_{rm} + \varepsilon_{rmi} \\ &= \tilde{\xi}_{0rm} + \tilde{\xi}_{1rm} s_{1i}^c + \xi_{2rm} s_{2i}^m + x_{wi}' \beta_{rm} + \varepsilon_{rmi}. \end{aligned} \tag{42}$$

Estimated equations

The first stage estimates the ASVAB equations (3) to (10), the education level choice equation (39), and the job type choice, equation (38) for high school and equation (41) for some college. The second stage estimates the GPA equations (32), (33), (14) and (15) and the college major choice (40). In the third stage, the general cognitive task intensity choice, equation (23) for high school and equation (24) for the other education groups, job relatedness choice (42), and log wage equations, equation (26) for high school, equation (34) for some college, and equation (35) for major $m = H, E, S$, are estimated.

Parameter estimates

Table 1: Skill equation parameter estimates

Variables	Cognitive (α_c)		Mechanical (α_{mech})	
Constant	-0.8231	(0.0542)	-0.1389	(0.0526)
Female	0.0944	(0.0262)	-0.5163	(0.0232)
Hispanic/Mixed race	0.3493	(0.0412)	0.2782	(0.0321)
Non-Black/Non-Hispanic	0.5917	(0.0329)	0.6341	(0.0294)
North central 1997	-0.0553	(0.0400)	0.1063	(0.0367)
South 1997	-0.1203	(0.0375)	0.0003	(0.0310)
West 1997	-0.1864	(0.0469)	0.0010	(0.0378)
Urban 1997	0.0464	(0.0316)	-0.1983	(0.0264)
Broken home 1997	-0.0839	(0.0264)	-0.0185	(0.0251)
Father's education	0.0869	(0.0148)	0.0149	(0.0114)
Mother's education	0.1229	(0.0170)	0.0394	(0.0133)
Household income (Quartile groups)	0.1091	(0.0116)	0.0447	(0.0109)
Number of siblings	-0.0360	(0.0071)	-0.0302	(0.0068)

Notes: General cognitive skill: $s^c_{1i} = x'_{si}\alpha^c + \theta^{mech}_i$ and mechanical skill: $s^{mech}_{1i} = x'_{si}\alpha^{mech} + \theta^{mech}_i$, where x_s is a vector of observed variables.
Standard errors are in parentheses.
Standard errors are calculated by bootstrap. The number of replication is 200.

Table 2: ASVAB test score equations estimates

	s_1^c		s_1^c		Error	
WK		1		0	$\log(Var(e_1))$	-1.1338
						(0.0247)
PC	(δ_{12})	1.0233		0	$\log(Var(e_2))$	-1.2388
		(0.0138)				(0.0285)
AR	(δ_{13})	1.0390		0	$\log(Var(e_3))$	-1.3212
		(0.0155)				(0.0295)
MK	(δ_{14})	1.0373		0	$\log(Var(e_4))$	-1.3090
		(0.0160)				(0.0269)
NO	(δ_{15})	0.7615		0	$\log(Var(e_5))$	-0.5003
		(0.0178)				(0.0230)
MC	(δ_{16})	0.5335	(δ_{26})	0.6397	$\log(Var(e_6))$	-1.0773
		(0.0218)		(0.0287)		(0.0285)
AI		1		1	$\log(Var(e_7))$	-0.7482
						(0.0319)
EI	(δ_{18})	0.4850	(δ_{28})	0.6929	$\log(Var(e_8))$	-1.0787
		(0.0209)		(0.0281)		(0.0294)
	$\log(Var(\theta^c))$	-0.8618	$\log(Var(\theta^{omech}))$	-1.5863	$Corr(\theta^c, \theta^{mech})$ (a)	0.5038
		(0.0278)		(0.0720)		(0.0179)

Notes: Equations (3) to (10).

Standard errors are in parentheses.

Standard errors are calculated by bootstrap. The number of replication is 200.

Table 3: Parameter estimates of education level choice equation

	High school		Some college		College	
Const	($\tilde{\eta}_{0HS}$)	0	($\tilde{\eta}_{0Some}$)	-1.0765	($\tilde{\eta}_{0College}$)	-0.9228
				(0.3237)		(0.4128)
s_1^c	(η_{1HS})	0	(η_{1Some})	0.7823	($\eta_{1College}$)	2.3326
				(0.1020)		(0.1509)
s_1^{mech}	(η_{2HS})	0	(η_{2Some})	-0.0791	($\eta_{2College}$)	-0.9353
				(0.1389)		(0.1574)
LUR	($\varphi_{LUR,HS}$)	-0.0077	($\varphi_{LUR,Some}$)	-0.0398	($\varphi_{LUR,College}$)	-0.4577
		(0.0316)		(0.0514)		(0.1523)

Notes: Base group is high school. LUR: Local unemployment rates. Equation (39).

Other parameter estimates are omitted in this table.

Standard errors are in parentheses.

Standard errors are calculated by bootstrap. The number of replication is 200.

Table 4: College major choice equation estimates

	Humanities&SS		Business&Econ		STEM	
Parameters						
Const	$(\tilde{\eta}_{0H})$	0	$(\tilde{\eta}_{0E})$	-1.5149	$(\tilde{\eta}_{0S})$	-2.2176
s_1^c	(η_{1H})	0	(η_{1E})	-0.0928	(η_{1S})	1.0917
s_2^m	(η_{2H})	0.8398	(η_{2E})	-0.6813	(η_{2S})	1.9520
Math relative score	$(\varphi_{M,H})$	0	$(\varphi_{M,E})$	2.1878	$(\varphi_{M,S})$	2.6633
Mechanical relative score	$(\varphi_{M,echH})$	0	$(\varphi_{Mech,H})$	0.2210	$(\varphi_{Mech,H})$	0.7569
Marginal effects at means*						
s_1^c		-0.1434		-0.0164		0.1598
s_2^m		0.1512		-0.0376		0.2841

Notes: Base group is Humanities&SS. Base group is Humanities&SS. Equation (40).

*θ^c is set to one standard deviation above 0 and θ^m is set to 0.

Other parameter estimates are omitted in this table.

Table 5: Job type choice

	High school		Some college	
s_1^c	(ξ_1^c)	0.8991	$(\tilde{\xi}_1^c)$	0.6876
		(0.1026)		(0.2363)
s_1^{mech}	(ξ_2^c)	-0.8371	$(\tilde{\xi}_2^c)$	-0.6065
		(0.1542)		(0.2588)
Const	$(\tilde{\xi}_0^c)$	0.6258	$(\tilde{\xi}_{0,Some}^c)$	0.8658
		(0.2728)		(0.4219)

Notes: Dependent variable = 0: Mechanical job; =1: Cognitive job.

Equation (38) for high school and equation (41) for some college.

Other parameter estimates are omitted in this table.

Standard errors are in parentheses.

Standard errors are calculated by bootstrap. The number of replication is 200.

Table 6: Parameter estimates: latent GPA equations

		Humanities&SS		Business&Econ		STEM	
		$\log Var(s_2^H)$	-0.0366	$\log Var(s_2^E)$	-0.1144	$\log Var(s_2^S)$	-0.3363
GPA_1^{*m}	Const	$(\tilde{\gamma}_{01}^H)$	-0.8868	$(\tilde{\gamma}_{01}^E)$	0.0272	$(\tilde{\gamma}_{01}^S)$	-1.4647
	s_1^c	$(\tilde{\gamma}_{11}^H)$	0.4675	$(\tilde{\gamma}_{11}^E)$	0.5077	$(\tilde{\gamma}_{11}^S)$	0.6538
	s_2^m		1		1		1
	Error	$(\log Var(e_1^H))$	-1.7119	$(\log Var(e_1^E))$	-2.2157	$(\log Var(e_1^S))$	-0.7235
GPA_2^{*m}	Const	$(\tilde{\gamma}_{02}^H)$	-0.9830	$(\tilde{\gamma}_{02}^E)$	-0.2183	$(\tilde{\gamma}_{02}^S)$	-1.2845
	s_1^c	$(\tilde{\gamma}_{12}^H)$	0.5599	$(\tilde{\gamma}_{12}^E)$	0.5842	$(\tilde{\gamma}_{12}^S)$	0.6864
	s_2^m	(γ_{22}^H)	0.6613	(γ_{22}^E)	0.6888	(γ_{22}^S)	1.0911
	Error	$(\log Var(e_2^H))$	-0.6090	$(\log Var(e_2^E))$	-0.7170	$(\log Var(e_2^S))$	-1.1686

Notes: Equations (32) and (33).

Other parameter estimates are omitted in this table.

Table 7: Job relatedness choice

	Humanities&SS		Business&Econ		STEM	
s_1^c	$(\tilde{\xi}_{1rH})$	-0.0950	$(\tilde{\xi}_{1rE})$	0.1419	$(\tilde{\xi}_{1rS})$	0.1402
s_2^m	(ξ_{2rH})	0.3108	(ξ_{2rE})	0.3588	(ξ_{2rS})	0.1678
Const	$(\tilde{\xi}_{0rH})$	-0.1280	$(\tilde{\xi}_{0rH})$	0.9856	$(\tilde{\xi}_{0rH})$	-2.1851

Notes: Dependent variable = 0: Unrelated job; =1: Related job.

Equation (42).

Other parameter estimates are omitted in this table.

Table 8: Cognitive skill growth parameter estimates; log wage point metric

Some college			Business&Economics		
Const	$(\pi_1\lambda^c_{0Some})$	0.1136	Const	$(\pi_1\lambda^c_{0E})$	0.3122
$\pi_1 s^c_1$	(λ^c_{1Some})	0.9827	$\pi_1 s^c_1$	(λ^c_{1E})	1.0188
Humanities&Social Sciences			**STEM**		
Const	$(\pi_1\lambda^c_{0H})$	0.2434	Const	$(\pi_1\lambda^c_{0S})$	0.3986
$\pi_1 s^c_1$	(λ^c_{1H})	0.0497	$\pi_1 s^c_1$	(λ^c_{1S})	1.5426

Notes: For $m_+ = Some, H, E, S$, post-college general cognitive skill in the log wage metric is $\pi_1 s_{2m_+,i} = \pi_1\lambda^c_{0m_+} + \lambda^c_{1m_+}\pi_1 s^c_{1i}$.

Table 9: Log wage equation estimates excluding task intensity

τ^c	(π_{21})	0			
			Business & Econ		
$(\tau^c)^2$	(π_{22})	0	s^c_1	(π_{1E})	0.0463
High school			Unrelated		
Const	$(\tilde{\pi}_0)$	1.8819	Const	$(\dot{\pi}_{0E,nr})$	2.4369
s^c_1	(π_1)	0.0536	s^E_2	$(\pi^E_{3,nr})$	0.0101
Some college			Related		
Const	$(\dot{\pi}_{0Some})$	2.1943	Const	$(\dot{\pi}_{0E,r})$	2.4957
s^c_1	(π_{1Some})	0.0608	s^E_2	$(\pi^E_{3,r})$	0.0440
Humanities & SS			STEM		
s^c_1	(π_{1H})	0.0059	s^c_1	(π_{1S})	0.1030
Unrelated			Unrelated		
Const	$(\dot{\pi}_{0H,nr})$	2.3602	Const	$(\dot{\pi}_{0S,nr})$	2.4481
s^H_2	$(\pi^H_{3,nr})$	-0.0036	s^S_2	$(\pi^S_{3,nr})$	-0.0047
Related			Related		
Const	$(\dot{\pi}_{0H,r})$	2.2911	Const	$(\dot{\pi}_{0S,r})$	2.5723
s^H_2	$(\pi^H_{3,r})$	0.0003	s^S_2	$(\pi^S_{3,r})$	0.1076

Notes: Equation (26) for high school, equation (34) for some college, and equation (35) for major m, $m = H, E, S$. Other parameter estimates are omitted in this table.